Chapter 3: Dielectric Waveguides and Optical Fibers

Symmetric planar dielectric slab waveguides

Modal and waveguide dispersion in planar waveguides

Step index fibers

Numerical aperture

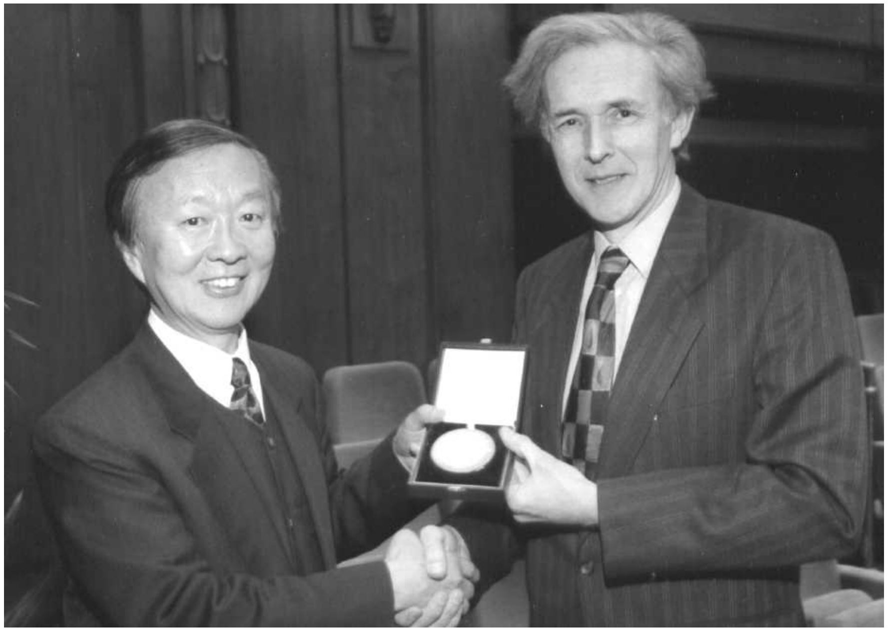
Dispersion in single mode fibers

Bit-rate, dispersion, and optical bandwidth

Graded index optical fibers

Light absorption and scattering

Attenuation in optical fibers



Professor Charles Kao, who has been recognized as the inventor of fiber optics, left, receiving an IEE prize from Professor John Midwinter (1998 at IEE Savoy Place, London, UK; courtesy of IEE).

Professor Charles Kao served as Vice Chancellor (President) of the Chinese University of Hong Kong from 1987 to 1996.

"The introduction of optical fiber systems will revolutionize the communications network. The low-transmission loss and the large bandwidth capability of the fiber system allow signals to be transmitted for establishing communications contacts over large distances with few or no provisions of intermediate amplification." — **Charles K. Kao**

Prof. Charles Kuen Kao (高錕)

Dr. Charles Kuen Kao is a pioneer in the use of fiber optics in telecommunications. He is recognized internationally as the "Father of Fiber Optic Communications".

He was born in Shanghai on November 4, 1933, and was awarded BSc in 1957 and PhD in 1965, both in electrical engineering, from the University of London.

He joined ITT in 1957 as an engineer at Standard Telephones and Cables Ltd., an ITT subsidiary in the United Kingdom.

In 1960, he joined Standard Telecommunications Laboratories Ltd., UK, ITT's central research facility in Europe. It was during this period that Dr. Kao made his pioneering contributions to the field of optical fibers for communications. After a four years' leave of absence spent at The Chinese University of Hong Kong, Kao returned to ITT in 1974 when the field of optical fibers was ready for the pre-product phase.

In 1982, in recognition of his outstanding research and management abilities, ITT named him the first ITT executive scientist.

From 1987 until 1996, Dr. Kao served as vice chancellor (president) of The Chinese University of Hong Kong.

Professor Charles Kao
Engineer and Inventor of Fibre Optics
Father of Fiber Optic Communications

He received L.M. Ericsson International Prize, Marconi International Fellowship, 1996 Prince Philip Medal of the Royal Academy of Engineers*, and 1999 Charles Stark Draper Prize**. He was elected a member of the National Academy of Engineering of USA in 1990.

*The Royal Academy of Engineering Medals
The Fellowship of Engineering Prince Philip Medal (solid gold)
"For his pioneering work which led to the invention of optical fibers and for his leadership in its engineering and commercial realization; and for his distinguished contribution to higher education in Hong Kong".

In 1989 HRH The Prince Philip, Duke of Edinburgh, Senior Fellow of The Fellowship of Engineering, agreed to the commissioning of a gold medal to be "awarded periodically to an engineer of any nationality who has made an exceptional contribution to engineering as a whole through practice, management or education", to be known as The Fellowship of Engineering Prince Philip Medal.

**NAE Awards

One of the NAE's goals is to recognize the superior achievements of engineers. Accordingly, election to Academy membership is one of the highest honors an engineer can receive. Beyond this, the NAE presents four awards to honor extraordinary contributions to engineering and society.

"For the conception and invention of optical fibers for communications and for the development of manufacturing processes that made the telecommunications revolution possible."

The development of optical fiber technology was a watershed event in the global telecommunications and information technology revolution. Many of us today take for granted our ability to communicate on demand, much as earlier generations quickly took for granted the availability of electricity. But this dramatic and rapid revolution would simply not be possible but for the development of silica fibers as a high bandwidth, light-carrying medium for the transport of voice, video, and data. The silica fiber is now as fundamental to communication as the silicon integrated circuit is to computing. Optical fiber is the "concrete" of the "information superhighway." By the end of 1998, there were more than 215 million kilometers of optical fibers installed for communications worldwide. Through their efforts, Kao, Maurer, and MacChesney created the basis of modern fiber optic communications. Their creative application of materials science and engineering and chemical engineering to every aspect of fiber materials composition, characterization, and manufacturing, their understanding of the stringent materials requirements placed on the fiber by the performance needs of the telecommunications system, and, above all, their dedication to achieving their vision, were all critical to their success.

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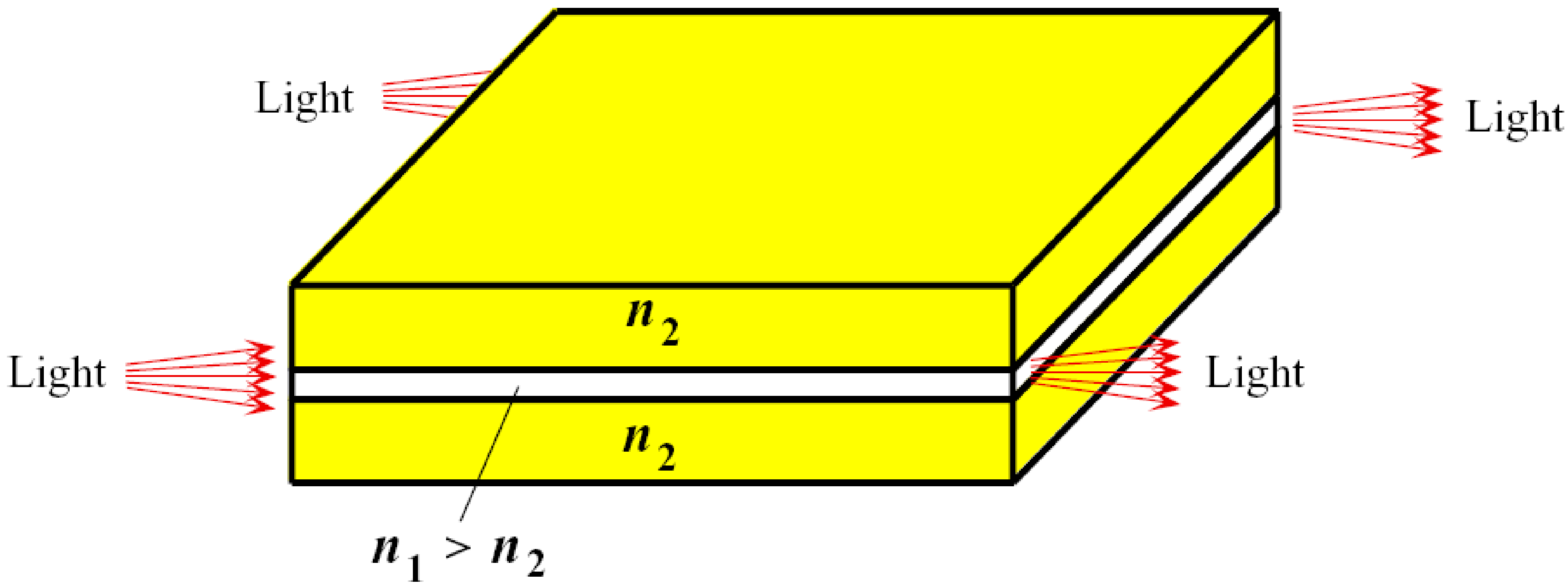
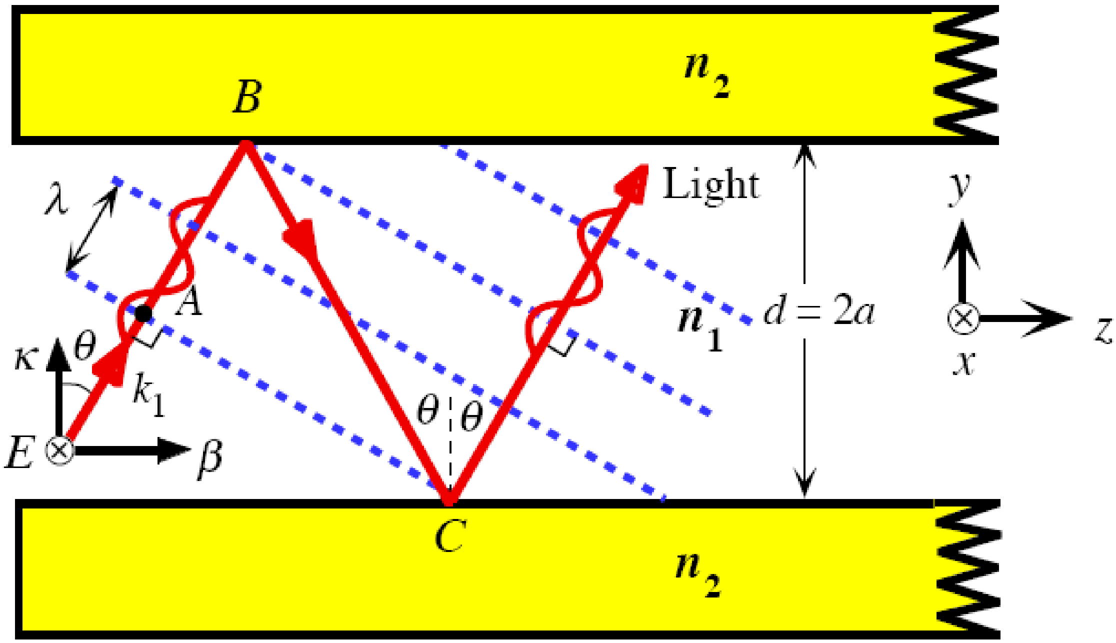
Symmetric Planar Dielectric Slab Waveguides



The region of higher refractive index (n_1) is called the **core**, and the region of lower refractive index (n_2) sandwiching the core is called the **cladding**.

Our goal is to find the conditions for light rays to propagate along such waveguides.



A and C are on the same wavefront. They must be in phase.

OR

Constructive interference occurs between A and C to achieve maximal transmitted light intensity.

$$\Delta\phi(AC) = m(2\pi),\ m=0,1,2,\ldots$$

$$\Delta\phi(AC) = k_1(AB + BC) - 2\phi$$

$$= k_1\left[BC\cos(2\theta) + BC\right] - 2\phi$$

$$= k_1 \frac{d}{\cos\theta}\left[2\cos^2\theta - 1 + 1\right] - 2\phi$$

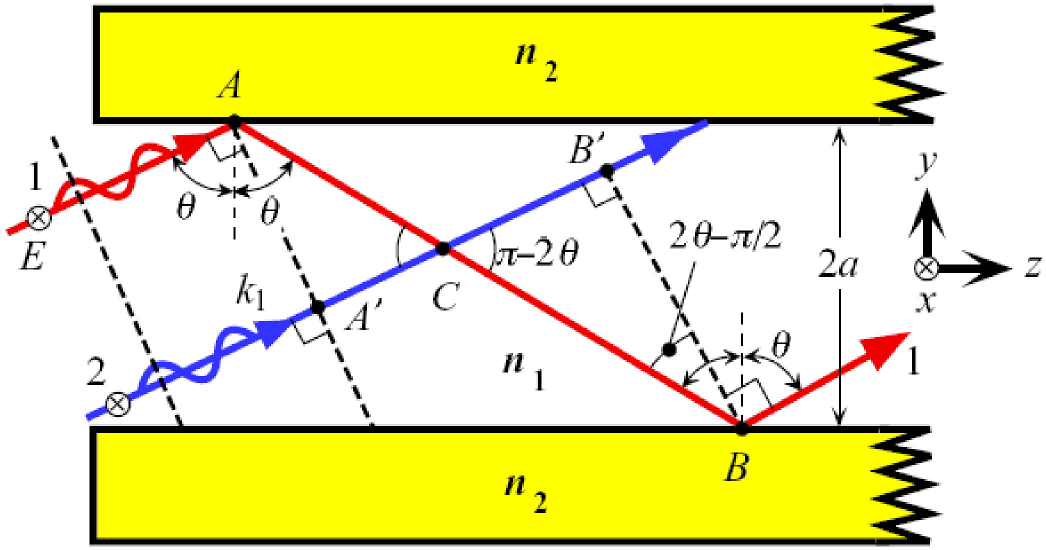
$$= k_1\left[2d\cos\theta\right] - 2\phi$$

$$\Rightarrow k_1\left[2d\cos\theta\right] - 2\phi = m(2\pi), m = 0,1,2,\ldots$$

Waveguide condition:

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

ϕ_m is a function of θ_m.



B and B' are on the same wavefront. They must be in phase.

OR

Constructive interference occurs between B and B' to achieve maximal transmitted light intensity.

$$\Delta\phi = k_1(AB) - 2\phi - k_1(A'B')$$
$$= k_1(AB) - k_1[AB\cos(\pi - 2\theta)] - 2\phi$$
$$= k_1(AB)(2\cos^2\theta) - 2\phi$$
$$= k_1 \frac{d}{\sin\left(\frac{\pi}{2} - \theta\right)}[2\cos^2\theta] - 2\phi$$
$$\Rightarrow k_1[2d\cos\theta] - 2\phi = m(2\pi), m = 0,1,2,\ldots$$

$$\Delta\phi(BB') = m(2\pi),\ m=0,1,2,\ldots$$

Waveguide condition:

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

To obtain the waveguide condition and solve the propagation modes for the symmetric planar dielectric waveguides:

(1) The wave optics approach

Solve Maxwell's equations. There is no approximations and the results are rigorous.

(2) The coefficient matrix approach

Straightforward. Not suitable for multilayer problems.

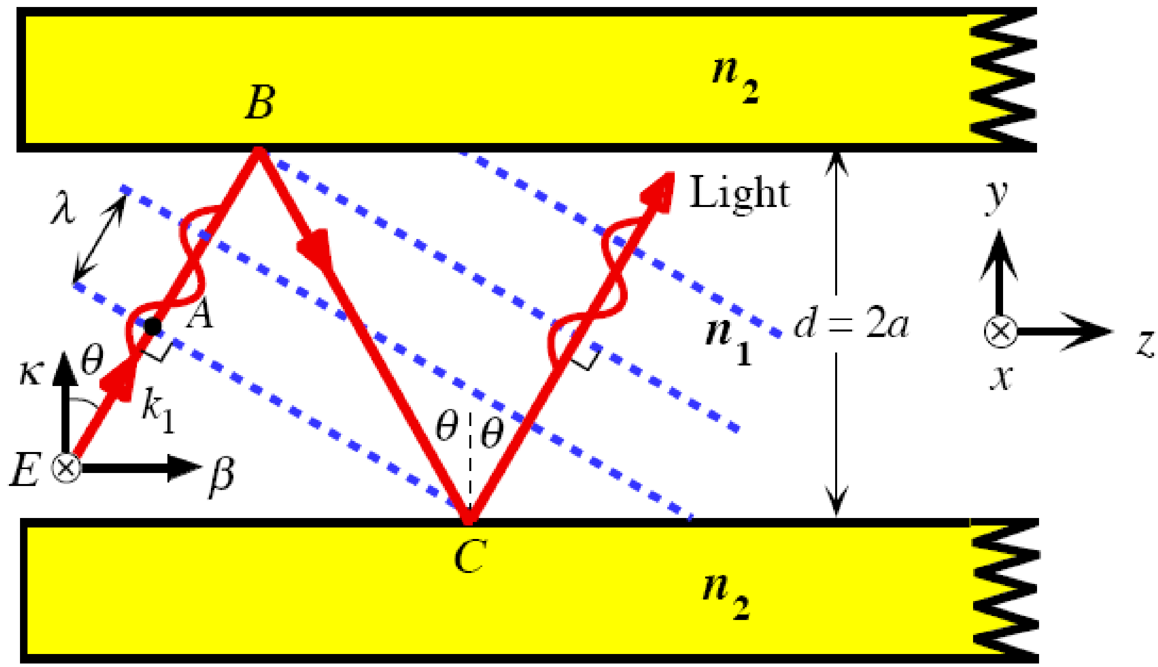
(3) The transmission matrix method

Suitable for multilayer waveguides.

(4) The modified ray model method

It is simple, but provides less information.

Propagation Constant along the Waveguide



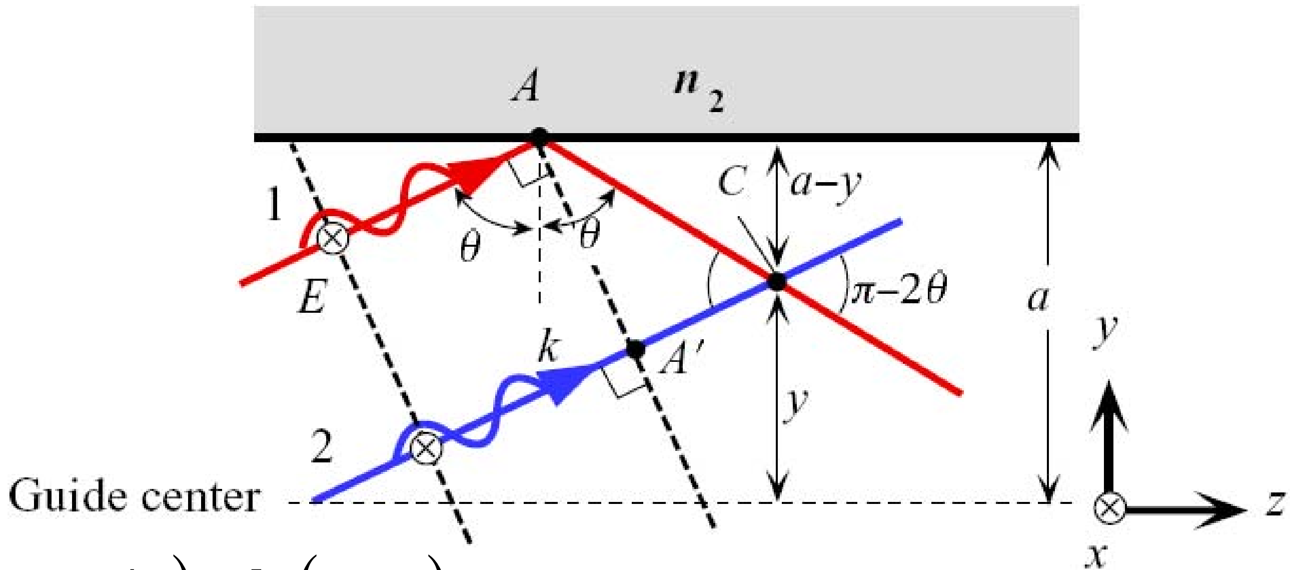
$$\begin{cases} \beta_m = k_1 \sin\theta_m = \left(\dfrac{2\pi n_1}{\lambda}\right)\sin\theta_m \\ \kappa_m = k_1 \cos\theta_m = \left(\dfrac{2\pi n_1}{\lambda}\right)\cos\theta_m \end{cases}$$

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

Larger m leads to smaller θ_m.

κ_m: transverse propagation constant

Electric Field Patterns

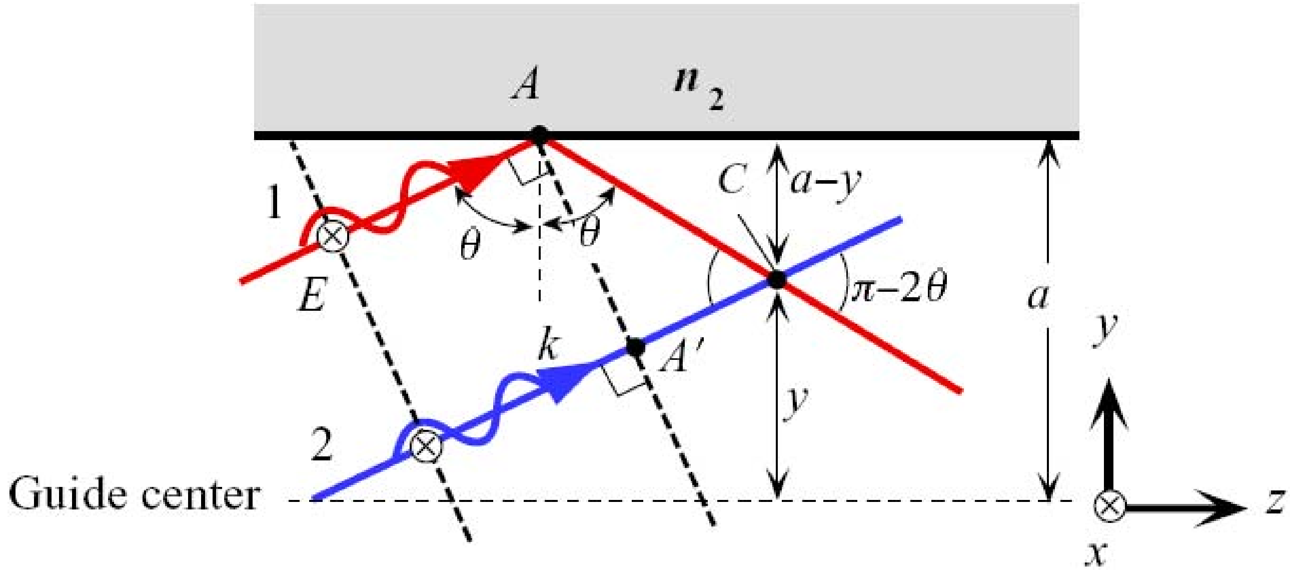


$$\Phi_m = (k_1 AC - \phi_m) - k_1(A'C)$$
$$= k_1 AC - k_1 AC\cos(\pi - 2\theta_m) - \phi_m$$
$$= k_1 AC(2\cos^2\theta_m) - \phi_m$$
$$= k_1 \frac{a-y}{\sin\left(\frac{\pi}{2} - \theta_m\right)}\left[2\cos^2\theta_m\right] - \phi_m$$
$$= 2k_1(a-y)\cos\theta_m - \phi_m$$

