Introduction to Mechanics, Heat, and Sound /FIC 318

Lecture II

- Math review (vectors cont.)
- Motion in one dimension
  - Position and displacement
  - Velocity
    - average
    - instantaneous
  - Acceleration
    - motion with constant acceleration
- Motion in two dimensions

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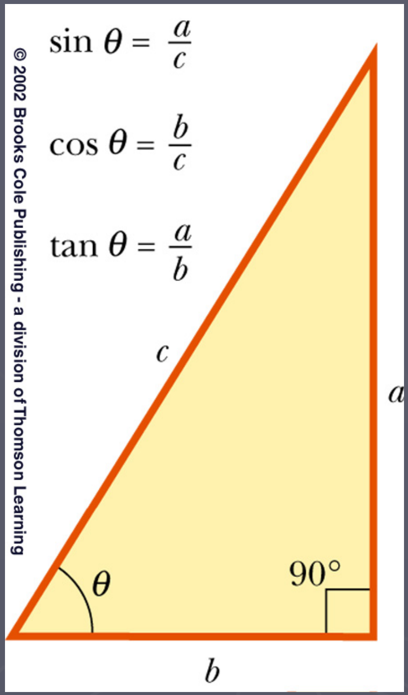
Lightning Review



Last lecture:

1. Math review: trigonometry
2. Math review: vectors, vector addition

Note: magnitudes do not add unless vectors point in the same direction

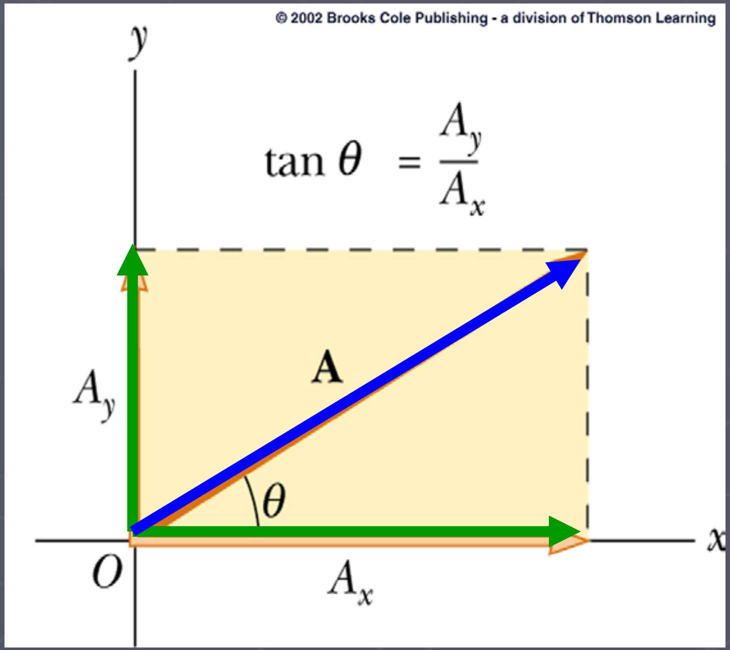
3. Physics introduction: units, measurements, etc.

Review Problem: How many beats would you detect if you take someone's pulse for 10 sec instead of a minute?

Hint: Normal heartbeat rate is 60 beats/minute.

Components of a Vector

- A **component** is a part
- It is useful to use **rectangular components**
  - These are the projections of the vector along the x- and y-axes
- Vector **A** is now a sum of its components:



$$\vec{\mathbf{A}} = \vec{A}_x + \vec{A}_y$$

What are $\vec{A}_x$ and $\vec{A}_y$?

Components of a Vector

- The components are the legs of the right triangle whose hypotenuse is **A**

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$

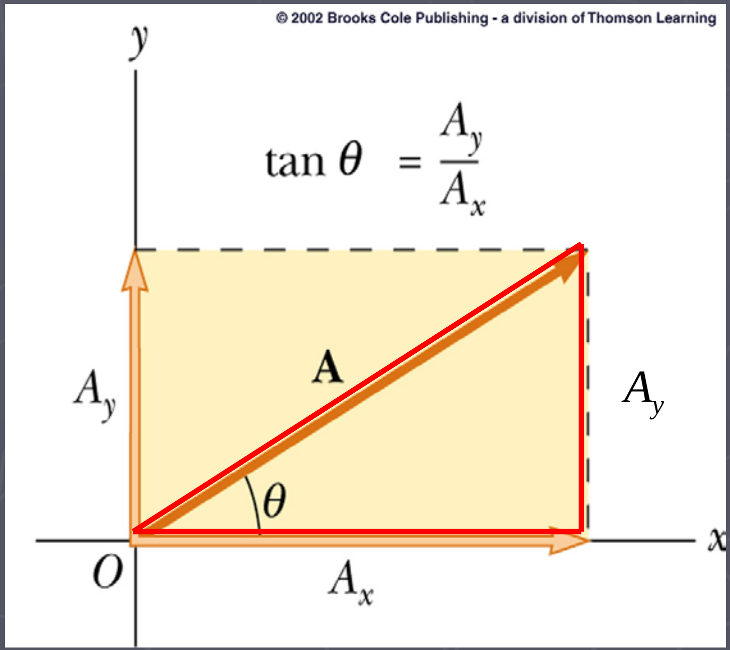
- The x-component of a vector is the projection along the x-axis