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

$$\Phi_m(y) = m\pi - \frac{y}{a}(m\pi + \phi_m)$$

Electric Field Patterns



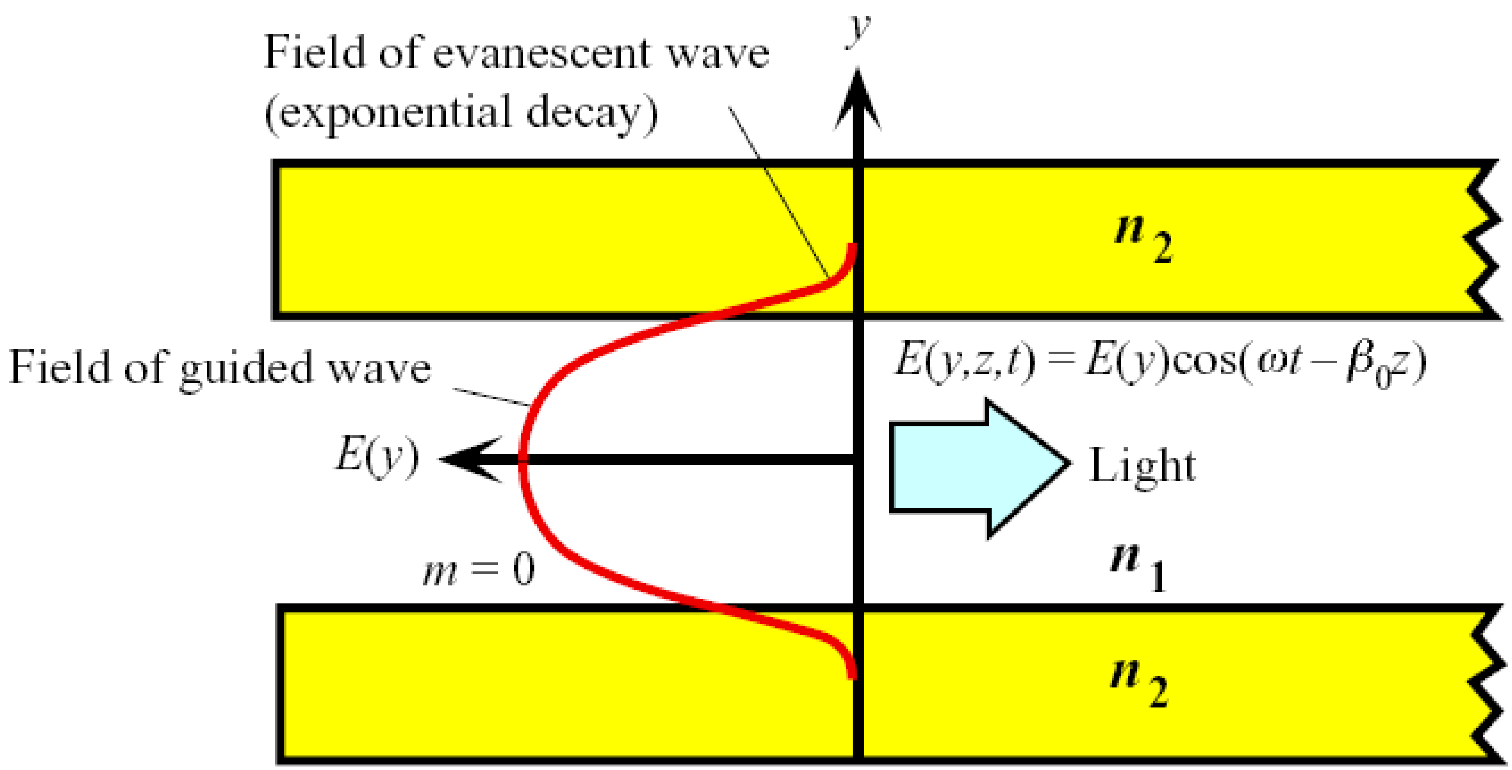
$$E_1(y,z,t) = E_0 \cos(\omega t - \beta_m z + \kappa_m y + \Phi_m)$$

$$E_2(y,z,t) = E_0 \cos(\omega t - \beta_m z - \kappa_m y)$$

$$E(y,z,t) = E_1 + E_2 = 2E_0 \cos\left(\kappa_m y + \frac{1}{2}\Phi_m\right)\cos\left(\omega t - \beta_m z + \frac{1}{2}\Phi_m\right)$$

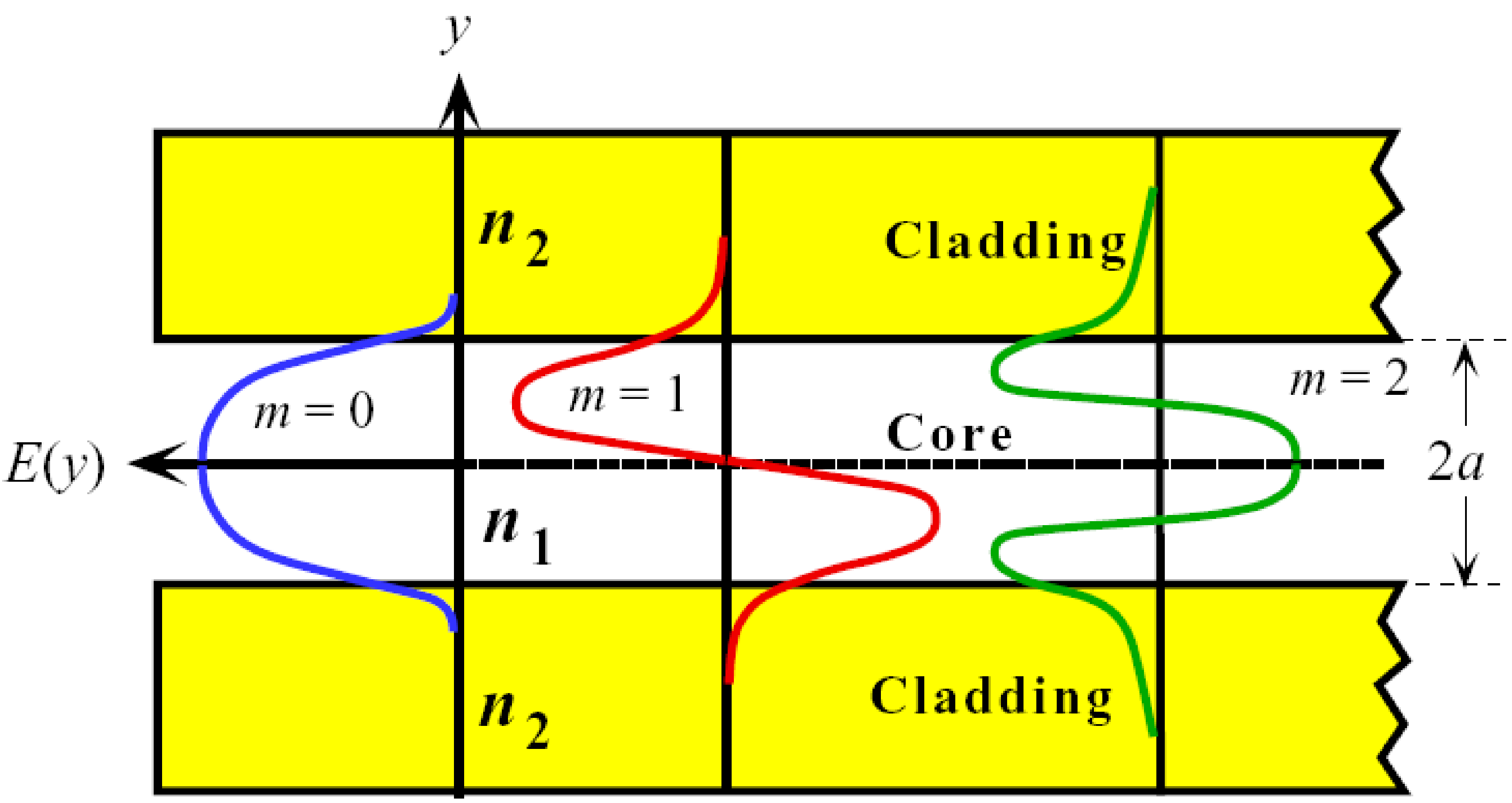
It is a stationary wave along the *y*-direction, which travels down the waveguide along *z*.

Electric Field Patterns

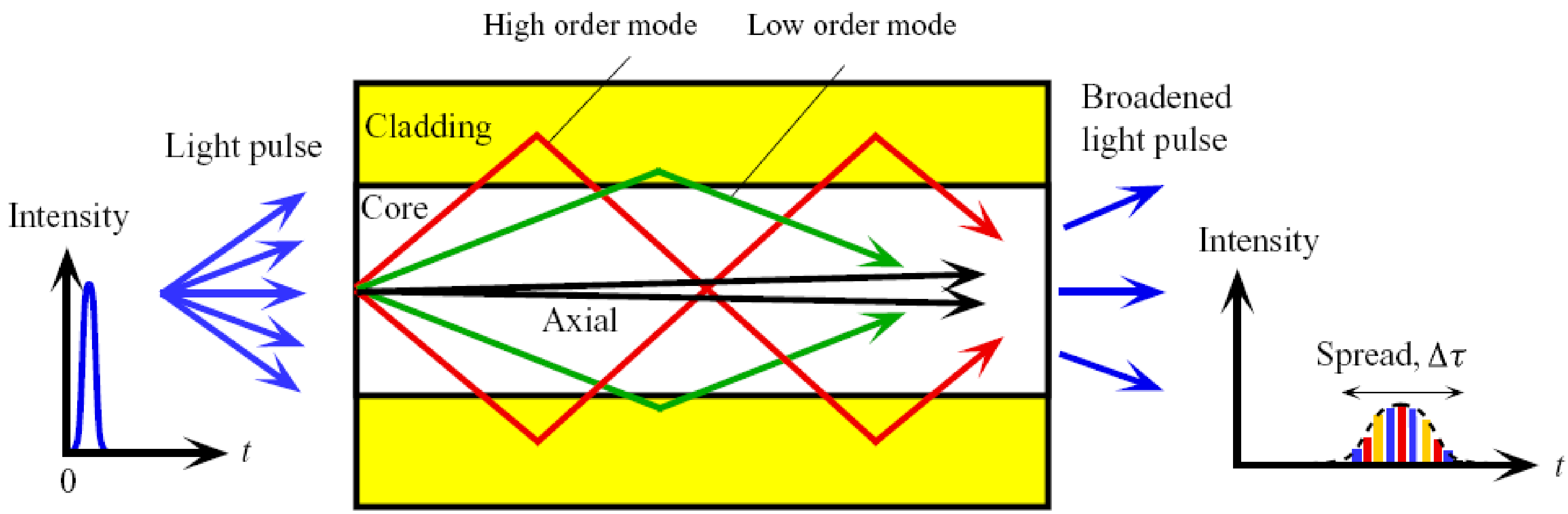


The lowest mode (m = 0) has a maximum intensity at the center and moves along z with a propagation constant of β_0. There is a propagating evanescent wave in the cladding near the boundary.

Electric Field Patterns



Broadening of the Output Pulse



Each light wave that satisfies the waveguide condition constitutes a **mode of propagation**. The integer *m* identifies these modes and is called the **mode number**.

θ_m is smaller for larger *m*. Higher modes exhibit more reflections and penetrate more into the cladding.

Short-duration pulses of light transmitted through the waveguide will be broadened in terms of time duration.

Single and Multimode Waveguides

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

$$\theta_m > \theta_c \Rightarrow$$

$$\sin\theta_m > \frac{n_2}{n_1} \Rightarrow$$

$$1-\cos^2\theta_m > \frac{n_2^2}{n_1^2} \Rightarrow$$

$$\left[\frac{m\pi+\phi_m}{4\pi a n_1/\lambda}\right]^2 - 1 < -\frac{n_2^2}{n_1^2} \Rightarrow$$

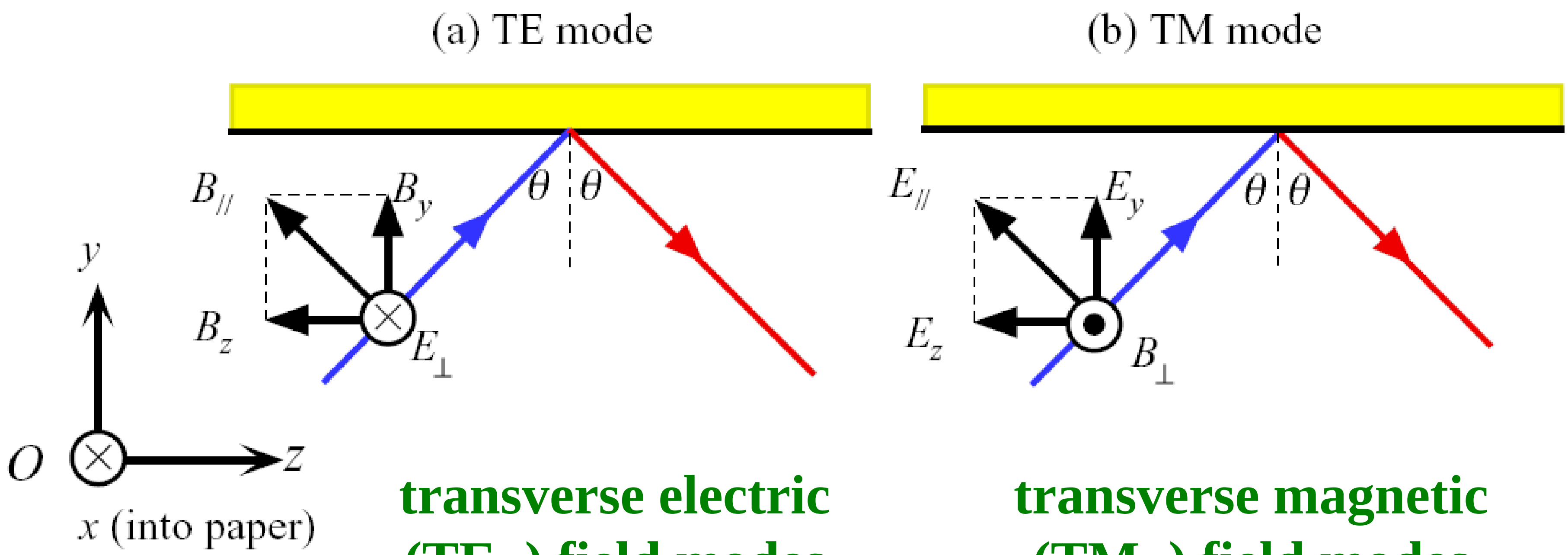
$$\frac{m\pi+\phi_m}{4\pi a/\lambda} < \left(n_1^2 - n_2^2\right)^{1/2}$$

$$\Rightarrow \begin{cases} V = \dfrac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} \\ m \le \left(2V - \phi_m\right)/\pi \end{cases}$$

V*-number** (V*-parameter, normalized thickness, normalized frequency**) is a characteristic parameter of the waveguide.

When $V < \pi/2$, $m = 0$ is the only possibility and only the fundamental mode ($m = 0$) propagates along the waveguide. It is termed as the **single mode** waveguide. λ_c that satisfies $V = \pi/2$ is the **cut-off wavelength**. Above this wavelength, only the fundamental mode will propagate.

TE and TM Modes



transverse electric (TE_m) field modes

transverse magnetic (TM_m) field modes

The phase change at TIR depends on the polarization of the electric field, and it is different for $E_\perp$ and $E_{//}$. These two fields require different angles θ_m to propagate along the waveguide.

Example: waveguide modes

Consider a planar waveguide with a core thickness 20 μm, n_1 = 1.455, n_2 = 1.440, light wavelength of 900 nm. Given the waveguide condition and the phase change in TIR for the TE mode, find angles θ_m for all the modes.

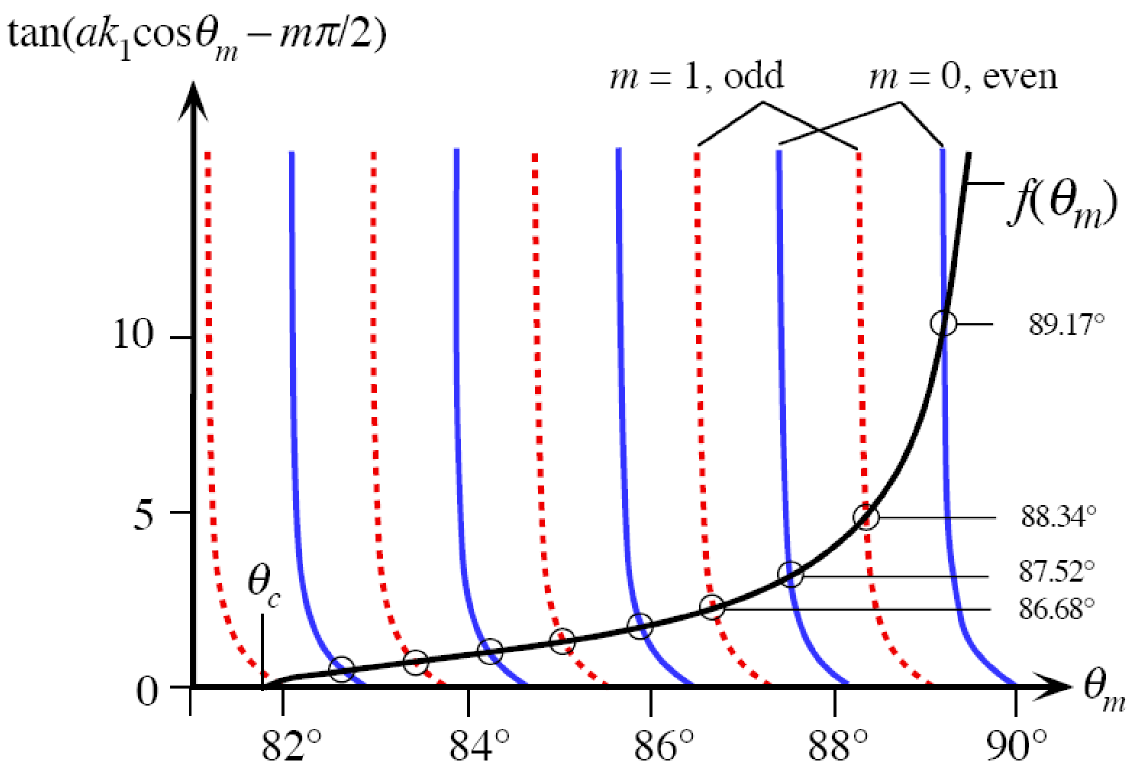
$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

$$\tan\left(\frac{1}{2}\phi_m\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_m}$$

$$\tan\left(\frac{2\pi n_1 a}{\lambda}\cos\theta_m - m\frac{\pi}{2}\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_m}$$

LHS (left-hand side): $\tan\left(\frac{2\pi n_1 a}{\lambda}\cos\theta_m - m\frac{\pi}{2}\right)$ versus θ_m

RHS (right-hand side): $f(\theta_m) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_m}$ versus θ_m



Penetration depth of the evanescent wave:

$$\frac{1}{\delta_m} = \alpha_m = \frac{2\pi n_2 \left[\left(\frac{n_1^2}{n_2^2} \right) \sin^2 \theta_m - 1 \right]^{1/2}}{\lambda}$$

m	0	1	2	3	4	5	6	7	8	9
θ_m	89.2°	88.3°	87.5°	86.7°	85.9°	85.0°	84.2°	83.4°	82.6°	81.9°
δ_m	0.691	0.702	0.722	0.751	0.793	0.866	0.970	1.15	1.57	3.83

δ_m in μm.

An accurate solution of the angle for the fundamental TE mode is 89.172°. The angle for the fundamental TM mode is 89.170°, which is almost identical to the angle for the TE mode.

Example: the number of modes

Estimate the number of modes that can be supported in a planar dielectric waveguide that is 100 μm wide and has $n_1 = 1.490$, $n_2 = 1.470$. The free-space light wavelength is 1 μm.

$$m \le \frac{2V - \phi}{\pi} \approx \frac{2V}{\pi}$$

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2}$$

$$= \frac{2\pi 50}{1}\left(1.490^2 - 1.470^2\right)^{1/2} = 76.44$$

$$m \le \frac{2 \times 76.44}{\pi} = 48.7$$

There are about 49 modes.

$$M = Int\left(\frac{2V}{\pi}\right) + 1$$

Int (x) is the integer function. It removes the decimal fraction of x.

Example: mode field width (MFW), 2w_0

The field distribution along y penetrates into the cladding. The extent of the electric field across the waveguide is therefore more than 2a. The penetrating field is due to the evanescent wave and it decays exponentially according to

$$E_{cladding}(y') = E_{cladding}(0)\exp(-\alpha_{cladding}\, y')$$

The penetration depth

$$\delta_{cladding} = \frac{1}{\alpha_{cladding}} = \frac{\lambda}{2\pi n_2}\left[\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1\right]^{-1/2}$$

For $m = 0$ mode (axial mode), $\theta_i \rightarrow 90°$

$$\delta_{cladding} \approx \frac{\lambda}{2\pi}\left[n_1^2 - n_2^2\right]^{-1/2} = \frac{a}{V}$$

The mode field width

$$2w_0 \approx 2a + 2\frac{a}{V} = 2a\frac{(V+1)}{V}$$

For optical fibers, it is called the **mode field diameter (MFD)**.

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Group Velocity

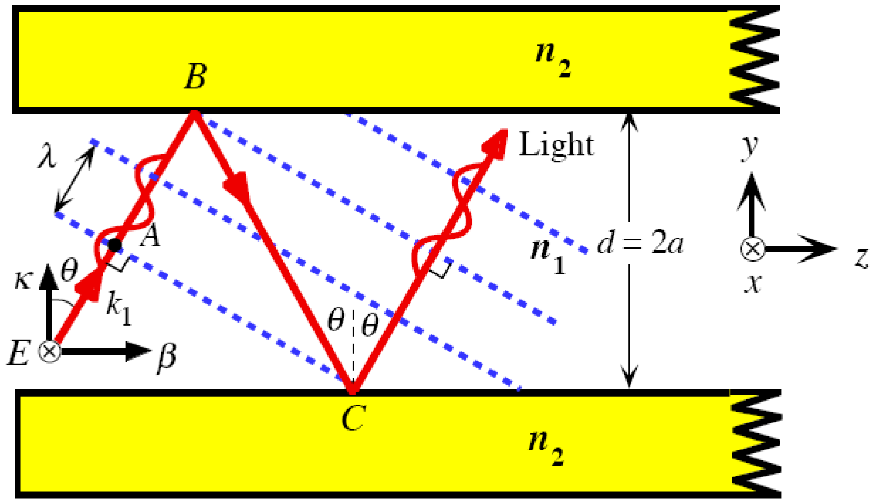
$$\textbf{Group velocity} = v_g = \frac{d\omega}{dk}$$

$$\textbf{Phase velocity} = \frac{\omega}{k} = v = \frac{c}{n}$$

The group velocity defines the speed with which energy or information is propagated because it defines the speed of the envelope of the amplitude variation.

To find out the group velocity, we need to know the change of ω with respect to *k*. The ω versus *k* characteristics is called the **dispersion relation** or **dispersion diagram**.

Waveguide Dispersion Diagram

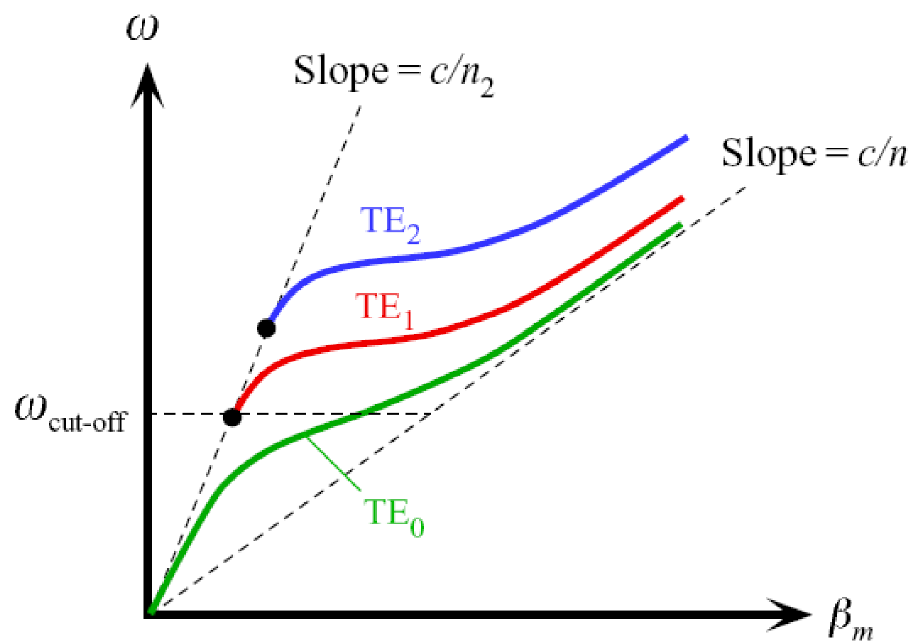


Waveguide condition

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

θ_m depends on the waveguide properties (n_1, n_2, and a) and the light frequency, ω.