$$A_x = A\cos\theta$$

- The y-component of a vector is the projection along the y-axis

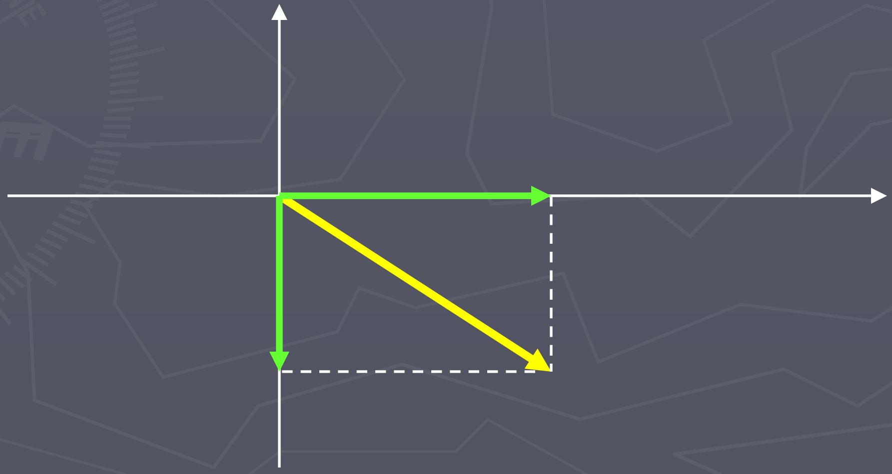
$$A_y = A\sin\theta$$

- Then, $\vec{\mathbf{A}} = \vec{A}_x + \vec{A}_y$



Notes About Components

- The previous equations are valid ***only if θ is measured with respect to the x-axis***
- The components can be positive or negative and will have the same units as the original vector



Example 1

A golfer takes two putts to get his ball into the hole once he is on the green. The first putt displaces the ball 6.00 m east, and the second, 5.40 m south. What displacement would have been needed to get the ball into the hole on the first putt?

Given:

$\Delta x_1 = 6.00$ m (east)
$\Delta x_2 = 5.40$ m (south)

Find:

R = ?

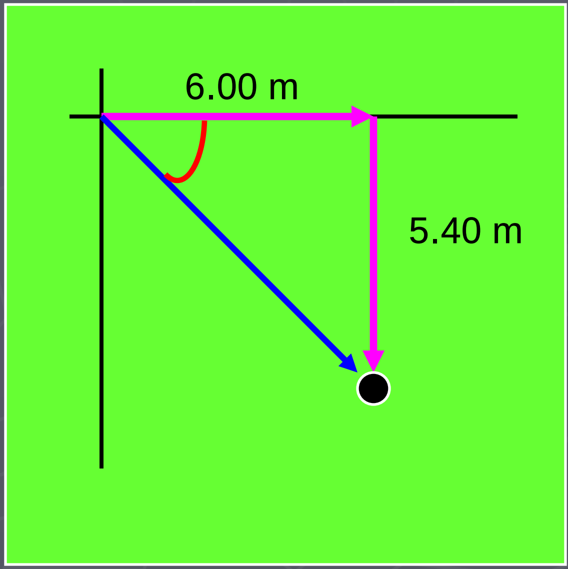
Solution:

1. Note right triangle, use Pythagorean theorem

$$R = \sqrt{(6.00\ \text{m})^2 + (5.40\ \text{m})^2} = 8.07\ \text{m}$$

2. Find angle:

$$\theta = \tan^{-1}\left(\frac{5.40\ \text{m}}{6.00\ \text{m}}\right) = \tan^{-1}(0.900) = 42.0^\circ$$



What Components Are Good For: Adding Vectors Algebraically

- Choose a coordinate system and sketch the vectors v_1, v_2, ...
- Find the x- and y-components of all the vectors
- Add all the x-components
  - This gives R_x:

$$R_x = \sum v_x$$

- Add all the y-components
  - This gives R_y:

$$R_y = \sum v_y$$

Magnitudes of vectors pointing in the same direction can be added to find the resultant!

Adding Vectors Algebraically (cont.)