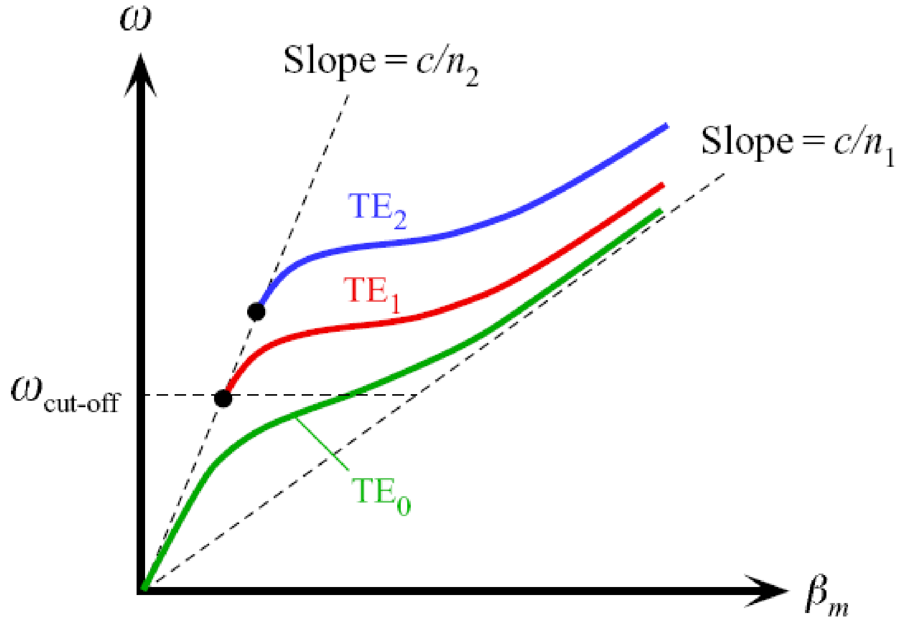
$$\beta_m = k_1 \sin\theta_m = \frac{2\pi n_1}{\lambda}\sin\theta_m$$



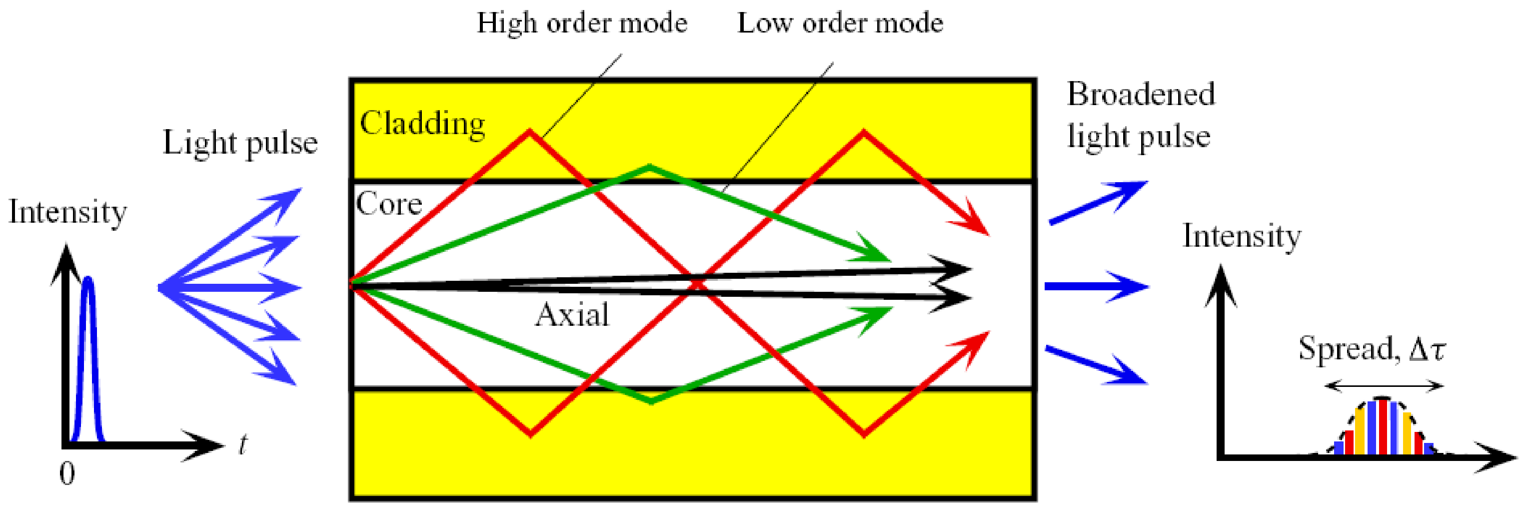
The slope $d\omega/d\beta_m$ at any frequency is the group velocity v_g.

ω is a function of β_m — waveguide dispersion diagram.

Intermodal Dispersion



The lowest mode ($m = 0$) has the slowest group velocity, close to c/n_1, because the lowest mode is contained mainly in the core, which has a larger refractive index n_1. The highest mode has the highest group velocity, close to c/n_2 because a portion of the field is carried by the cladding, which has the smaller refractive index n_2.
Different modes take different time to travel the length of the fiber (for an exact monochromatic light wave). This phenomenon is called **intermodal dispersion**.



A direct consequence is that a short duration light pulse signal that is coupled into the waveguide will travel along the guide via the various allowed modes with different group velocities. The reconstruction of the light pulse at the receiving end from the various modes will result in a broadened signal. The intermodal dispersion can be estimated by

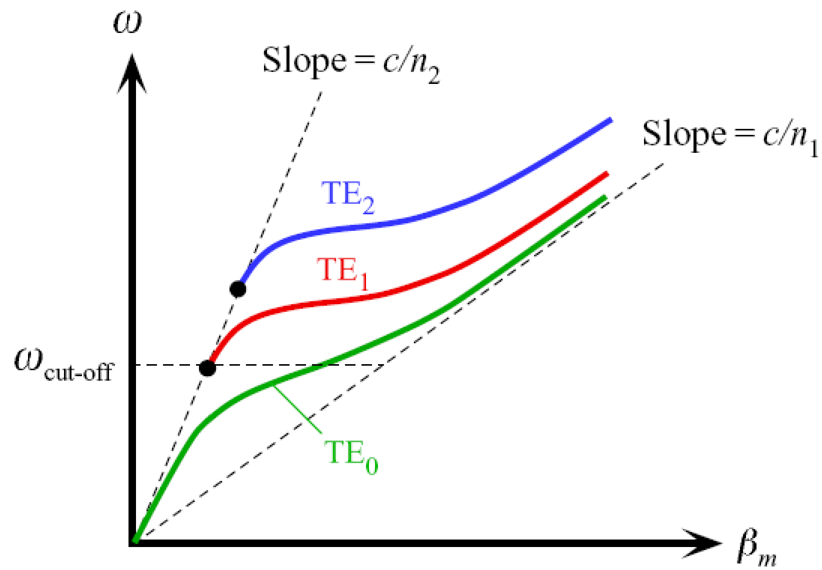
$$\Delta\tau = \frac{L}{v_{g\min}} - \frac{L}{v_{g\max}}$$

$$v_{g\min} \approx \frac{c}{n_1}, v_{g\max} \approx \frac{c}{n_2}$$

$$\Rightarrow \frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c}$$

$$n_1 = 1.48$$
$$n_2 = 1.46$$
$$\frac{\Delta\tau}{L} \approx 67 ns / km$$

Intramodal Dispersion



The group velocity $d\omega/d\beta_m$ of a single mode changes with the frequency ω. If the light source contains various frequencies (there is no perfect monochromatic wave), different frequencies will travel at different velocities. This is called **waveguide dispersion**.

The refractive index of a material is usually a function of the light frequency. The n–ω dependence also results in the change in the group velocity of a given mode. This is called **material dispersion**. Waveguide dispersion and material dispersion combined together are called **intramodal dispersion**.

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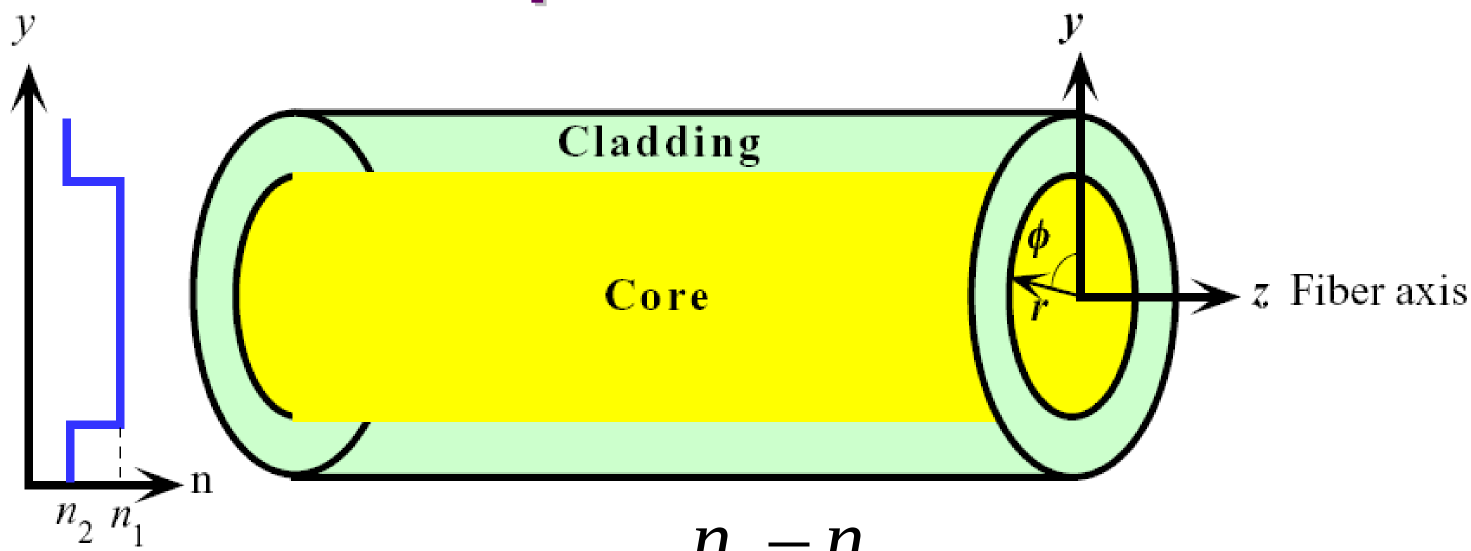
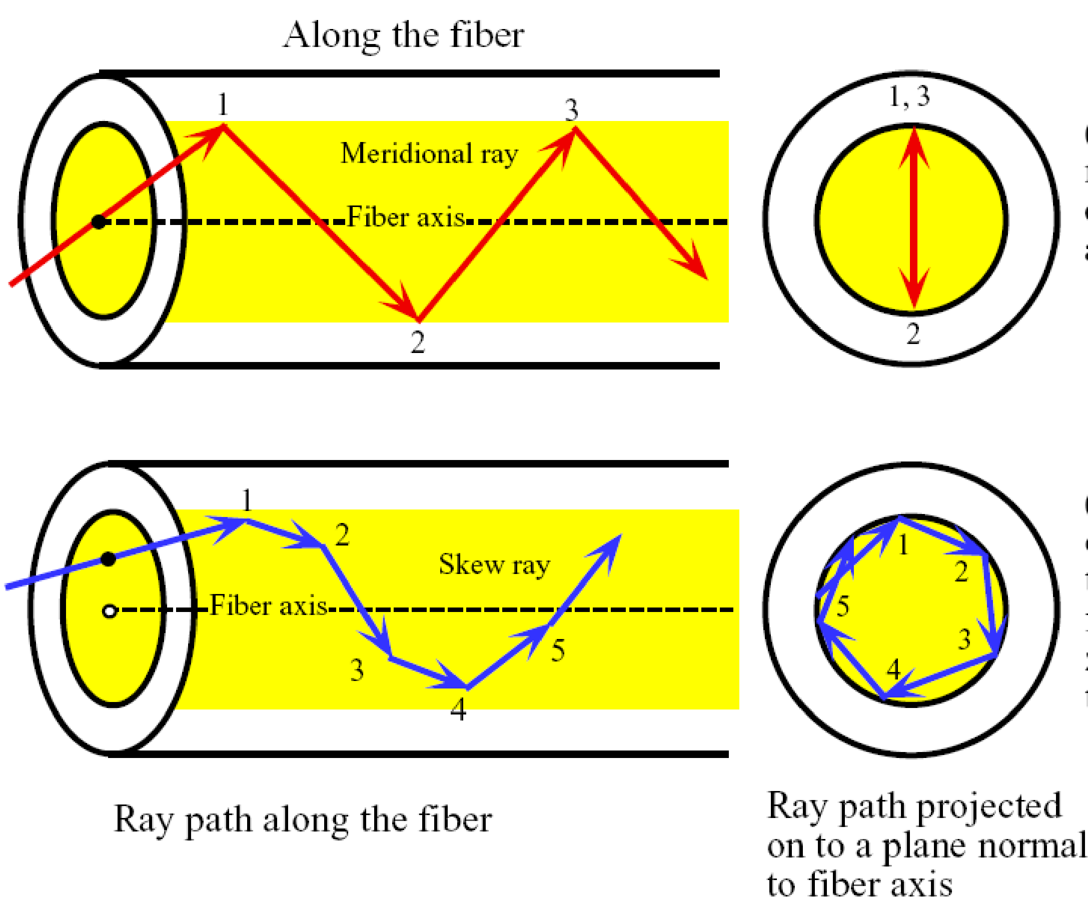


Normalized index difference $\Delta = \dfrac{n_1 - n_2}{n_1}$ **for practical fibers, $\Delta << 1$**

The general ideas for guided wave propagation in planar waveguides can be extended to step indexed optical fibers with certain modifications.

The planar waveguide is bounded only in one dimension. Distinct modes are labeled with one integer, *m*. The cylindrical fiber is bounded in two dimensions. Two integers, *l* and *m*, are required to label all the possible guided modes.



(a) A meridional ray always crosses the fiber axis.

(b) A skew ray does not have to cross the fiber axis. It zigzags around the fiber axis.

There are two types of light rays for cylindrical fibers. A **meridional** ray enters the fiber through the axis and also crosses the fiber axis on each reflection as it zigzags down the fiber. A **skew** ray enters the fiber off the fiber axis and zigzags down the fiber without crossing the axis. It has a helical path around the fiber axis.
Guided meridional rays are either TE or TM type. Guided skew rays can have both electric and magnetic field components along z, which are called hybrid modes (EH or HE modes).

We usually consider **linearly polarized** light waves that are guided in step index fibers. A guided mode along the fiber is represented by the propagation of an electric field distribution $E_{lm}(r, \varphi)$ along z.

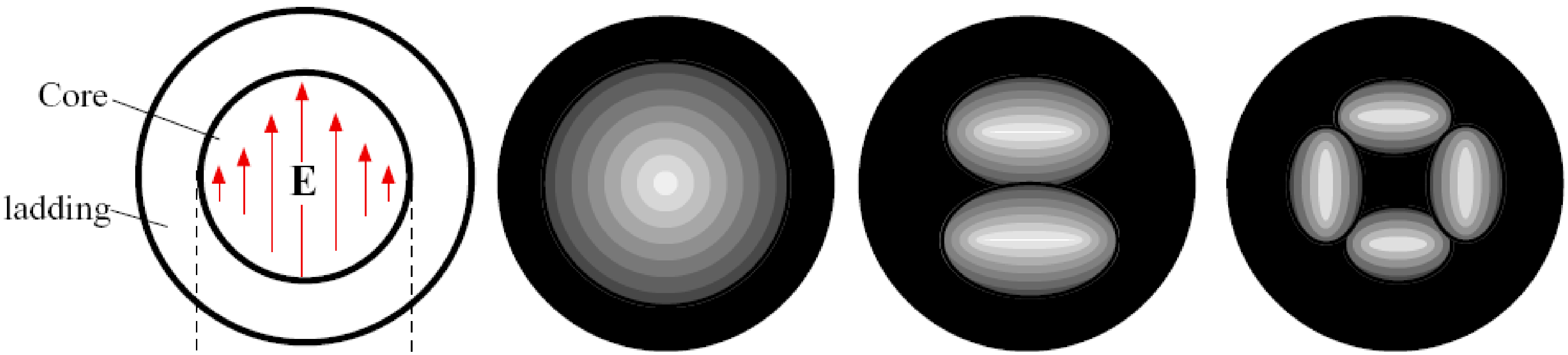
$$E_{LP} = E_{lm}(r,\varphi)\exp\left[j\left(\omega t - \beta_{lm} z\right)\right]$$

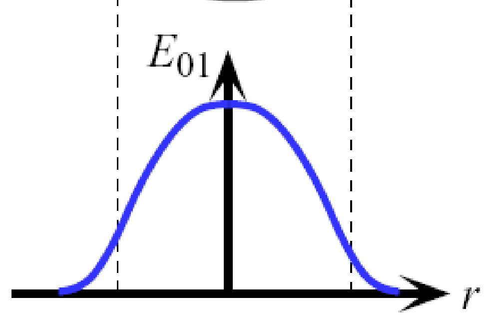
(a) The electric field of the fundamental mode

(b) The intensity in the fundamental mode LP_{01}

(c) The intensity in LP_{11}

(d) The intensity in LP_{21}





The electric field distribution of the fundamental mode in the transverse plane to the fiber axis z. The light intensity is greatest at the center of the fiber. Intensity patterns in LP_{01}, LP_{11} and LP_{21} modes.

V-Number of Step Index Fibers

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2}$$

$$\Delta = \left(n_1 - n_2\right)/n_1 \qquad n_1 > n_2$$

$$n = \left(n_1 + n_2\right)/2 \qquad \Delta << 1$$

$$V = \frac{2\pi a}{\lambda}\left[\left(n_1 + n_2\right)\left(n_1 - n_2\right)\right]^{1/2} = \frac{2\pi a}{\lambda}\left(2n_1 n\Delta\right)^{1/2}$$

When the *V*-number is smaller than **2.405**, only the fundamental mode (LP_{01}) can propagate through the fiber core (**single mode fiber**). The cut-off wavelength λ_c above which the fiber becomes single mode is given by

$$V_{cut-off} = \frac{2\pi a}{\lambda_c}\left(n_1^2 - n_2^2\right)^{1/2} = 2.405$$

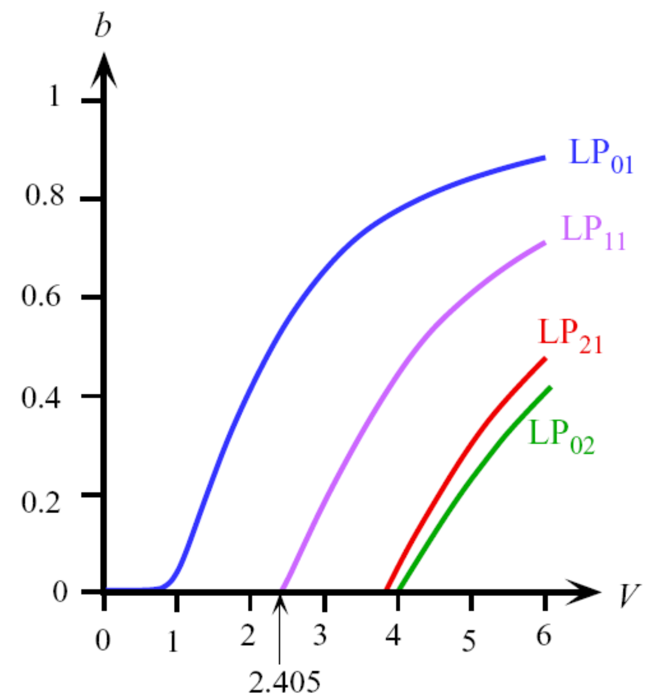
The number of modes M in a step index fiber can be estimated by

$$M \approx \frac{V^2}{2}$$

Since the propagation constant β_{lm} depends on the waveguide properties and the wavelength, a **normalized propagation constant** is usually defined. This normalized propagation constant, b, depends only on the V-number.

$$b = \frac{\left(\frac{\lambda\beta_{lm}}{2\pi}\right)^2 - n_2^2}{n_1^2 - n_2^2}$$

$b = 0$ corresponds to $\beta_{lm} = 2\pi n_1/\lambda$
$b = 1$ corresponds to $\beta_{lm} = 2\pi n_2/\lambda$



Example: A Multimode Fiber

A step index fiber has a core of refractive index 1.468 and diameter 100 μm, a cladding of refractive index of 1.447. If the source wavelength is 850 nm, calculate the number of modes that are allowed in this fiber.

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{\pi 100}{0.85}\sqrt{1.468^2 - 1.447^2} = 91.44$$

$$M \approx \frac{V^2}{2} = \frac{91.44^2}{2} = 4181$$

Example: A Single Mode Fiber

What should be the core radius of a single mode fiber that has a core refractive index of 1.468 and a cladding refractive index of 1.447, and is to be used for a source wavelength of 1.3 μm?

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{1.3}\sqrt{1.468^2 - 1.447^2} \le 2.405$$

$$a \le 2.01\mu\text{m}$$

Example: Single Mode Cut-Off Wavelength

What is the cut-off wavelength for single mode operation for a fiber that has a core with a diameter of 7 μm, a refractive index of 1.458, and a cladding of refractive index of 1.452? What is the *V*-number and the mode field diameter (MFD) when operating at $\lambda = 1.3$ μm?

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{\pi 7}{\lambda}\sqrt{1.458^2 - 1.452^2} \leq 2.405$$

$$\lambda \geq 1.208\mu\text{m}$$

When $\lambda = 1.3$ μm:

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{\pi 7}{1.3}\sqrt{1.458^2 - 1.452^2} = 2.235$$

$$2w_0 \approx (2a)\frac{V+1}{V} = 7 \times \frac{2.235+1}{2.235} = 10.13\mu\text{m}$$

Example: Group Velocity and Delay

Consider a single mode fiber with core and cladding indices of 1.448 and 1.440, core radius of 3 μm, operating at 1.5 μm. Given that we can approximate the fundamental mode normalized propagation constant by

$$b \approx \left(1.1428 - \frac{0.996}{V}\right)^2 \qquad (1.5 < V < 2.5)$$

Calculate the propagation constant β. Change the operating wavelength to λ' by a small amount, say 0.01%, and then recalculate the new propagation constant β'. Then determine the group velocity of the fundamental mode at 1.5 μm, and the group delay τ_g over 1 km of fiber.

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} \qquad k = \frac{2\pi}{\lambda}$$

$$b = \frac{(\beta / k)^2 - n_2^2}{n_1^2 - n_2^2} \qquad \omega = \frac{2\pi c}{\lambda}$$

λ (μm)	V	k (m^{-1})	ω (rad s^{-1})	b	β (m^{-1})
1.500000	1.910088	4188790	1.255768×10^{15}	0.3860858	6.044817×10^{6}
1.500150	1.909897	4188371	1.255642×10^{15}	0.3860210	6.044211×10^{6}

The group velocity is

$$v_g = \frac{\omega'-\omega}{\beta'-\beta} = \frac{(1.255768-1.255642)\times10^{15}}{(6.044817-6.044211)\times10^{6}} = 2.0792\times10^{8}\,\mathrm{m/s}$$

The group delay is

$$\tau_g = \frac{L}{v_g} = \frac{1000}{2.0792\times10^{8}} = 4.81\,\mu\mathrm{s}$$

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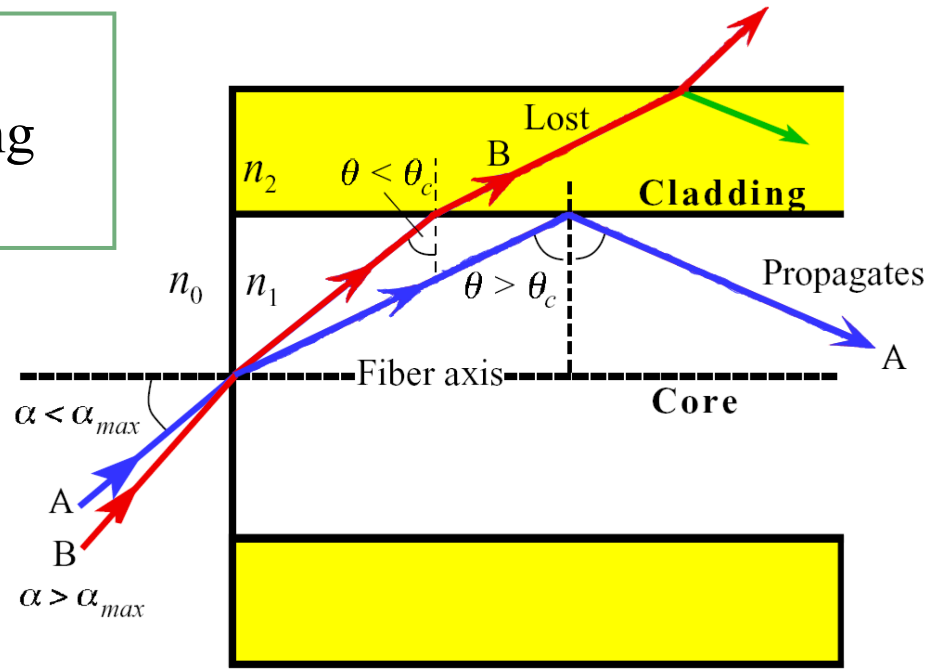
Numerical Aperture

Only the rays that fall within a certain cone at the input of the fiber can propagate through the optical fiber.

Maximum acceptance angle $\alpha_{\max}$ is that which just gives TIR at the core-cladding interface.



$$n_0 \sin \alpha_{\max} = n_1 \sin(90° - \theta_c)$$

$$\sin \theta_c = n_2 / n_1$$

$$\sin \alpha_{\max} = \frac{n_1}{n_0} \cos \theta_c = \frac{\left(n_1^2 - n_2^2\right)^{1/2}}{n_0}$$

$$NA = \left(n_1^2 - n_2^2\right)^{1/2}$$

$$\sin \alpha_{\max} = \frac{NA}{n_0}$$

NA: **numerical aperture**

$\alpha_{\max}$: **maximum acceptance angle**

$2\alpha_{\max}$: **total acceptance angle**

Example: A Multimode Fiber and Total Acceptance Angle

A step index fiber has a core diameter of 100 μm and a refractive index of 1.480. The cladding has a refractive index of 1.460. Calculate the numerical aperture of the fiber, acceptance angle from air, and the number of modes sustained when the source wavelength is 850 nm.

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.480^2 - 1.460^2} = 0.2425$$

$$\sin\alpha_{\max} = \frac{NA}{n_0} = \frac{0.2425}{1.0}$$

$$\alpha_{\max} = 14^\circ$$

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda} NA = \frac{\pi 100}{0.85} 0.2425 = 89.63$$

$$M \approx \frac{V^2}{2} = \frac{89.63^2}{2} = 4017$$

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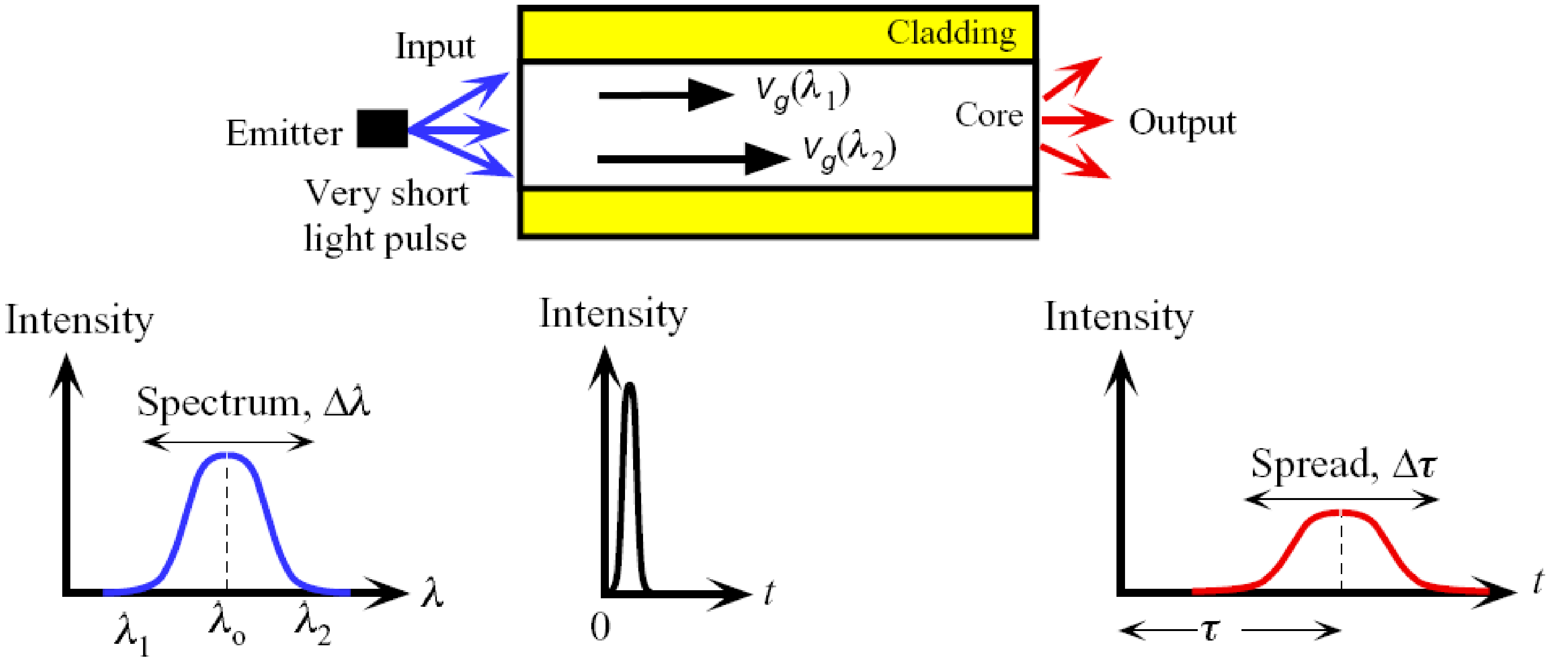
Material Dispersion

There is no intermodal dispersion in single-mode fibers.
Material dispersion is due to the dependence of the refractive index on the free space wavelength.

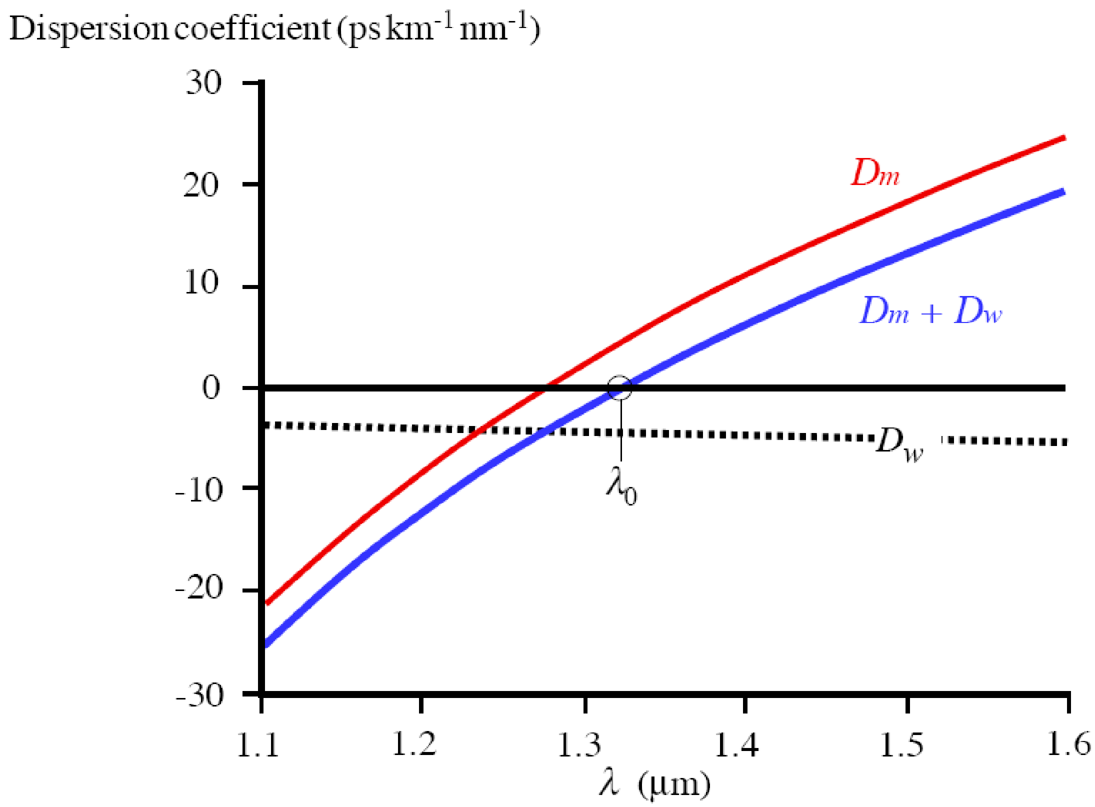


$$\frac{\Delta\tau}{L} = |D_m|\Delta\lambda$$

D_m is called the **material dispersion coefficient**. Dispersion is expressed as spread per unit length because slower waves fall further behind the faster waves over a longer distance.

Material Dispersion

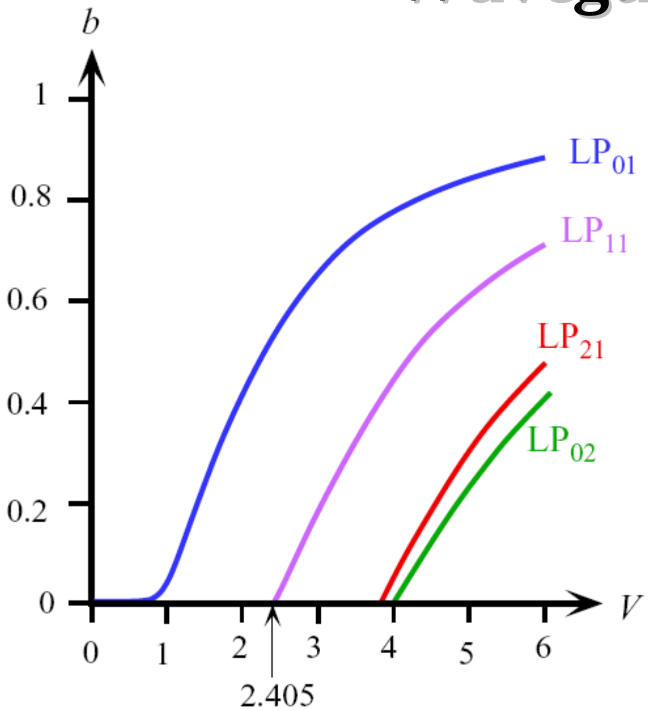
$$D_m \approx -\frac{\lambda}{c}\left(\frac{d^2 n}{d\lambda^2}\right)$$



The transit time τ of a light pulse represents a delay between the output and the input. The signal delay time per unit distance, τ/L, is called the **group delay** (τ_g).

$$\tau_g = \frac{\tau}{L} = \frac{1}{v_g} = \frac{d\beta}{d\omega}$$

Waveguide Dispersion



$$b = \frac{\left(\frac{\lambda \beta_{lm}}{2\pi}\right)^2 - n_2^2}{n_1^2 - n_2^2}$$

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

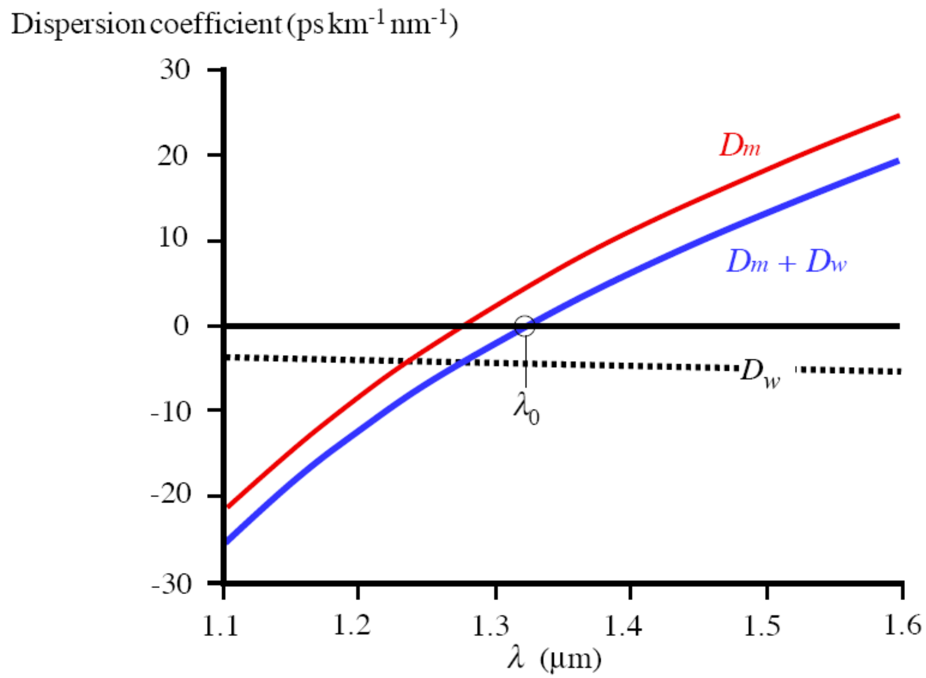
Waveguide dispersion is due to that the group velocity $d\omega/d\beta$ varies as a function of λ.

$$\frac{\Delta \tau}{L} = |D_w| \Delta \lambda$$

D_w is called the **waveguide dispersion coefficient**.

Waveguide Dispersion

$$D_w \approx \frac{1.984 N_{g2}}{(2\pi a)^2 2cn_2^2} \qquad \text{for } 1.5 < V < 2.4$$



D_m and D_w have opposite tendencies.

Profile Dispersion

There is an additional dispersion mechanism called the **profile dispersion** that arises because the group velocity of the fundamental mode, $v_{g,01}$, also depends on the normalized index difference. Δ is dependent on the wavelength due to material dispersion characteristics, *i.e.*, n_1 versus λ and n_2 versus λ behavior. Therefore, in reality, profile dispersion originates from material dispersion.

$$\Delta = \frac{n_1 - n_2}{n_1} \qquad \frac{\Delta\tau}{L} = |D_p|\Delta\lambda$$

D_p is called the **profile dispersion coefficient**.

D_p is less than 1 ps km^{-1} nm^{-1}, much smaller than D_m and D_w.

Chromatic Dispersion

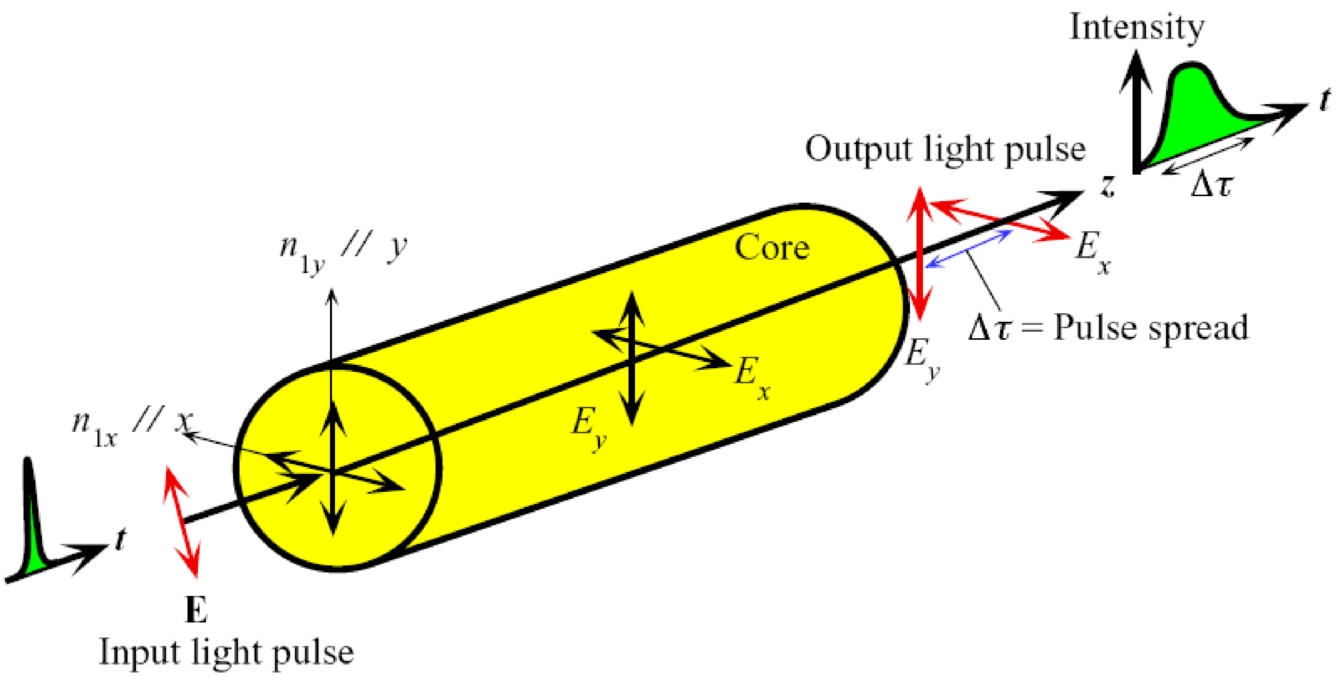
In single-mode fibers, the dispersion of a propagating pulse arises because of the finite width $\Delta\lambda$ of the source spectrum. This type of dispersion caused by a range of source wavelengths is generally termed **chromatic dispersion**, including material, waveguide, and profile dispersion, since they are all dependent on $\Delta\lambda$.

$$\frac{\Delta\tau}{L} = \left| D_m + D_w + D_p \right| \Delta\lambda$$

$$D_{ch} = D_m + D_w + D_p$$

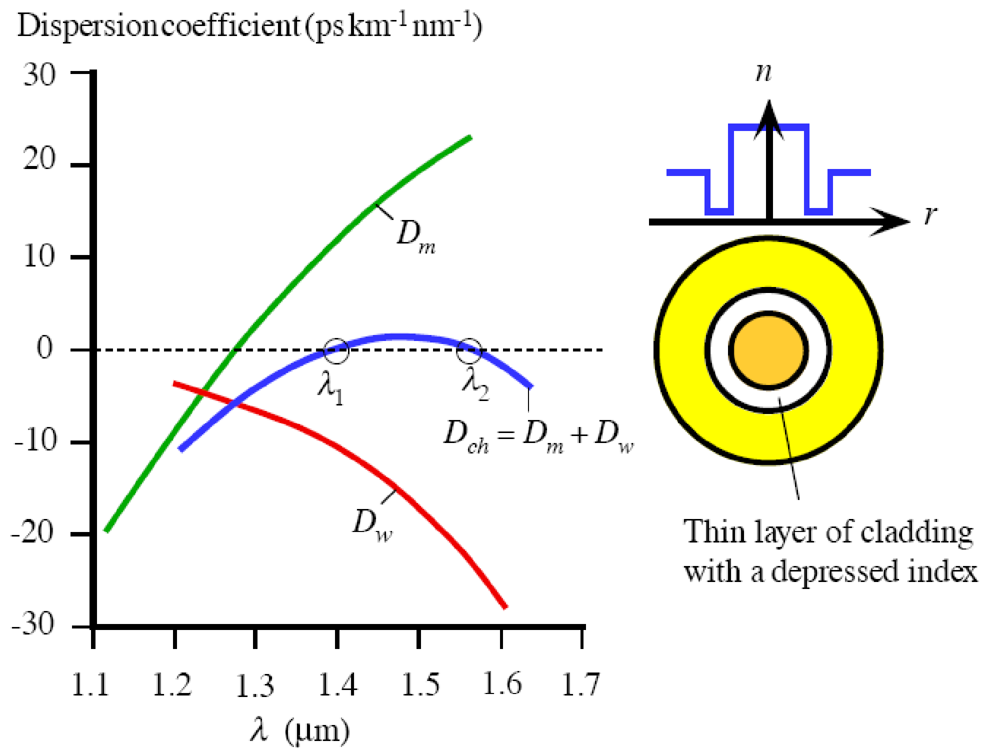
Polarization Dispersion

Polarization dispersion arises when the refractive index is not *isotropic*. When the refractive index depends on the direction of the electric field, the propagation constant of a given mode depends on the polarization. The anisotropic n_1 and n_2 may result from the fabrication process (changes in the glass composition, geometry, and induced local strains). Typically, polarization dispersion is less than a fraction of 1 ps km^{-1}. Polarization dispersion **scales roughly with L^2**.



Dispersion Flattened Fibers

D_w can be adjusted by changing the waveguide geometry, for example, using fibers with multiple cladding layers (such fibers are more difficult to manufacture). It is often desirable to have minimum dispersion over a range of wavelengths. For example, fibers with dispersion of 1–3 ps km^{-1} nm^{-1} over the wavelength range of 1.3–1.6 μm allow for wavelength multiplexing, e.g., using a number of wavelengths as communication channels.



Example: Material, Waveguide, and Chromatic Dispersion

A single-mode fiber has a core of SiO_2-13.5%GeO_2 for which the material and waveguide dispersion coefficients are shown in the figure. This fiber is excited from a 1.5 μm laser source with a linewidth $\Delta\lambda_{1/2}$ of 2 nm. What is the dispersion per km of the fiber if the core diameter 2*a* is 8 μm? What should be the core diameter for zero chromatic dispersion at λ = 1.5 μm?

At λ = 1.5 μm, D_m = +10 ps km^{-1} nm^{-1}

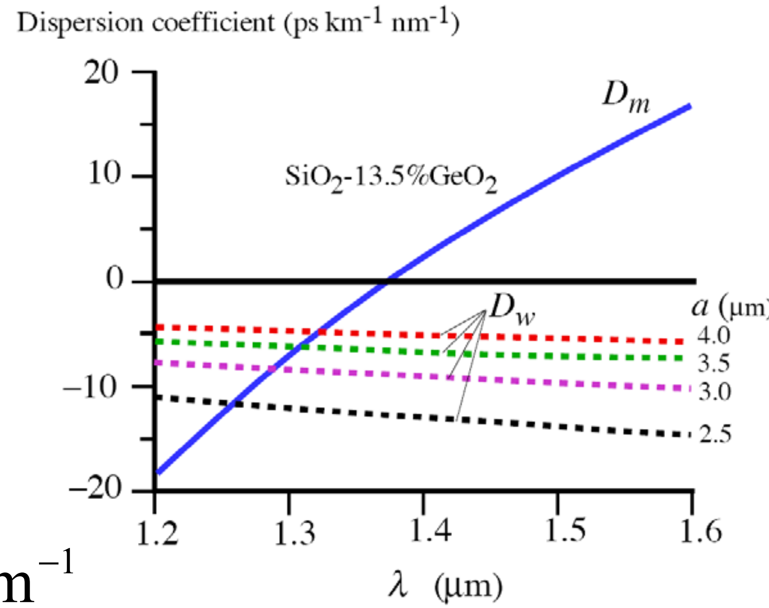
With *a* = 4 μm, D_w = –6 ps km^{-1} nm^{-1}

The chromatic dispersion coefficient is

$$D_{ch} = D_m + D_w = 10 - 6 = 4\text{pskm}^{-1}\text{nm}^{-1}$$

The chromatic dispersion is

$$\Delta\tau_{1/2}/L = |D_{ch}|\Delta\lambda_{1/2}$$
$$= (4\text{pskm}^{-1}\text{nm}^{-1})(2\text{nm}) = 8\text{pskm}^{-1}$$



The chromatic dispersion will be zero at 1.5 μm when $D_w = -D_m$ or when D_w = –10 ps km^{-1} nm^{-1}. The core radius should therefore be about 3 μm. **The dispersion is zero only at one wavelength**.

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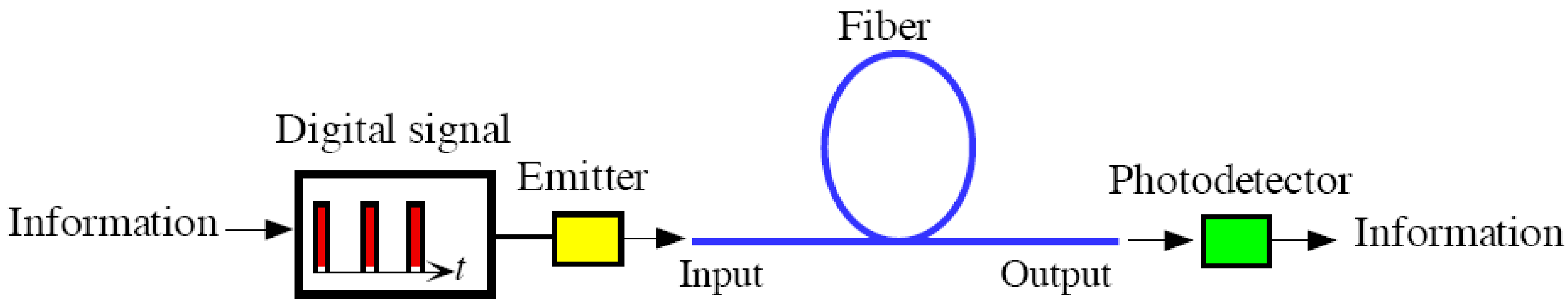
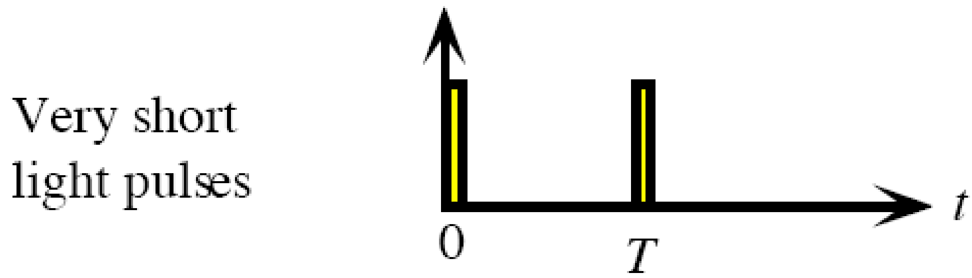
Bit Rate and Dispersion

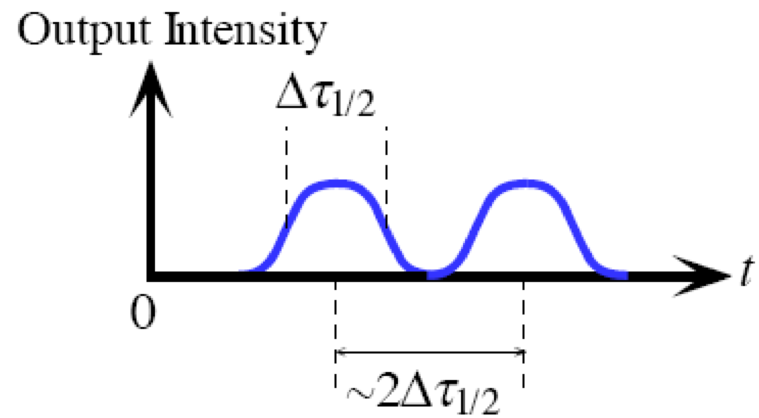


In digital communications, signals are generally sent as light pulses along an optical fiber. Information is first converted to an electrical signal in the form of pulses that represent bits of information. The electrical signal drives a laser diode whose light output is coupled into a fiber for transmission. The light output at the destination end of the fiber is coupled to a photodetector that converts the light signal back to an electrical signal. The information bits are then decoded from this electrical signal.

Engineers are interested in the maximum rate at which the digital data can be transmitted along the fiber. This rate is called the **bit rate capacity *B*** (bits per second) of the fiber.

Bit Rate and Dispersion

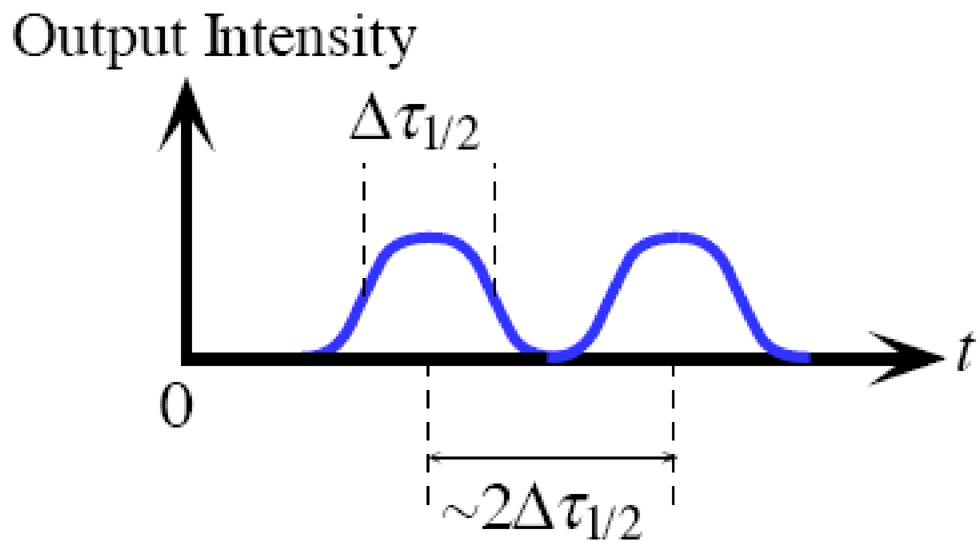




Suppose we feed light pulses of short duration into the fiber. The output pulses will be broadened due to various dispersion mechanisms. The dispersion is typically measured between half-power (or intensity) points and is called **full width at half power (FWHP)**, or **full width at half maximum (FWHM)**.

To clearly distinguish two consecutive pulses, that is no **intersymbol interference**, requires that they be separated from peak to peak by at least $2\Delta\tau_{1/2}$ (intuitively).

Bit Rate and Dispersion



There are two types of bit rates. One is called the **return-to-zero (RZ) bit rate**, for which a pulse representing the binary information 1 must return to zero before the next binary information. The other is called the **non-return-to-zero (NRZ) bit rate**, for which two consecutive binary 1 pulses don't have to return to zero in between, that is, the two pulses can be brought closer.

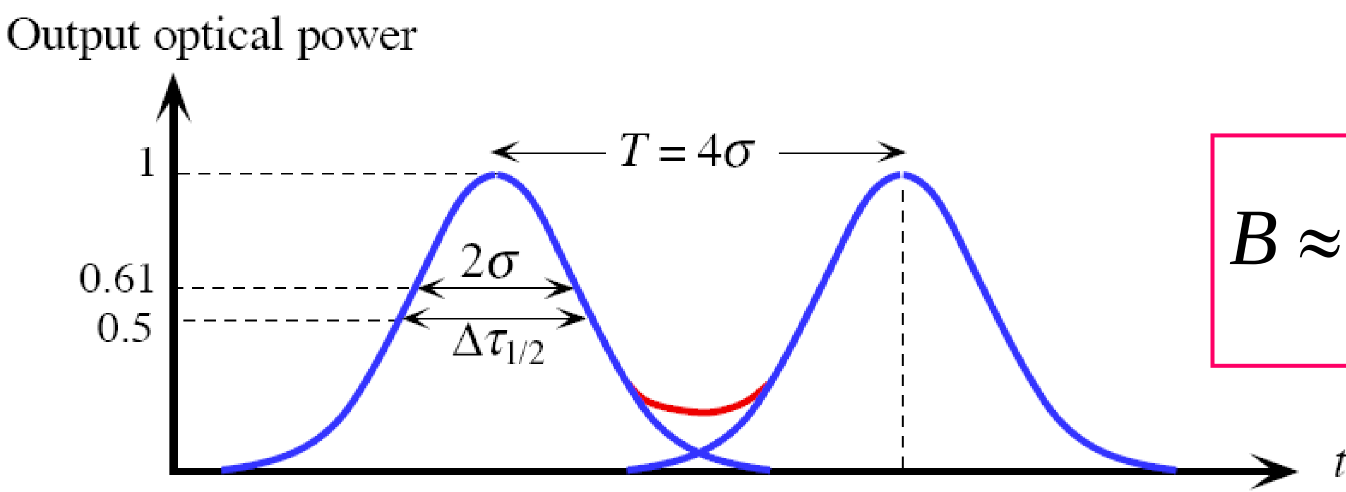
In most cases we refer to the RZ bit rate.

Bit Rate and Dispersion

The maximum bit rate depends on the input pulse shape, fiber dispersion characteristics (hence the output pulse shape), and the modulation scheme of information bits. For Gaussian output light pulses $h(t)$ centered at 0:

$$h(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{t^2}{2\sigma^2}}$$

Standard deviation $\sigma = 0.425\Delta\tau_{1/2}$



$$B \approx \frac{0.25}{\sigma} = \frac{0.59}{\Delta\tau_{1/2}}$$

Bit Rate and Dispersion

Dispersion increases with fiber length *L* and also with the wavelength range of the source, $\sigma_\lambda = 0.425\Delta\lambda_{1/2}$. It is therefore more appropriate to specify the product of the bit rate *B* and the fiber length *L* at the operating wavelength.

$$\mathrm{BL} \approx \frac{0.25\mathrm{L}}{\sigma_{\text{output}}} = \frac{0.25\mathrm{L}}{\mathrm{L}\left|\mathrm{D}_{\text{ch}}\right|\sigma_{\lambda,\text{input}}} = \frac{0.25}{\left|\mathrm{D}_{\text{ch}}\right|\sigma_{\lambda,\text{input}}}$$

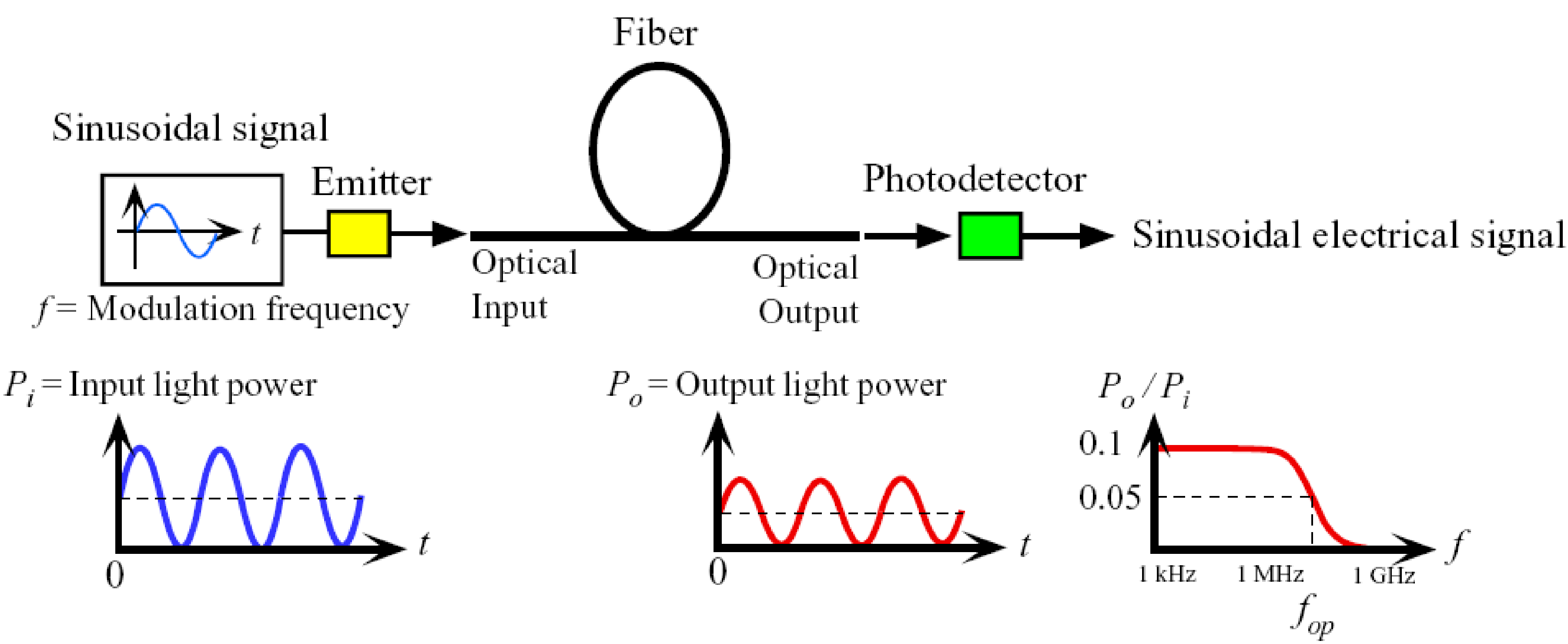
BL is a characteristic of the fiber, through D_{ch}, and also of the wavelength range of the source. In specifications, the fiber length is taken as 1 km and its unit is therefore Gb s^{-1} km.

When both chromatic (intramodal) and intermodal dispersion are present and needed to be taken into account. The overall dispersion can be found according to

$$\sigma_{\text{overall}}{}^2 = \sigma_{\text{intermodal}}{}^2 + \sigma_{\text{intramodal}}{}^2$$

Optical and Electrical Bandwidth

The input light intensity into the fiber can be modulated to be sinusoidal. The light output intensity at the fiber destination should also be sinusoidal with a phase shift due to the time it takes for the signal to travel along the fiber. We can determine the transfer characteristics of the fiber by feeding in light intensity signals with various frequencies.

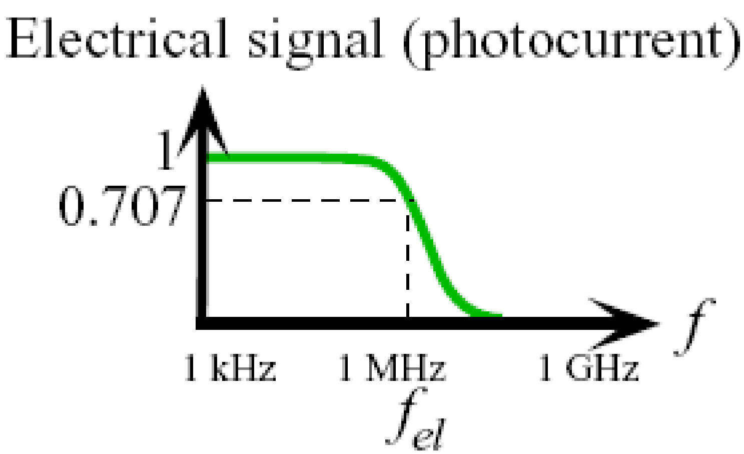


Optical and Electrical Bandwidth

The response, as defined by P_o/P_i, is flat and falls with frequency when the frequency becomes too large so that dispersion effects smear out the light at the output. The frequency f_{op} at which the output intensity is 50% below the flat region defines the **optical bandwidth** of the fiber and hence the useful frequency range in which modulated optical signals can be transferred along the fiber. For Gaussian output light pulses, we have

$$f_{op} \approx 0.75B \approx \frac{0.19}{\sigma}$$

The electrical signal from the photodetector (current or voltage) is proportional to the fiber output light intensity. The **electrical bandwidth**, f_{el}, is usually defined as the frequency at which the electrical signal is 70.7% of its low frequency value.



Relationship between dispersion parameters, maximum bit rates, and bandwidths

Dispersed output pulse shape	FWHM, $\Delta\tau_{1/2}$	B (RZ)	B' (NRZ)	f_{op}	f_{el}
Gaussian with standard deviation σ	$\sigma = 0.425\Delta\tau_{1/2}$	$0.25/\sigma$	$0.5/\sigma$	$0.75B = 0.19/\sigma$	$0.71f_{op} = 0.13/\sigma$
Rectangular with full width ΔT	$\sigma = 0.29\Delta T = 0.29\Delta\tau_{1/2}$	$0.25/\sigma$	$0.5/\sigma$	$0.69B = 0.17/\sigma$	$0.73f_{op} = 0.13/\sigma$

Example: Bit Rate and Dispersion

Consider an optical fiber with a chromatic dispersion coefficient 8 ps km^{-1} nm^{-1} at an operating wavelength of 1.5 μm. Calculate the bit rate-distance product (*BL*), and the optical and electrical bandwidths for a 10 km fiber if a laser diode source with a FWHP linewidth $\Delta\lambda_{1/2}$ of 2 nm is used.

The FWHP dispersion at the output side is

$$\Delta\tau_{1/2}/L = |D_{ch}|\Delta\lambda_{1/2} = \left(8\text{pskm}^{-1}\text{nm}^{-1}\right)\left(2\text{nm}\right) = 16\text{pskm}^{-1}$$

Assume a Gaussian light pulse shape, the RZ bit rate-distance product is

$$BL = \frac{0.25L}{\sigma} = \frac{0.25L}{0.425\Delta\tau_{1/2}} = \frac{0.25}{0.425\times 16} = 36.8\text{Gbs}^{-1}\text{km}$$

The optical and electrical bandwidths for a 10 km distance are

$$f_{op} = 0.19/\sigma = 0.19/\left(0.425\Delta\tau_{1/2}\right) = 0.19/\left(0.425\times 16\times 10\right) = 2.8\text{GHz}$$

$$f_{el} = 0.71 f_{op} = 0.71\times 2.8 = 2.0\text{GHz}$$

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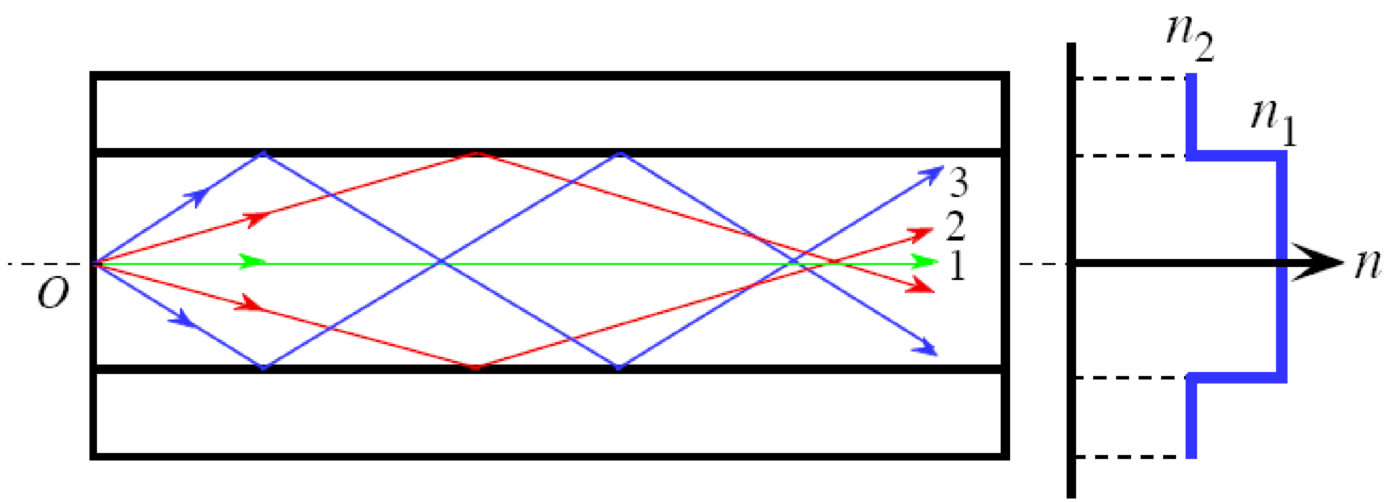
Drawback of Single Mode Step Index Fibers

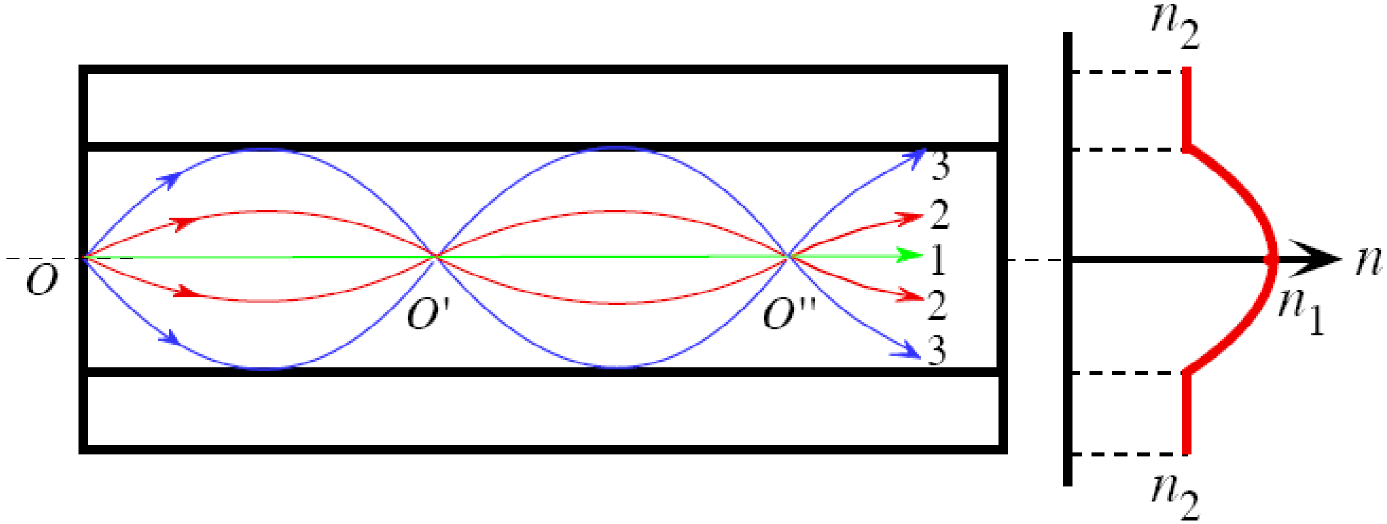


Single mode step index fibers have small *NA* and the amount of light coupled into a fiber is limited.

Multimode step index fibers have large *NA* and core diameters, which allow for more light power launched into a fiber. However, they suffer from intermodal dispersion.

Intuitively, those rays that experience less reflections will arrive at the end of the fiber earlier.

Graded Index (GRIN) Fibers



In the **graded index (GRIN)** fiber, the refractive index is not constant within the core but decreases from n_1 at the center, as a power law, to n_2 at the cladding. Such a refractive index profile is capable of minimizing intermodal dispersion.

Intuitively, the velocity, c/n, is not constant and increases away from the center. A ray such as 2 that has a longer path than ray 1 experiences a larger velocity during a part of its journey to enable it to catch up with ray 1. Similarly, ray 3 experiences a larger velocity than ray 2 during part of its propagation to catch up with ray 2.

Graded Index (GRIN) Fibers

The refractive index profile can be described by a power law with an index γ, which is called the **profile index** (or the **coefficient of index grating**).

$$\begin{cases} n = n_1\left[1 - 2\Delta\left(r/a\right)^{\gamma}\right]^{1/2} & r < a \\ n = n_2 & r = a \end{cases}$$

The intermodal dispersion is minimized when (Δ is small)

$$\gamma = \frac{4+2\Delta}{2+3\Delta} = \frac{2+\Delta}{1+\frac{3}{2}\Delta} \approx \left(2+\Delta\right)\left(1-\frac{3}{2}\Delta\right) \approx 2\left(1-\Delta\right)$$

With the optimal profile index, the dispersion in the output light pulse per unit length is given by

$$\boxed{\frac{\sigma_{\text{intermodal}}}{L} \approx \frac{n_1}{20\sqrt{3}c}\Delta^2}$$

TABLE 2.3 Comparison of typical characteristics of multimode step-index, single-mode step-index, and graded-index fibers. (Typical values combined from various sources.)

Property	Multimode step-index fiber	Single-mode step-index fiber	Graded index fiber
$\Delta = (n_1 - n_2)/n_1$	0.02	0.003	0.015
Core diameter (μm)	100	8.3 (MFD = 9.3 μm)	62.5
Cladding diameter (μm)	140	125	125
NA	0.3	0.1	0.26
Bandwidth × distance or Dispersion	20 − 100 MHz km.	$<$3.5 ps km^{-1} nm^{-1} at 1.3 μm $>$100 Gb s^{-1} km in common use	300 MHz km − 3 GHz km at 1.3 μm at 1.3 μm
Attenuation of light	4 − 6 dB km^{-1} at 850 nm 0.7 − 1 dB km^{-1} at 1.3 μm	1.8 dB km^{-1} at 850 nm 0.34 dB km^{-1} at 1.3 μm 0.2 dB km^{-1} at 1.55 μm	3 dB km^{-1} at 850 nm 0.6 − 1 dB km^{-1} at 1.3 μm 0.3 dB km^{-1} at 1.55 μm
Typical light source	Light emitting diode (LED)	Lasers, single mode injection lasers	LED, lasers
Typical applications	Short haul or subscriber local network communications	Long haul communications	Local and wide-area networks. Medium haul communications

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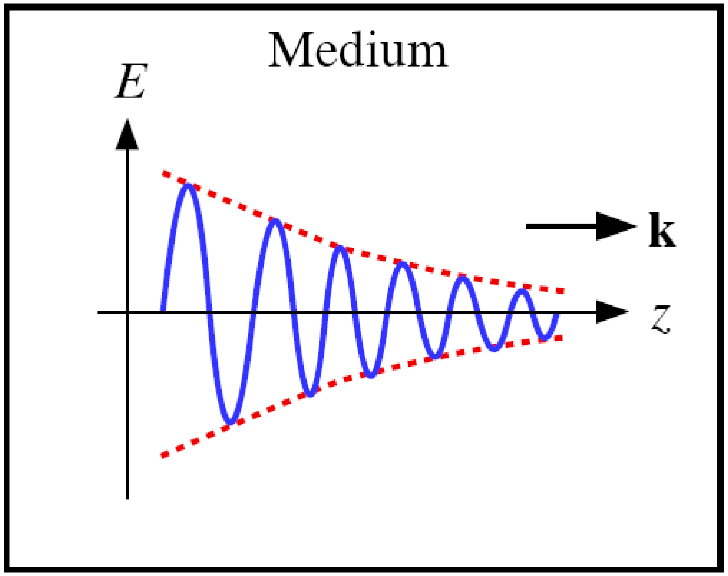
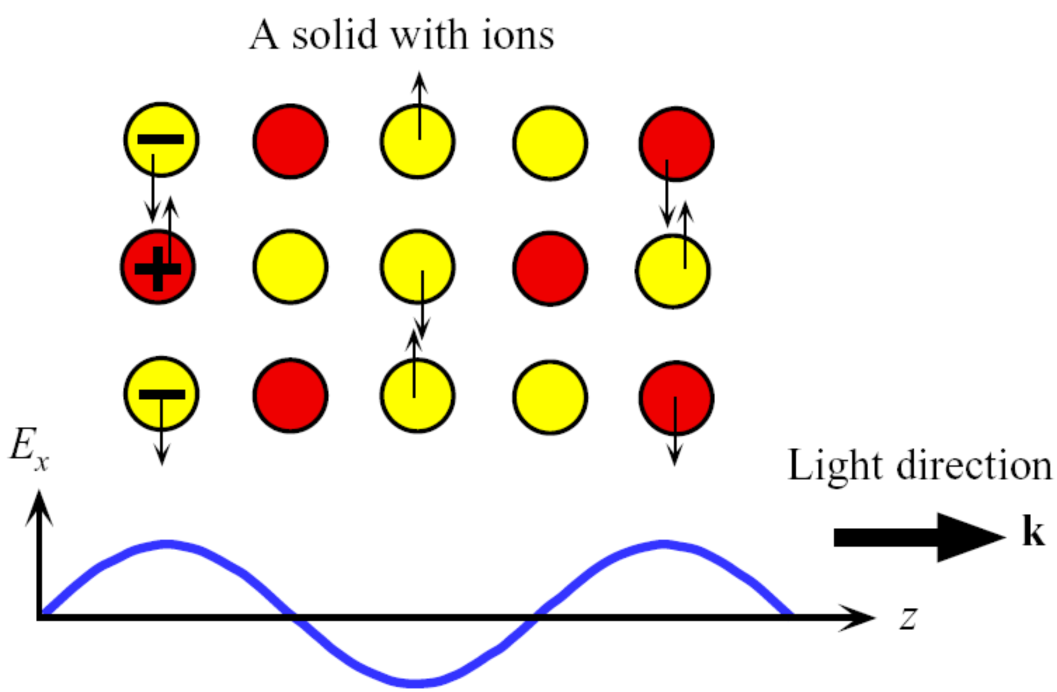
Absorption

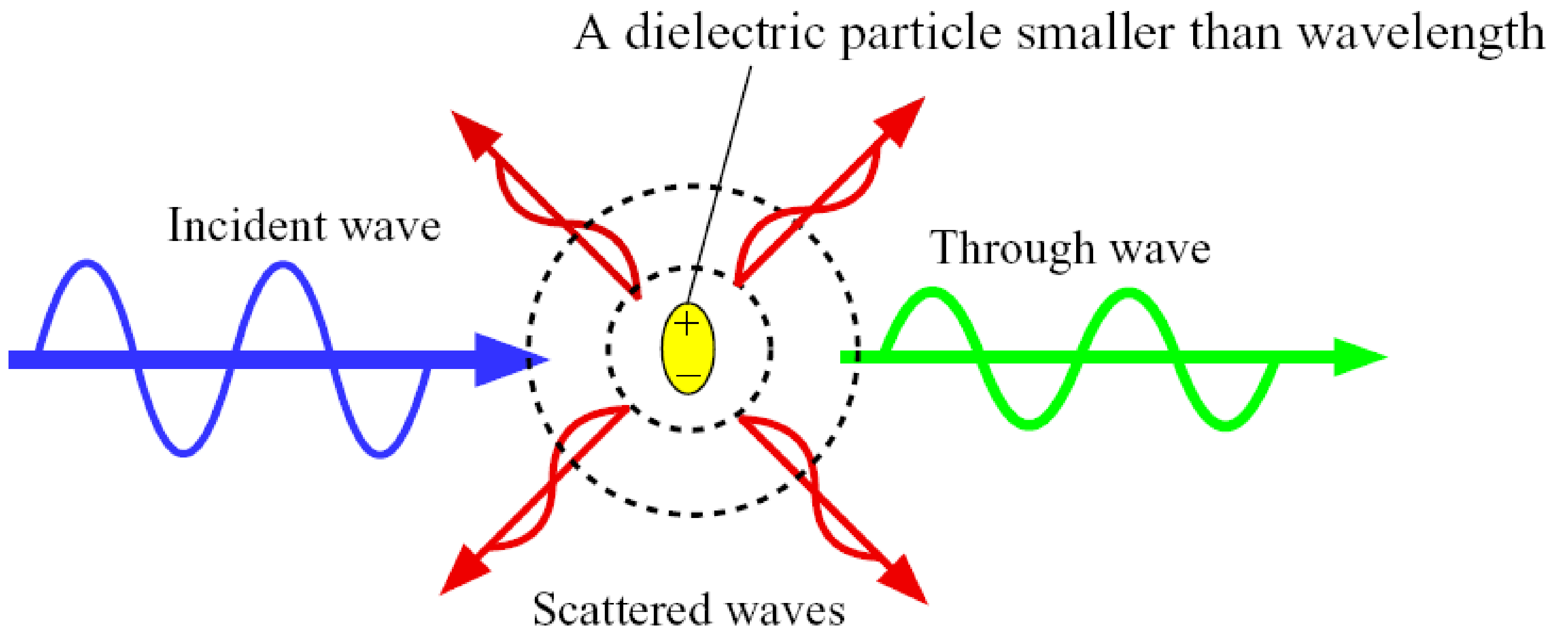


In general, light propagating through a material becomes *attenuated* in the direction of propagation.



In absorption, some of the energy from the propagating wave is converted to other forms of energy, for example, to heat by the generation of lattice vibrations.

Scattering



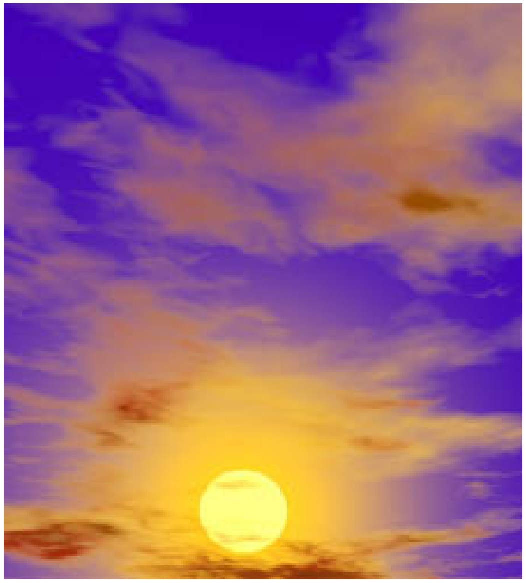
When a propagating wave encounters a small dielectric particle or a small inhomogeneous region whose refractive index is different from the average refractive index of the medium, the field forces dipole oscillations in the dielectric particle or region by polarizing it, leading to the emission of electromagnetic waves in many directions so that a portion of the light energy is directed away from the incident beam.

Scattering

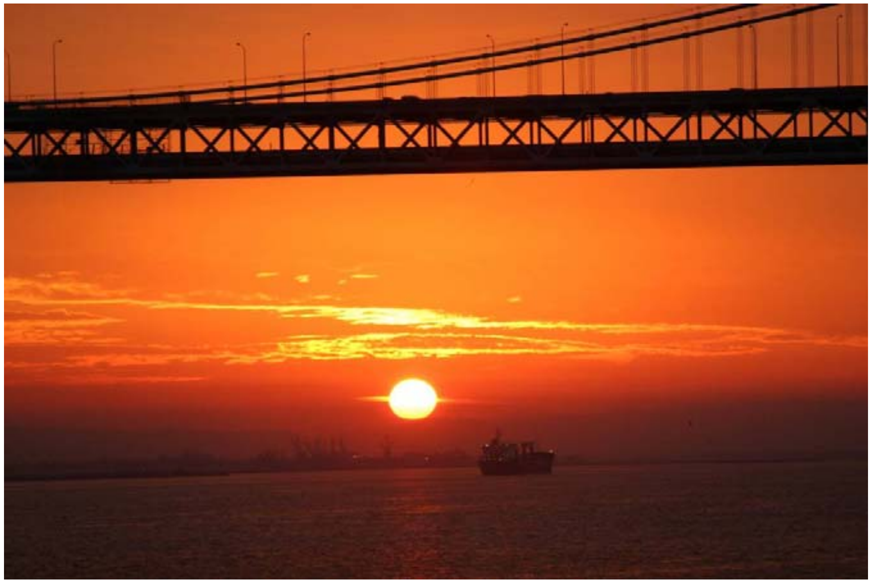
Whenever the size of a scattering region, whether an inhomogeneity or a small particle, is much smaller ($<\lambda/10$) than the wavelength of the incident wave, the scattering process is generally termed **Rayleigh scattering**.

Lord Rayleigh, an English physicist (1877 – 1919) and a Nobel laureate (1904), made a number of contributions to wave physics of sound and optics.

blue sky



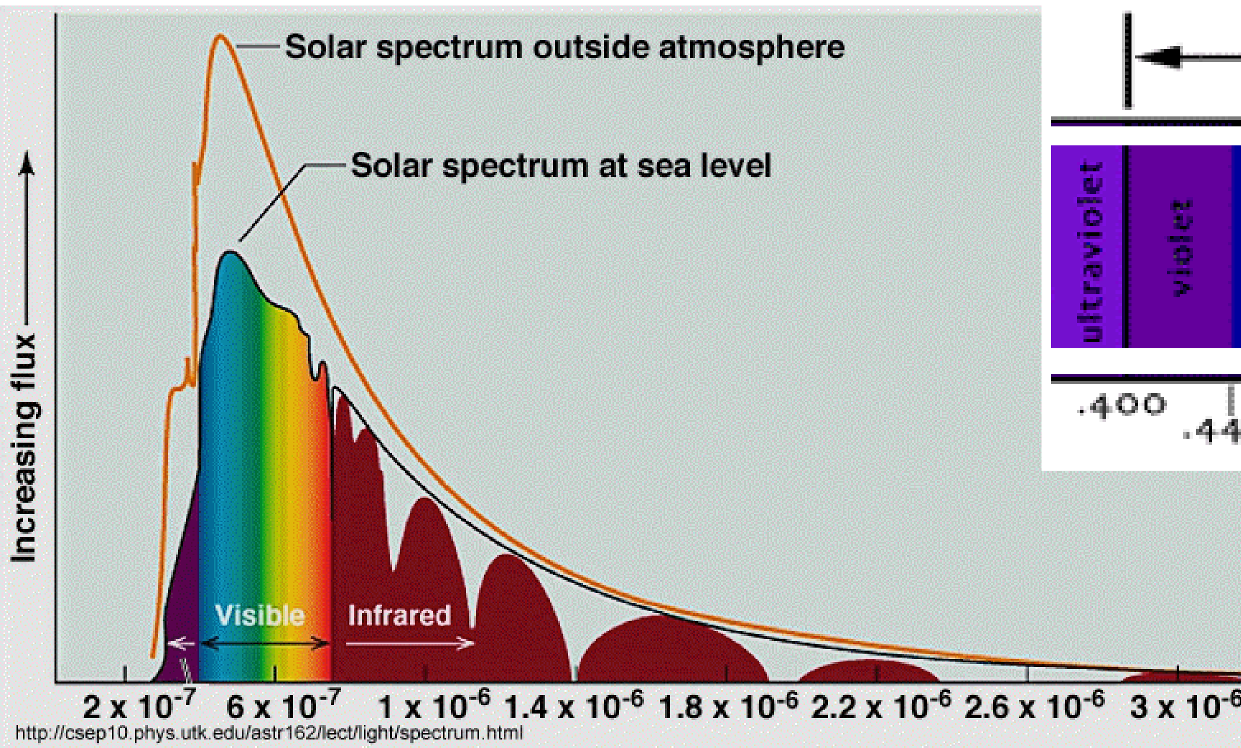
yellow sun

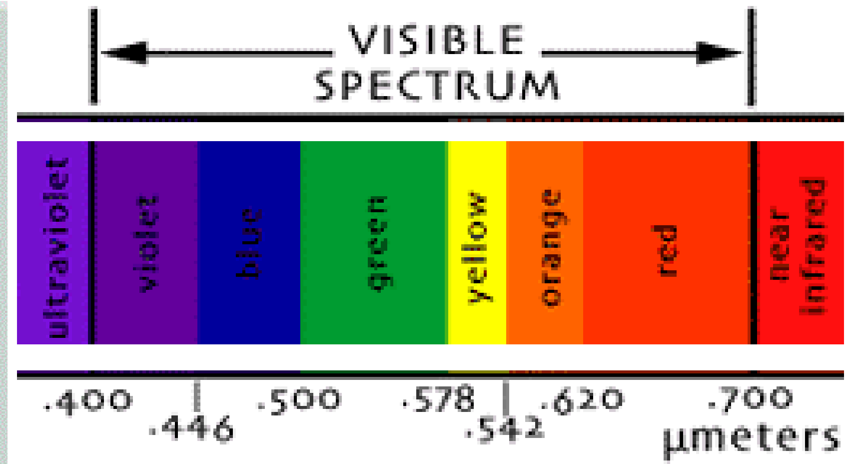


sunrise

sunset

- **Why the sky is blue?**
- **Why does the sun look yellow if we look at the sun directly?**
- **Why does the sky around the sun appear red during sunrise and sunset?**





Rayleigh scattering becomes more severe as the frequency of light increases (the wavelength decreases). Blue light that has a shorter wavelength than red light is scattered more strongly by particles in air.

When we look at the sun directly, it appears yellow because the blue light has been scattered.

When we look at the sky in any direction but the sun, our eyes receive scattered light, which appears blue.

At sunrise and sunset, the rays from the sun have to travel the longest distance through the atmosphere and have the most blue light scattered, which gives the sky around the sun its red color at these times.

Chapter 3: Dielectric Waveguides and Optical Fibers

Symmetric planar dielectric slab waveguides

Modal and waveguide dispersion in planar waveguides

Step index fibers

Numerical aperture

Dispersion in single mode fibers

Bit-rate, dispersion, and optical bandwidth

Graded index optical fibers

Light absorption and scattering

√ Attenuation in optical fibers

Attenuation in Optical Fibers

Assume a fiber of length L. The input optical power is P_{in}. The optical power is attenuated to P_{out} at the end of the fiber. We define an **attenuation coefficient** α for the fiber.

$$dP = -\alpha P dx$$

$$\frac{dP}{P} = -\alpha dx$$

$$\alpha = \frac{1}{L}\ln\left(\frac{P_{in}}{P_{out}}\right)$$

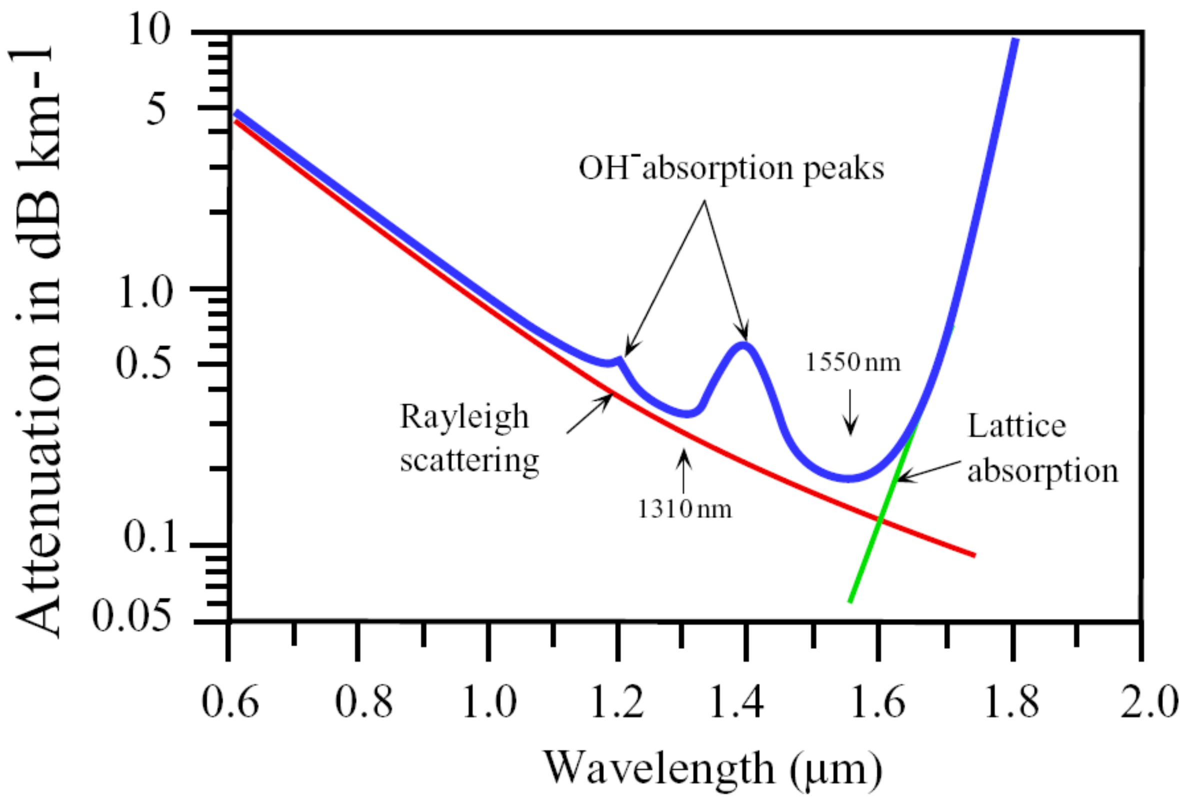
$$P_{out} = P_{in}\exp(-\alpha L)$$

Optical power attenuation in optical fibers is generally expressed in terms of **decibels** per unit length of fiber, typically as dB per km.

$$\alpha_{dB} = \frac{1}{L}10\log\left(\frac{P_{in}}{P_{out}}\right)$$

$$\alpha_{dB} = \frac{10}{\ln(10)}\alpha = 4.34\alpha$$

Attenuation in Optical Fibers



- The sharp increase in attenuation at wavelengths beyond 1.6 μm is due to energy absorption by lattice vibrations of silica.
- Two peaks at 1.4 and 1.24 μm are due to OH^- ions in silica glasses.
- The overall background is due to Rayleigh scattering because of the amorphous structure of silica glasses (impossible to eliminate Rayleigh scattering in glasses).

Attenuation in Optical Fibers

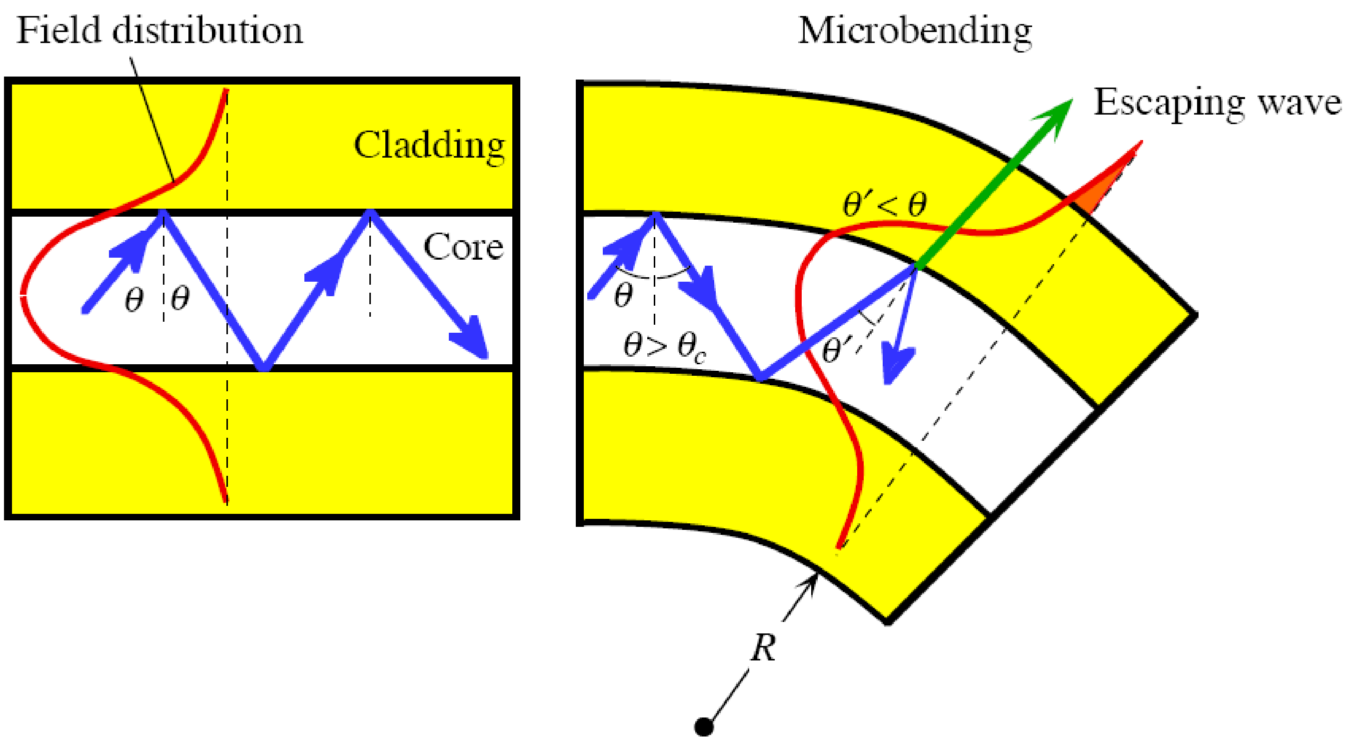
The attenuation α_R in a single component glass due to Rayleigh scattering is approximately given by

$$\alpha_R \approx \frac{8\pi^3}{3\lambda^4}\left(n^2 - 1\right)^2 \beta_T k_B T_f$$

λ is the free space wavelength. T_f is called the **fictive temperature**, at which the liquid structure during the cooling of the fiber is frozen to become the glass structure. β_T is the isothermal compressibility of the glass at T_f.

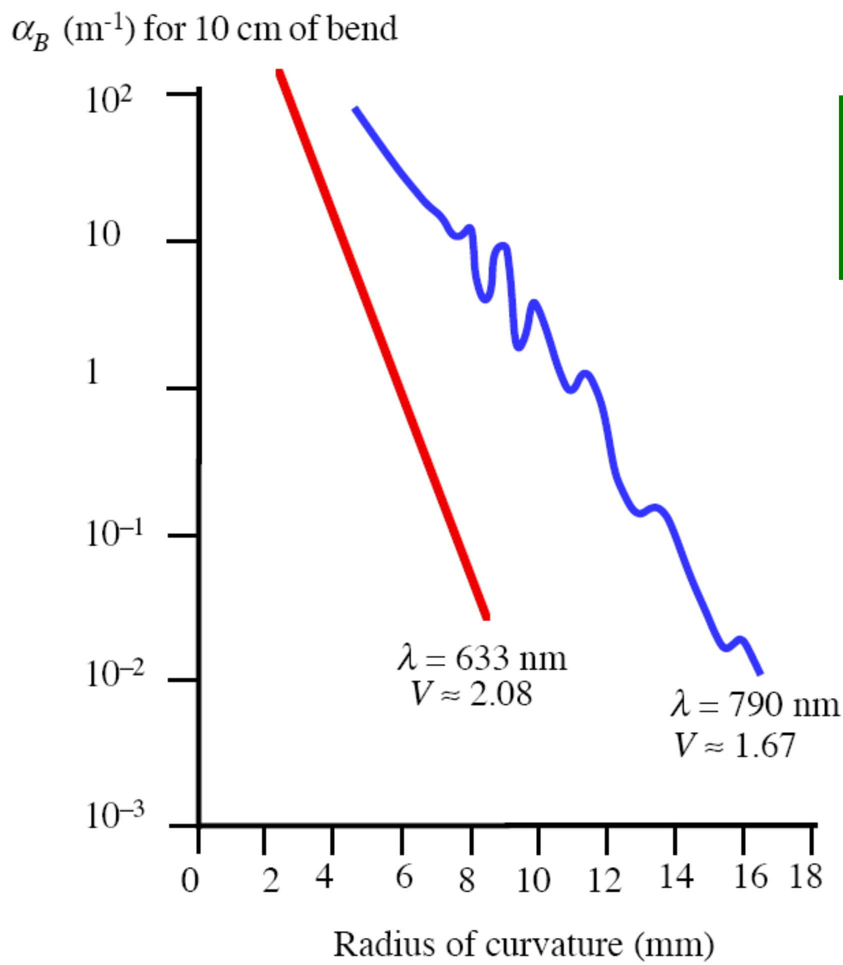
Microbending and Macrobending Losses

Microbending is due to a sharp local bending of the fiber that changes the guide geometry and refractive index profile locally, which leads to some of the light energy radiating away from the guiding direction.



Local bending leads to an increase in the incidence angle, which induces either an increase in the penetration depth into the cladding or a loss of total internal reflection.

Microbending and Macrobending Losses



Measured microbending loss for a 10-cm fiber bent with different amounts of radius of curvature. Single mode fiber with a core diameter of 3.9 μm, cladding radius 48 μm, Δ = 0.00275, $NA \approx 0.11$, $V \approx 1.67$ and 2.08.

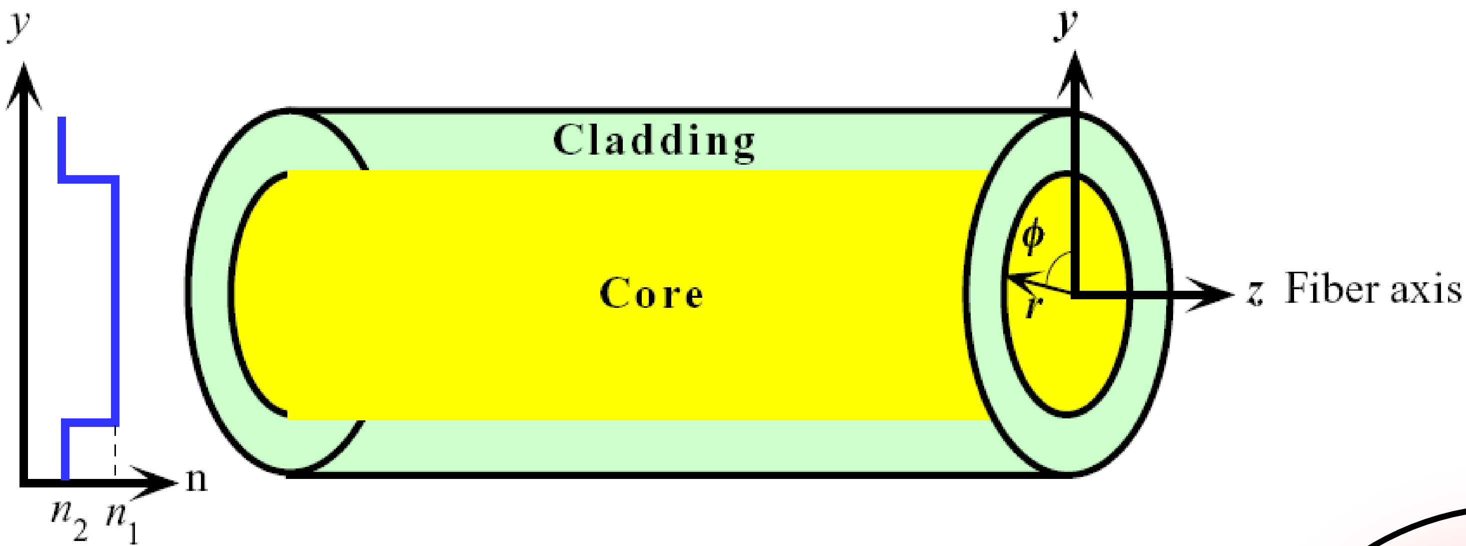
Microbending loss increases sharply with decreasing radius of curvature.

Macrobending loss is due to small changes in the refractive index of the fiber due to induced strain when it is bent during its use, for example, when it is cabled and laid. Typically, when the radius of curvature is close to a few centimeters, macrobending loss crosses over into the regime of microbending loss.

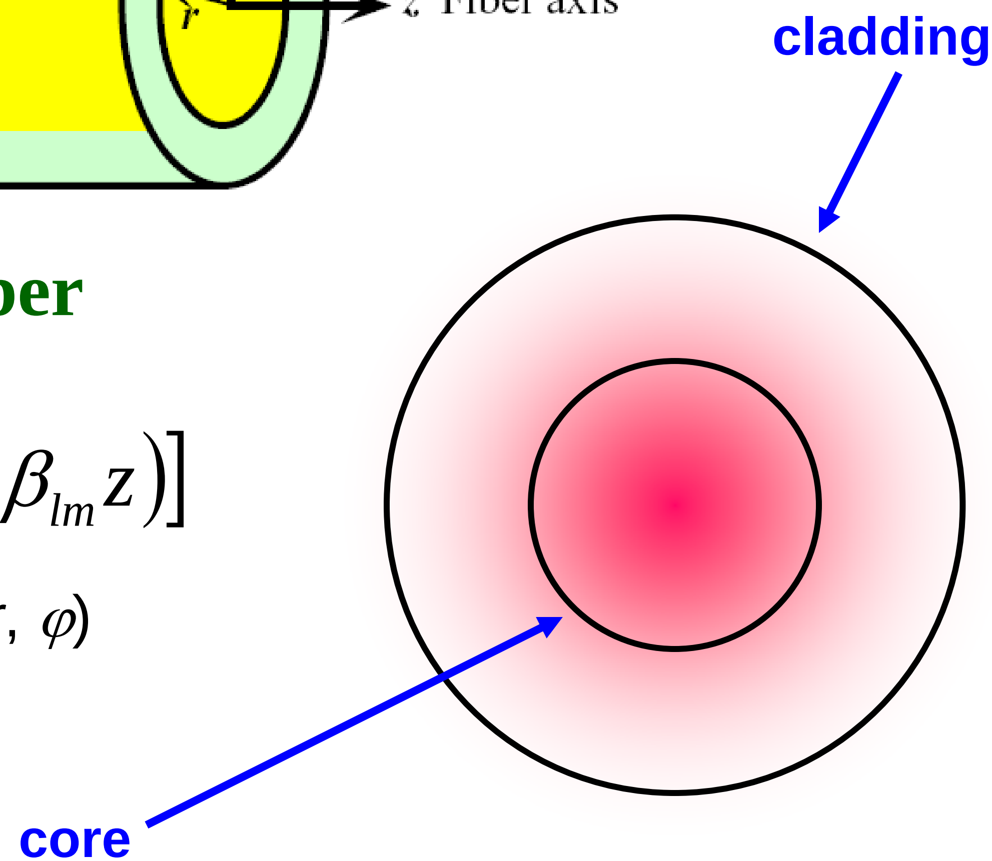
TABLE 2.3 Comparison of typical characteristics of multimode step-index, single-mode step-index, and graded-index fibers. (Typical values combined from various sources.)

Property	Multimode step-index fiber	Single-mode step-index fiber	Graded index fiber
$\Delta = (n_1 - n_2)/n_1$	0.02	0.003	0.015
Core diameter (μm)	100	8.3 (MFD = 9.3 μm)	62.5
Cladding diameter (μm)	140	125	125
NA	0.3	0.1	0.26
Bandwidth × distance or Dispersion	20 − 100 MHz km.	<3.5 ps km^{-1} nm^{-1} at 1.3 μm >100 Gb s^{-1} km in common use	300 MHz km − 3 GHz km at 1.3 μm at 1.3 μm
Attenuation of light	4 − 6 dB km^{-1} at 850 nm 0.7 − 1 dB km^{-1} at 1.3 μm	1.8 dB km^{-1} at 850 nm 0.34 dB km^{-1} at 1.3 μm 0.2 dB km^{-1} at 1.55 μm	3 dB km^{-1} at 850 nm 0.6 − 1 dB km^{-1} at 1.3 μm 0.3 dB km^{-1} at 1.55 μm
Typical light source	Light emitting diode (LED)	Lasers, single mode injection lasers	LED, lasers
Typical applications	Short haul or subscriber local network communications	Long haul communications	Local and wide-area networks. Medium haul communications

How Small A Fiber Can Be?



step index fiber



$$E_{LP} = E_{lm}(r,\varphi)\exp[j(\omega t - \beta_{lm} z)]$$

An electric field pattern $E_{lm}(r, \varphi)$ propagates along the fiber.

LP_{01} mode

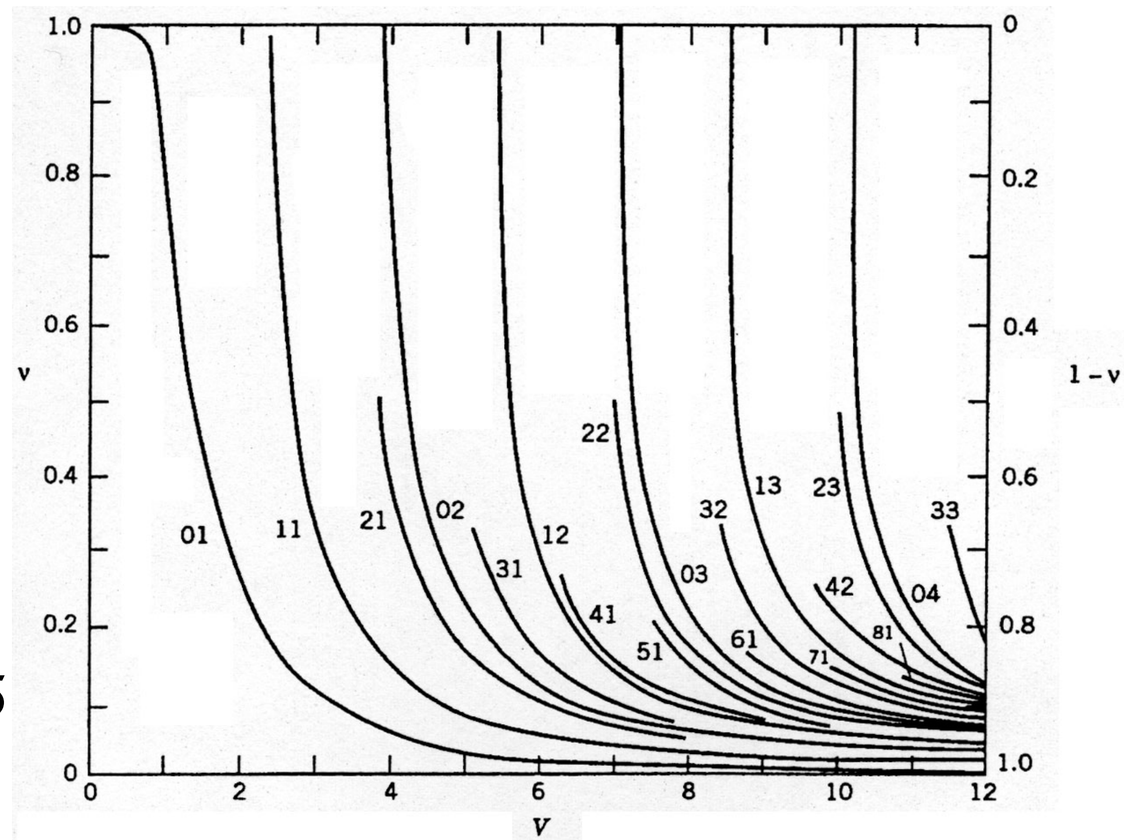
How Small A Fiber Can Be?

Due to the presence of the evanescent wave in the cladding, not all of the optical power propagating along the fiber is confined inside the core. The extent to which a propagating mode is confined to the fiber core can be measured by the ratio of the power carried in the cladding to the total power that propagates in the mode.

$$\nu = \frac{P_{\text{cladding}}}{P_{\text{core}} + P_{\text{cladding}}}$$

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2}$$

$$V_{\text{cut-off}}\,(\text{single mode}) = 2.405$$



How Small A Fiber Can Be?

Consider a step index fiber with a silica core (n_1 = 1.45). The cladding of this fiber is simply air (n_2 = 1.0). The laser light source is from an argon ion laser with a wavelength of 514.5 nm (green light). If the radius of this fiber is equal to the light wavelength (a = 514.5 nm), then

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} = 2\pi\sqrt{1.45^2 - 1.0^2} = 6.6 \qquad \boxed{\nu = 0.02}$$

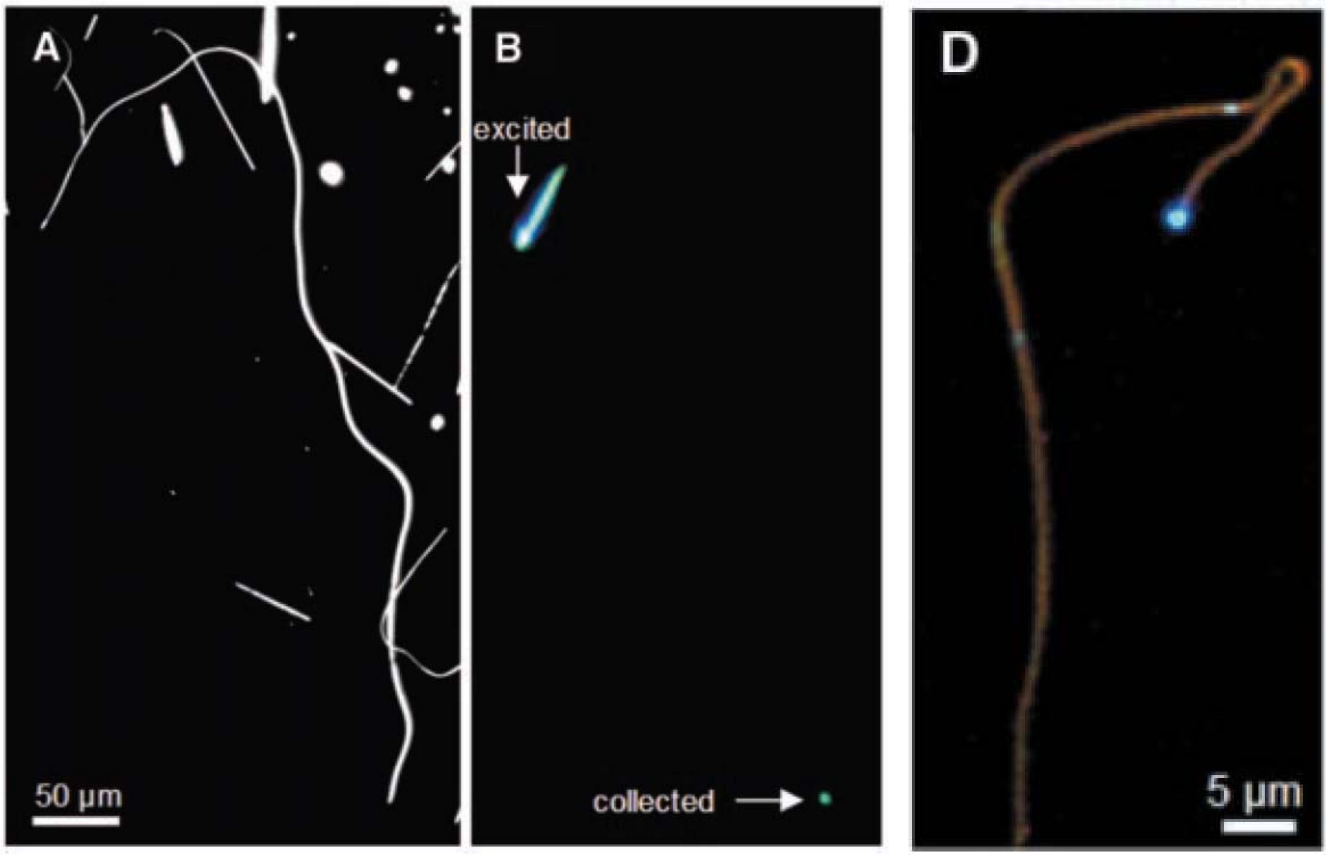
If the diameter of this fiber is equal to the light wavelength (2a = 514.5 nm), then

$$V = 3.3 \qquad \boxed{\nu = 0.1}$$

If the diameter of this fiber is equal to half the light wavelength (4a = 514.5 nm), then

$$V = 1.6 \qquad \boxed{\nu = 0.5}$$

Subwavelength Waveguides



SnO_2 nanoribbons
350 nm wide
245 nm thick
λ_0 ~525 nm

M. Law, D. J. Sirbuly, J. C. Johnson, J. Goldberger, R. J. Saykally, P. D. Yang, *Science* **2004**, 305, 1269.

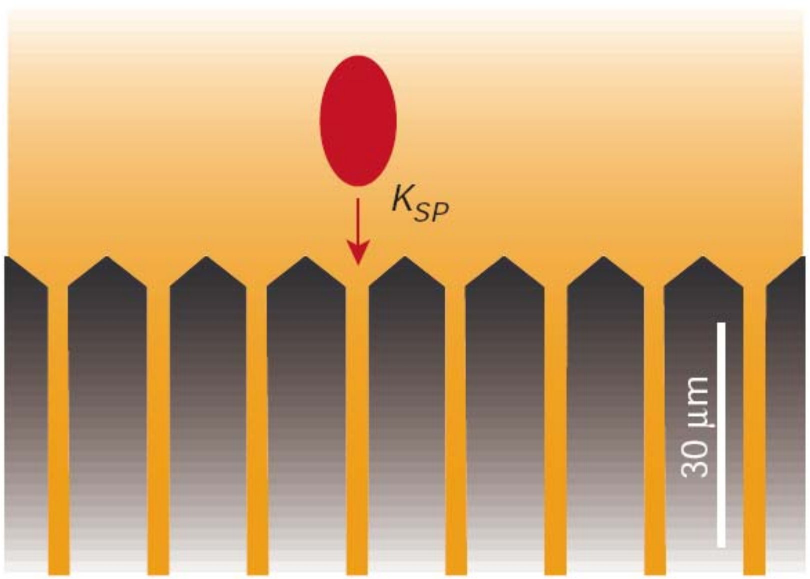
For such small fibers, difficulties lie in fabrication, precise positioning, and light coupling. Propagation loss might not be a problem if our goal is to fabricate high-density integrated photonic circuits.

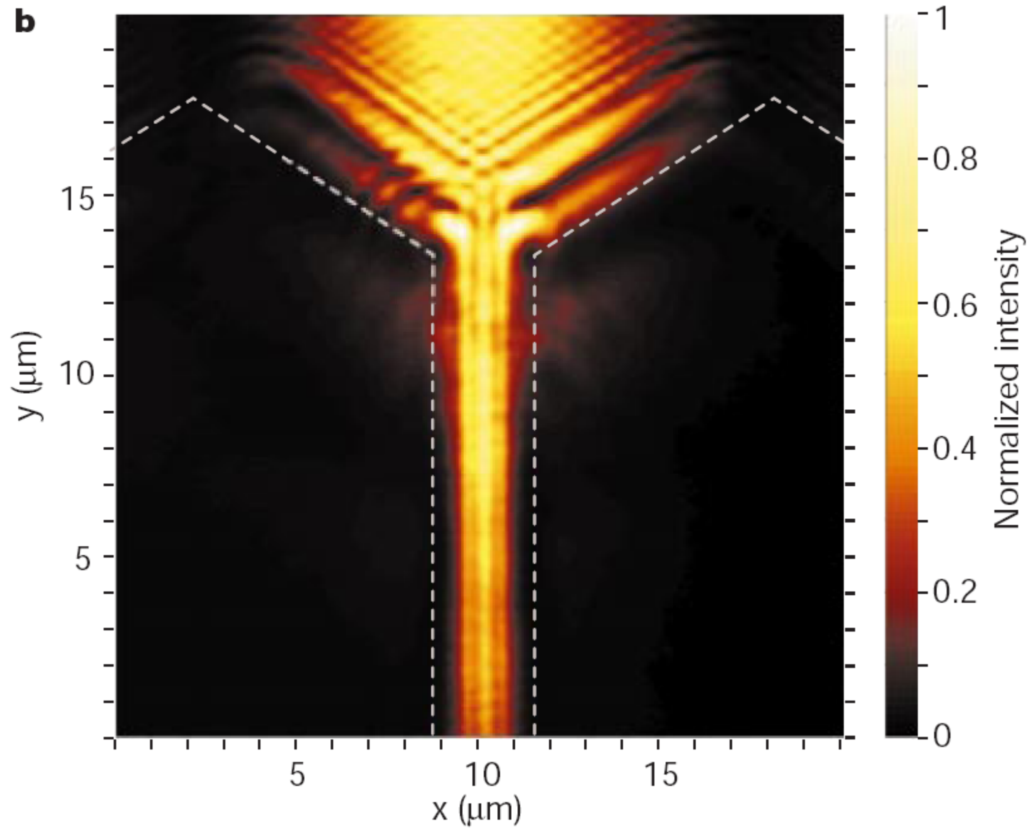
Plasmonic Waveguides

There has been great interest in the use of optical interconnects to exchange digital information between electronic microprocessors. One severe limit on the integration of optical and electronic circuits is their respective sizes. Electronic circuits can be fabricated with sizes below 100 nm, while the minimum sizes of optical structures are limited by optical diffraction to the order of 1000 nm.

Surface plasmon-based photonics, plasmonics, may offer a solution to this dilemma. The resonant interaction between the electron oscillations near the metallic surface and the electromagnetic field of light creates surface plasmons, which are bound to the metallic surface with exponentially decaying fields in both neighboring media. This feature of surface plasmons provides the possibility of the localization and guiding of light in sub-wavelength metallic structures, and it can be used to construct miniaturized optoelectronic circuits with sub-wavelength components.

Plasmonic Waveguides —– Thin Metal Films

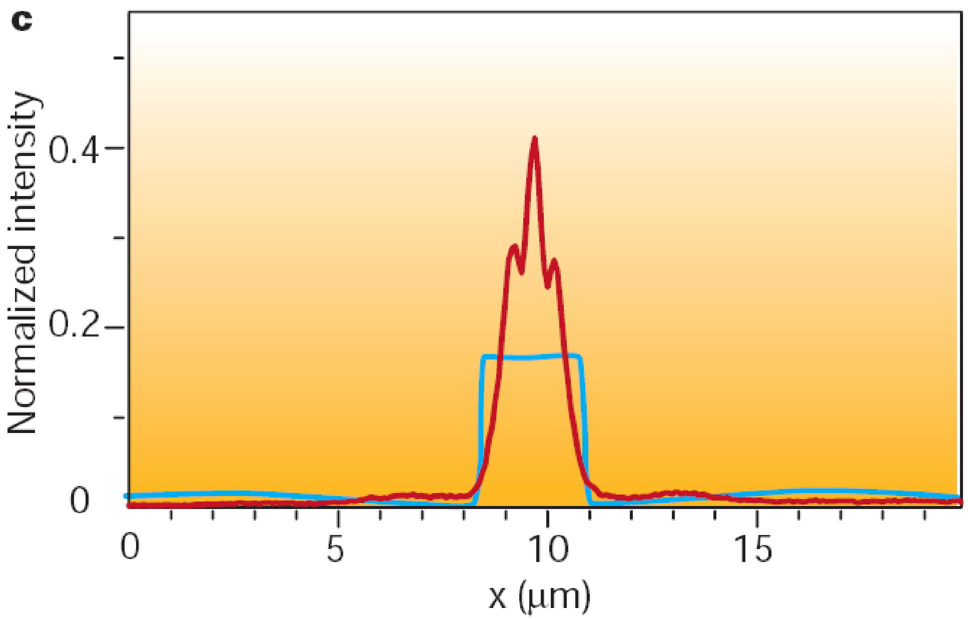




40-nm thick and 2.5-μm wide Au stripes lying on a glass substrate. One of the surface plasmon eigen modes is excited by total internal reflection illumination at a wavelength of 800 nm.

W. L. Barnes, A. Dereux, T. W. Ebbesen, *Nature* **2003**, *424*, 824; E. Ozbay, *Science* **2006**, *311*, 189.

Plasmonic Waveguides —– Thin Metal Films



The guided surface plasmon mode has three maxima.
Both the height and the square root of the waveguide cross section features a sub-wavelength size, implying that the guided mode is essentially bound to the metal surface rather than being a standing wave confined inside the metal volume.

Advantages: (1) Optical signals and electric currents can be carried through the same thin metal guides. (2) Light can be transported along these metallic thin films over a distance from several micrometers to several millimeters, depending the sizes of metallic structures and light wavelengths.

Disadvantages: There exists ohmic resistive heating within the metal, which limits the maximum propagation length within these structures.

Plasmons

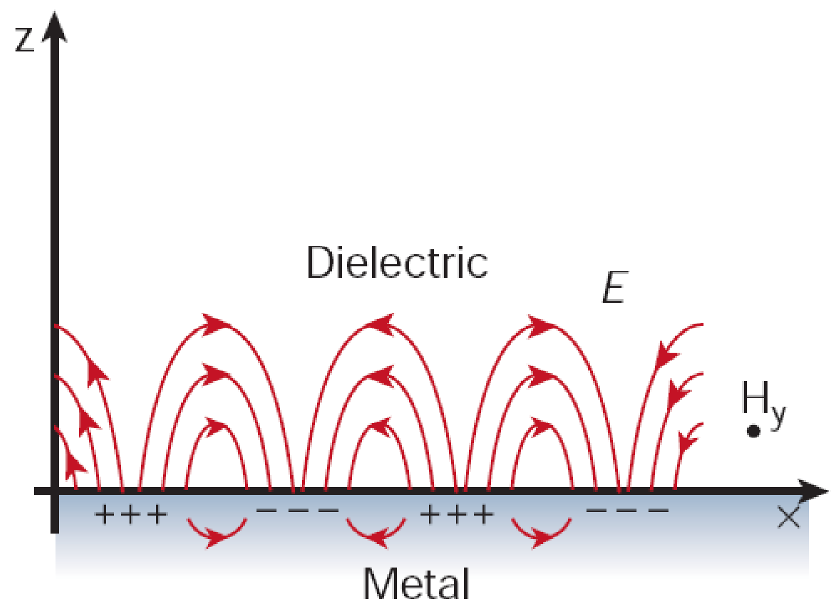
Plasmonic behavior is a physical concept that describes the collective oscillation of conduction electrons in a metal. Many metals can be treated as free-electron systems whose electronic and optical properties are determined by the conduction electrons alone. In the Drude–Lorentz model, such a metal is denoted as a plasma, because it contains equal numbers of positive ions (fixed in position) and conduction electrons (free and highly mobile). Under the irradiation of an electromagnetic wave, the free electrons are driven by the electric field to coherently oscillate at a *plasma frequency* of ω_p relative to the lattice of positive ions. For a bulk metal with infinite sizes in all three dimensions, ω_p is related to the number density of electrons. Quantized plasma oscillations are called *plasmons*.

$$\omega_p = \left(\frac{Ne^2}{\varepsilon_0 m_e} \right)^{1/2}$$

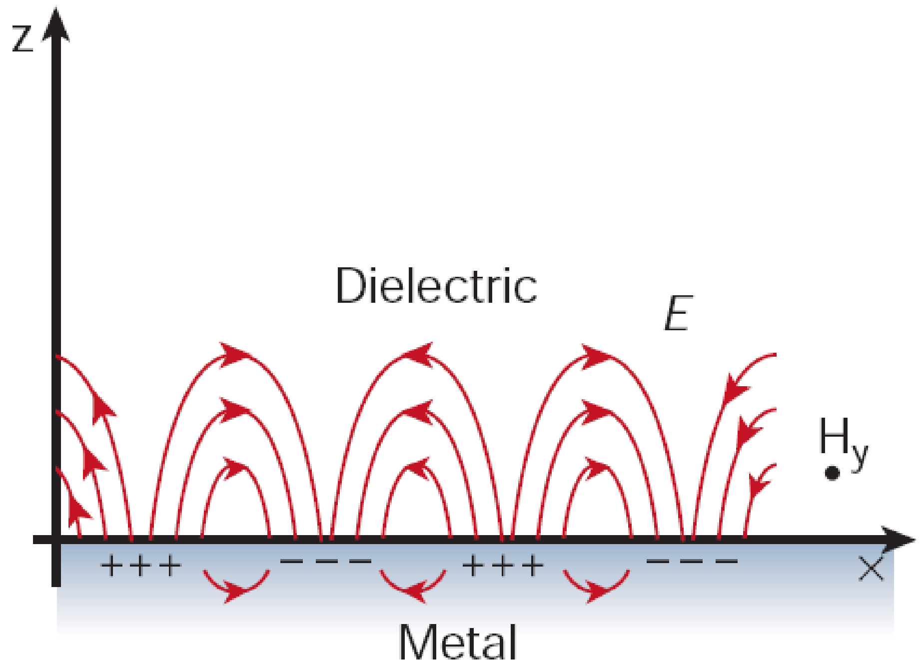
Metal	Theoretical λ_p
Na	200 nm
Al	77 nm
Ag	140 nm
Au	140 nm

Propagating Surface Plasmons (PSPs)

In reality, metallic structures are of finite dimensions and are surrounded by materials with different dielectric properties. Since an EM wave impinging on a metal surface only has a certain penetration depth, just the electrons on the surface are the most significant. Their collective oscillations are properly termed *surface plasmon polaritons* (SPPs), but are often referred to as surface plasmons (SPs). For a metal–vacuum interface, application of the boundary condition results in an SP mode of $\omega_p/2^{1/2}$ in frequency. Such an SP mode represents a longitudinal surface charge density wave that can travel across the surface. For this reason, these SPs are also widely known as *propagating* SPs (or PSPs).

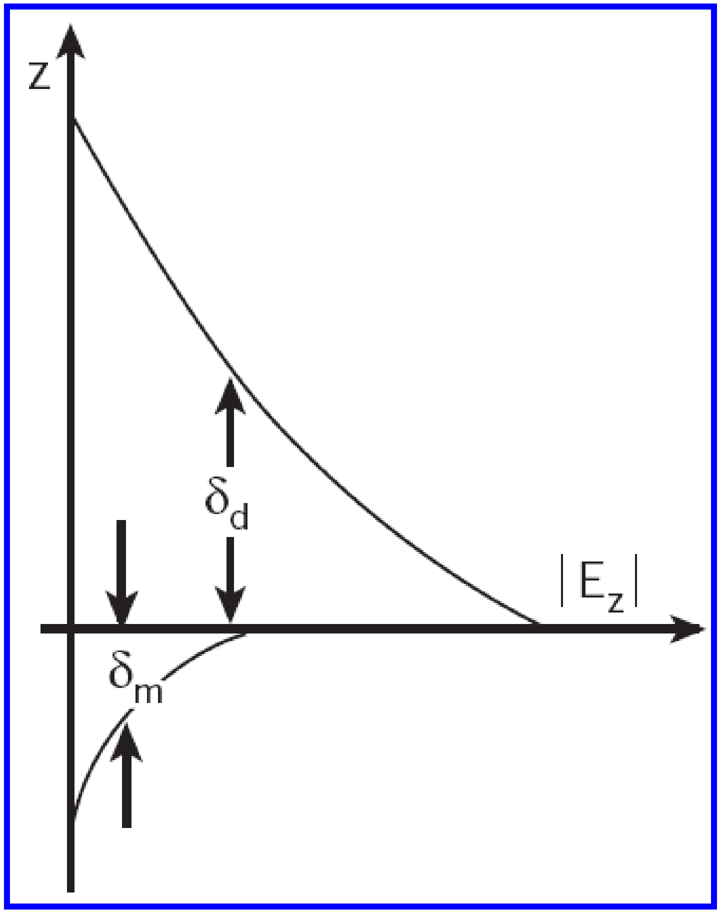


Propagating Surface Plasmons



Propagating surface plasmons at a metal-dielectric interface have a combined electromagnetic wave and surface charge character. There is an enhanced field component perpendicular to the interface and decaying exponentially away from the interface. This evanescent wave reflects the bound, non-radiative nature of surface plasmons and prevents power from propagating away from the interface.

Decay Lengths of PSPs



The decay length, δ_d, in the dielectric medium above the metal, typically air or glass, is of the order of half the wavelength of involved light.

The decay length in the metal, δ_m, is typically between one and two orders of magnitude smaller than the wavelength involved, which highlights the need for good control of fabrication of surface plasmon-based devices at the nanometer scale.

Propagation Length of PSPs

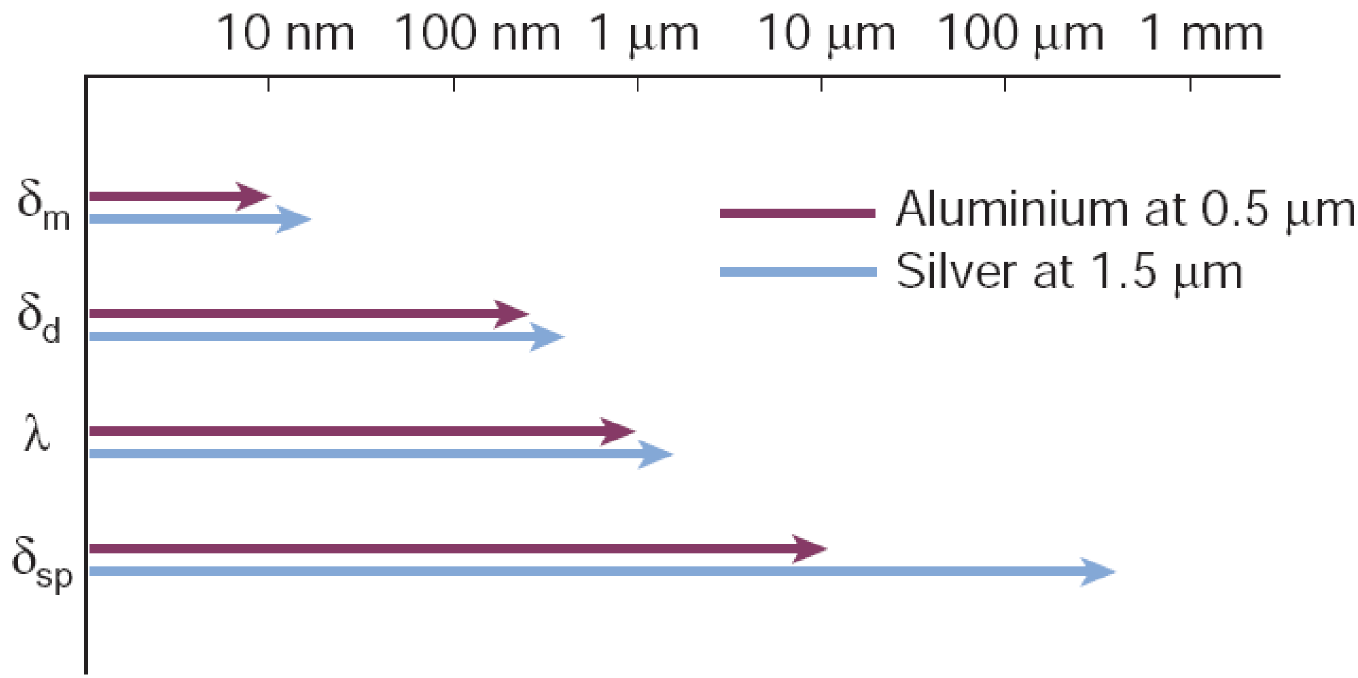
$$\delta_{SP} = \frac{c}{\omega}\left(\frac{\varepsilon_m^{'} + \varepsilon_d}{\varepsilon_m^{'}\varepsilon_d}\right)^{\frac{3}{2}} \frac{\left(\varepsilon_m^{'}\right)^2}{\varepsilon_m^{''}}$$

ε_d is the dielectric constant of the dielectric material.
$\varepsilon_m = \varepsilon'_m + \varepsilon''_m$ is the dielectric function of the metal.

Silver has the lowest absorption losses (smallest ε''_m) in the visible spectrum. The propagation length for silver is typically in the range of 10 – 100 μm, and increases to 1 mm as the wavelength moves into the 1.5 μm near-infrared telecommunication band.

In the past, absorption by metals was seen as such a significant problem that surface plasmons were not considered as viable for photonic elements. This view is now changing due primarily to recent demonstrations of surface plasmon-based components that are significantly smaller than the propagation length. Such developments open the way to integrate surface plasmon-based devices into circuits before propagation losses become too significant.

Characteristic Length Scales of PSPs

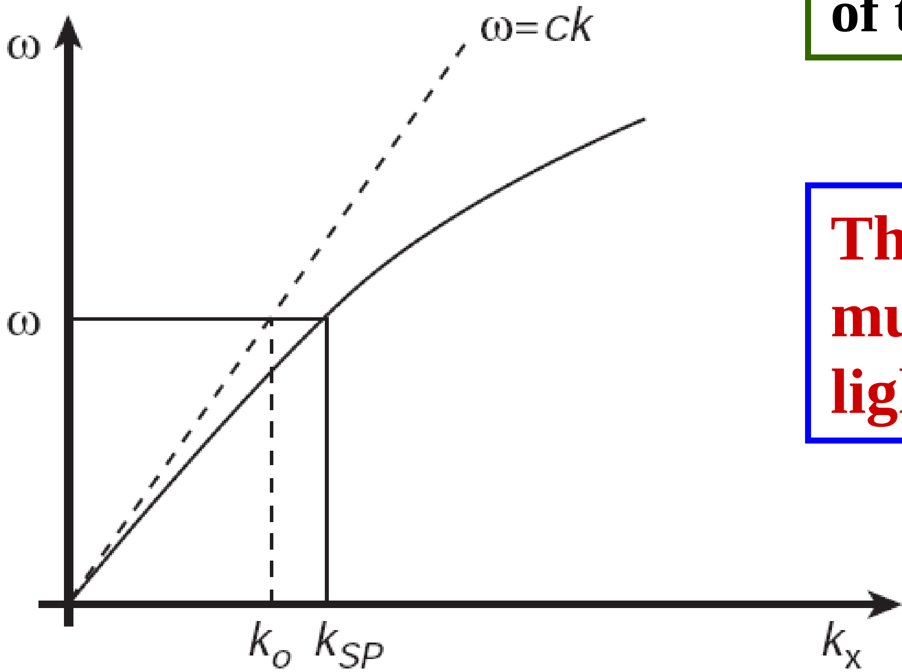


These characteristic length scales are important for PSP-based photonics in addition to the associated light wavelength. The propagation length sets the upper size limit for any photonic circuit based on PSPs. The decay length in the dielectric material, δ_d, dictates the maximum heights of any individual feature that might be used to control surface plasmons. The decay length in the metal, δ_m, determines the minimum feature size that can be used.

Momentum Mismatch between PSPs and Free-Space Light Wave

$$k_{SP} = k_0 \sqrt{\frac{\varepsilon_d \varepsilon_m}{\varepsilon_d + \varepsilon_m}}$$

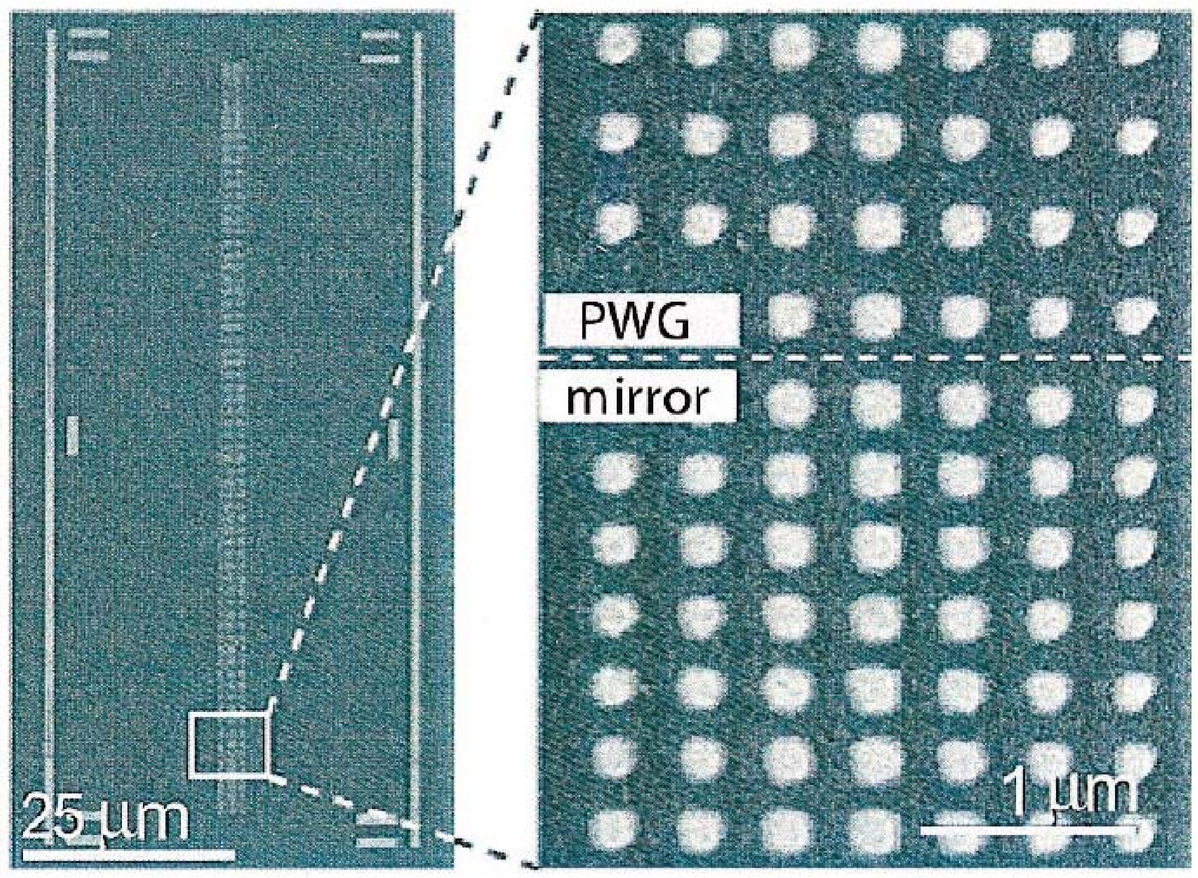
The interaction between the surface charge density and the electromagnetic field results in the momentum of the surface plasmon mode, $\hbar k_{sp}$, being larger than that of a free-space photon of the same frequency, $\hbar k_0$.



The momentum mismatch problem must be overcome in order to couple light and surface plasmons together.

Plasmonic Waveguides ––– Arrays of Metal Nanoparticles

One can use an array of metal nanoparticle resonators in order to avoid ohmic heating. The resonant structures of nanoparticles can be used to guide light, whereas the reduced metallic volume means a substantial reduction in ohmic losses.



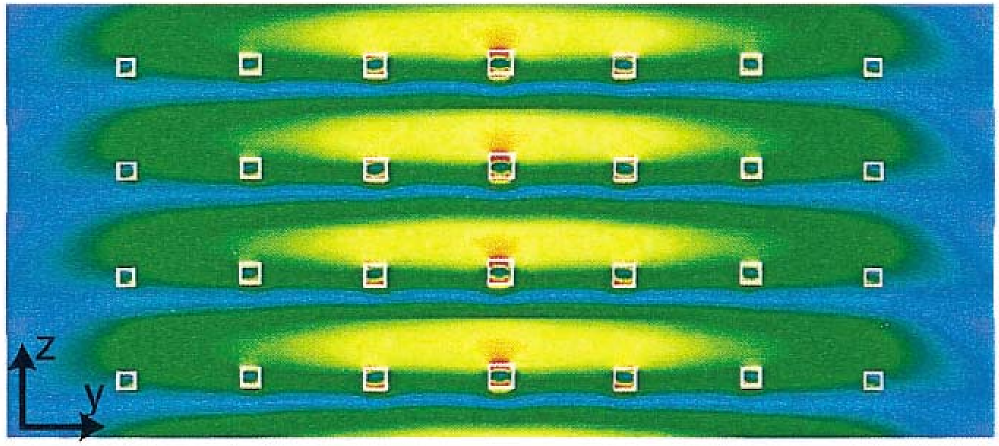
Gold nanodots:

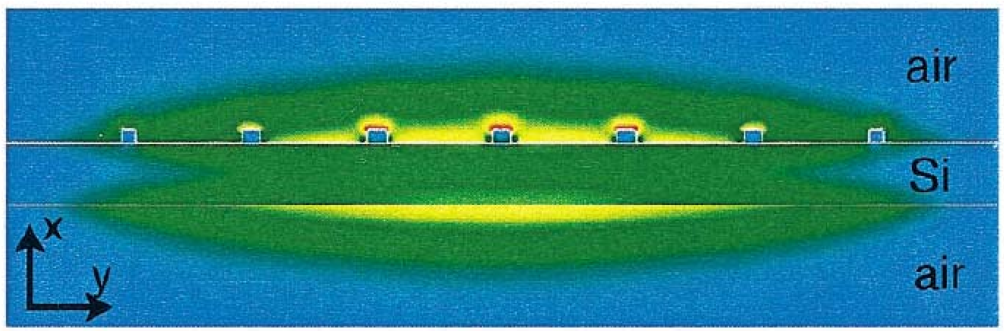
Lattice constant = 500 nm
Height = 50 nm
Edge sizes = 80 × 80 nm^2 (at the center, decreasing to 50 × 50 nm^2 at both sides)

The waveguide is terminated at both ends by mirrors consisting of a compressed lattice.

Designed for the guiding of the light at a wavelength of 1.6 µm

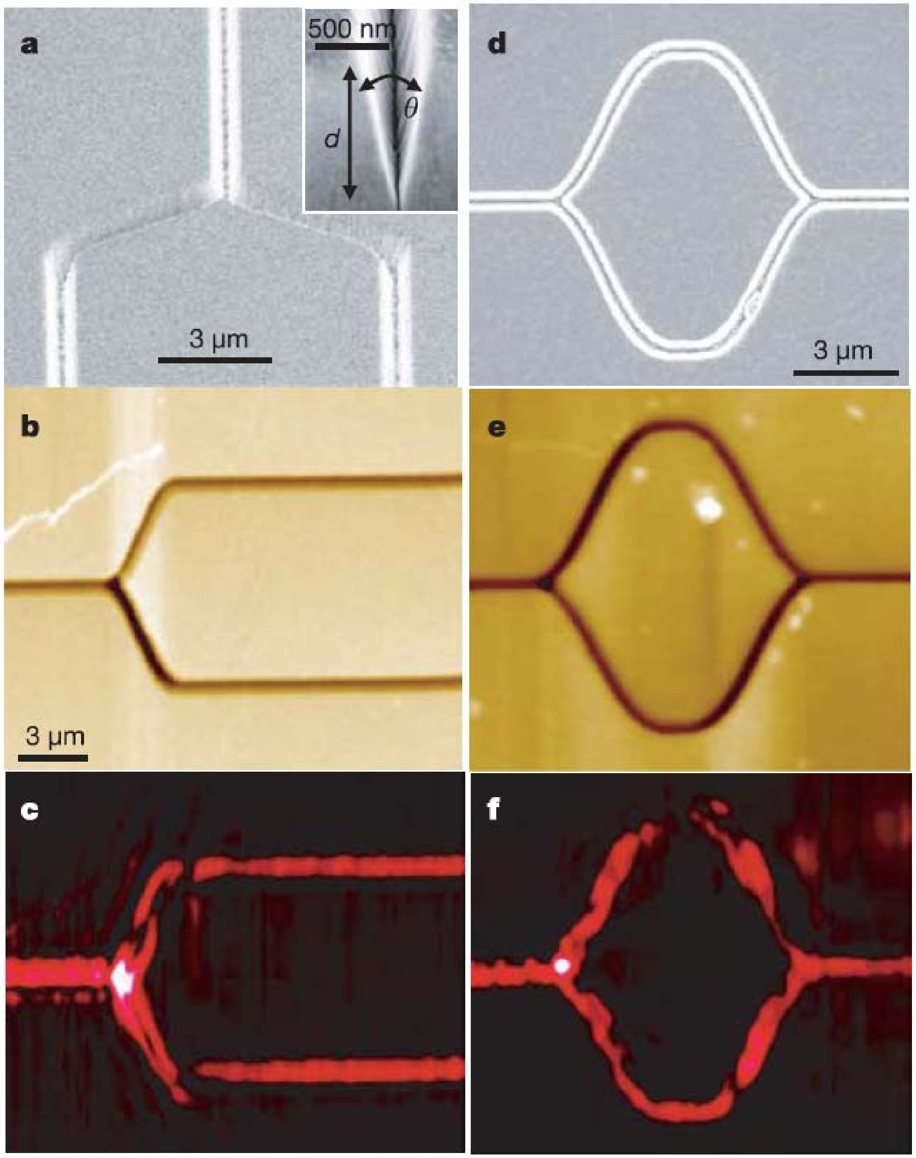
Plasmonic Waveguides —– Arrays of Metal Nanoparticles





This structure has been shown to have a decay length of about 50 μm, whereas finite-difference time-domain (FDTD) simulations predict a 1/*e* energy attenuation length of 320 μm.

Plasmonic Waveguides —– Grooves in Metal Films



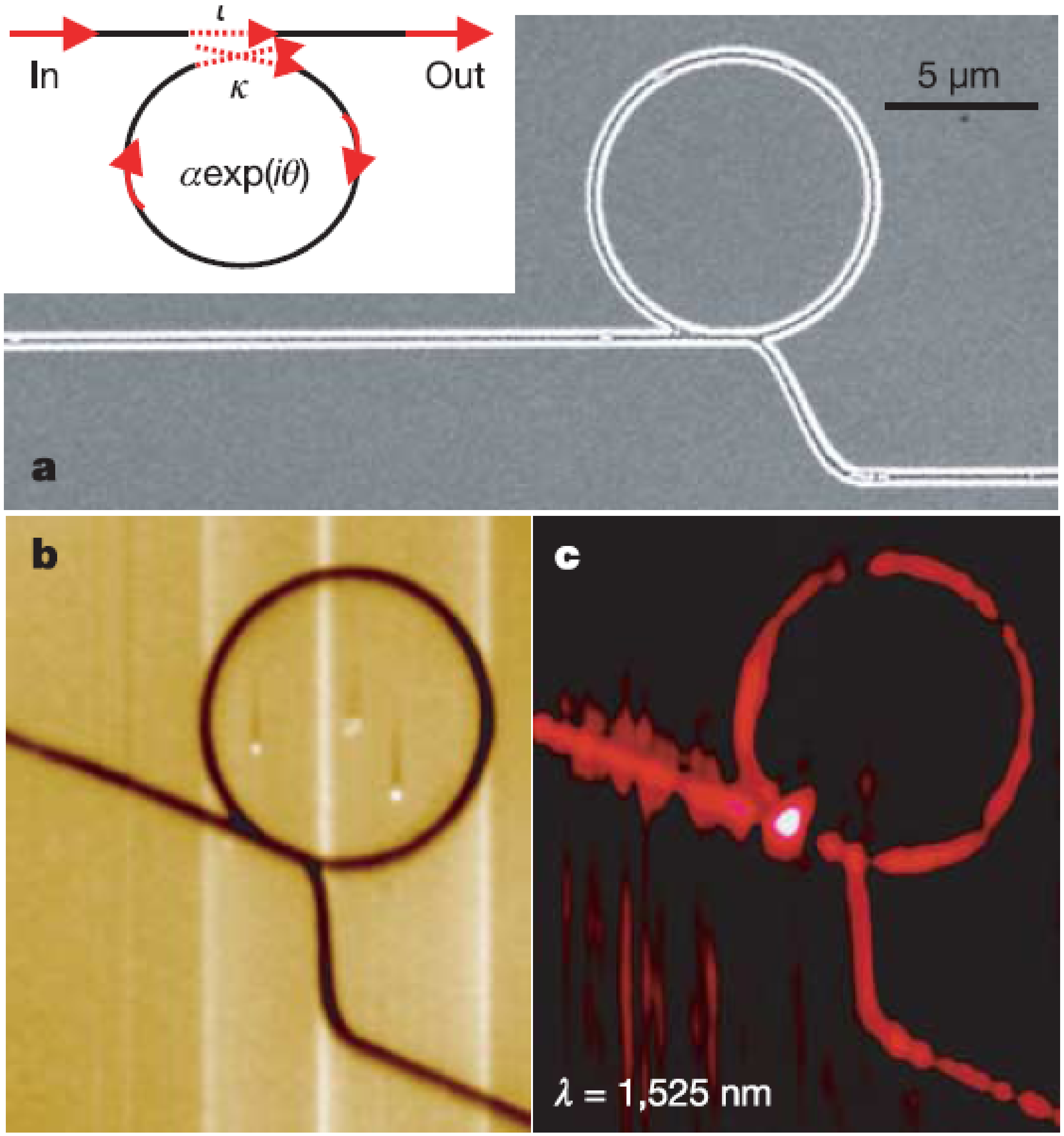
Surface plasmons are bound to and propagate along the bottom of V-shaped grooves milled in gold films.

λ = 1600 nm
d = 1.1–1.3 μm
2θ = ~25°

Y-splitter
Mach-Zehnder interferometer

S. I. Bozhevolnyi, V. S. Volkov, E. Devaux, J.-Y. Laluet, T. W. Ebbesen, *Nature* **2006**, *440*, 508.

Plasmonic Waveguides —— Grooves in Metal Films



Waveguide-ring resonator

Reading Materials

S. O. Kasap, “Optoelectronics and Photonics: Principles and Practices”, Prentice Hall, Upper Saddle River, NJ 07458, 2001, Chapter 2, “Dielectric Waveguides and Optical Fibers”.