- Use the Pythagorean Theorem to find the magnitude of the Resultant:

$$R = \sqrt{R_x^2 + R_y^2}$$

- Use the inverse tangent function to find the direction of R:

$$\theta = \tan^{-1}\frac{R_y}{R_x}$$

IV. Motion in One Dimension

Dynamics

- The branch of physics involving the motion of an object and the relationship between that motion and other physics concepts

- ***Kinematics*** is a part of dynamics
  - In kinematics, you are interested in the *description* of motion
  - *Not* concerned with the cause of the motion

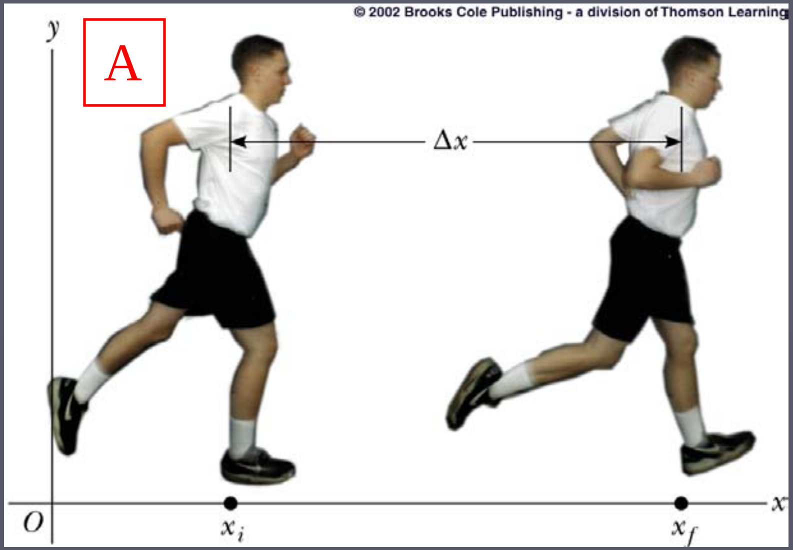
Position and Displacement

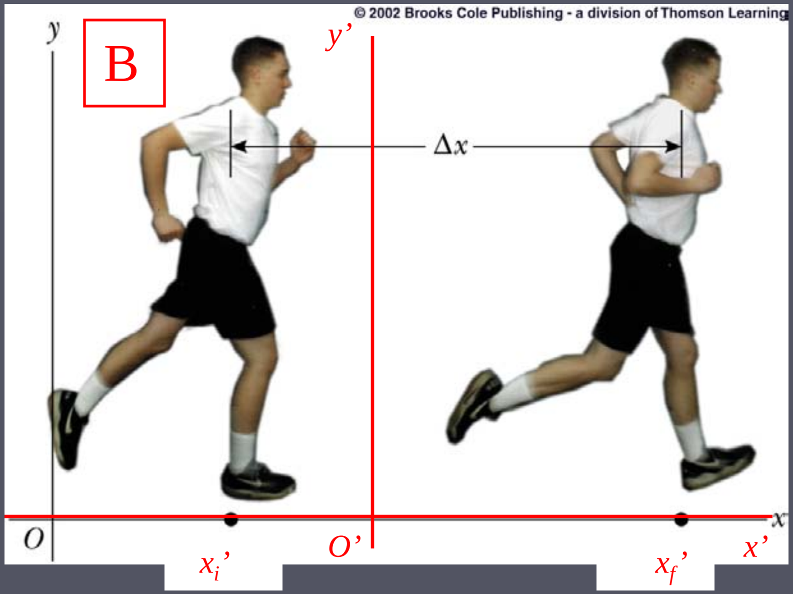
- Position is defined in terms of a **frame of reference**

Frame A: $x_i>0$ and $x_f>0$

Frame B: $x'_i<0$ but $x'_f>0$

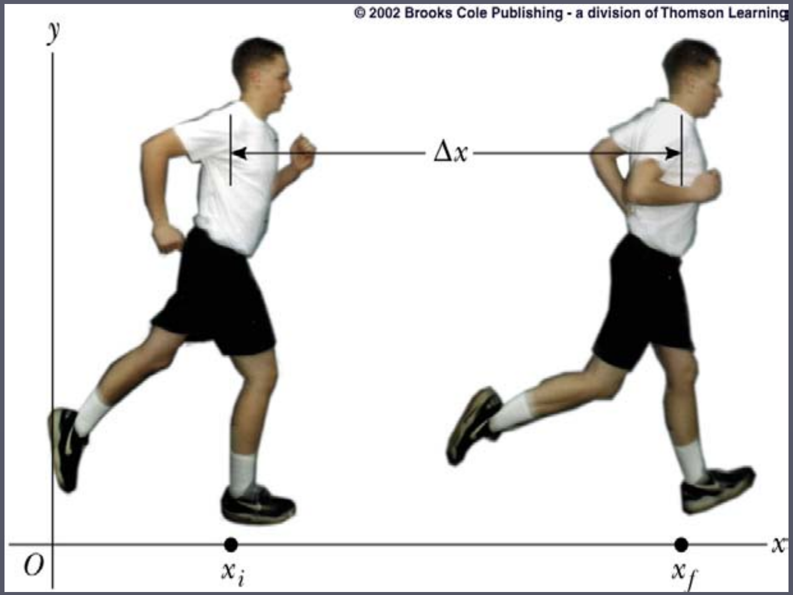
- One dimensional, so generally the x- or y-axis





Position and Displacement

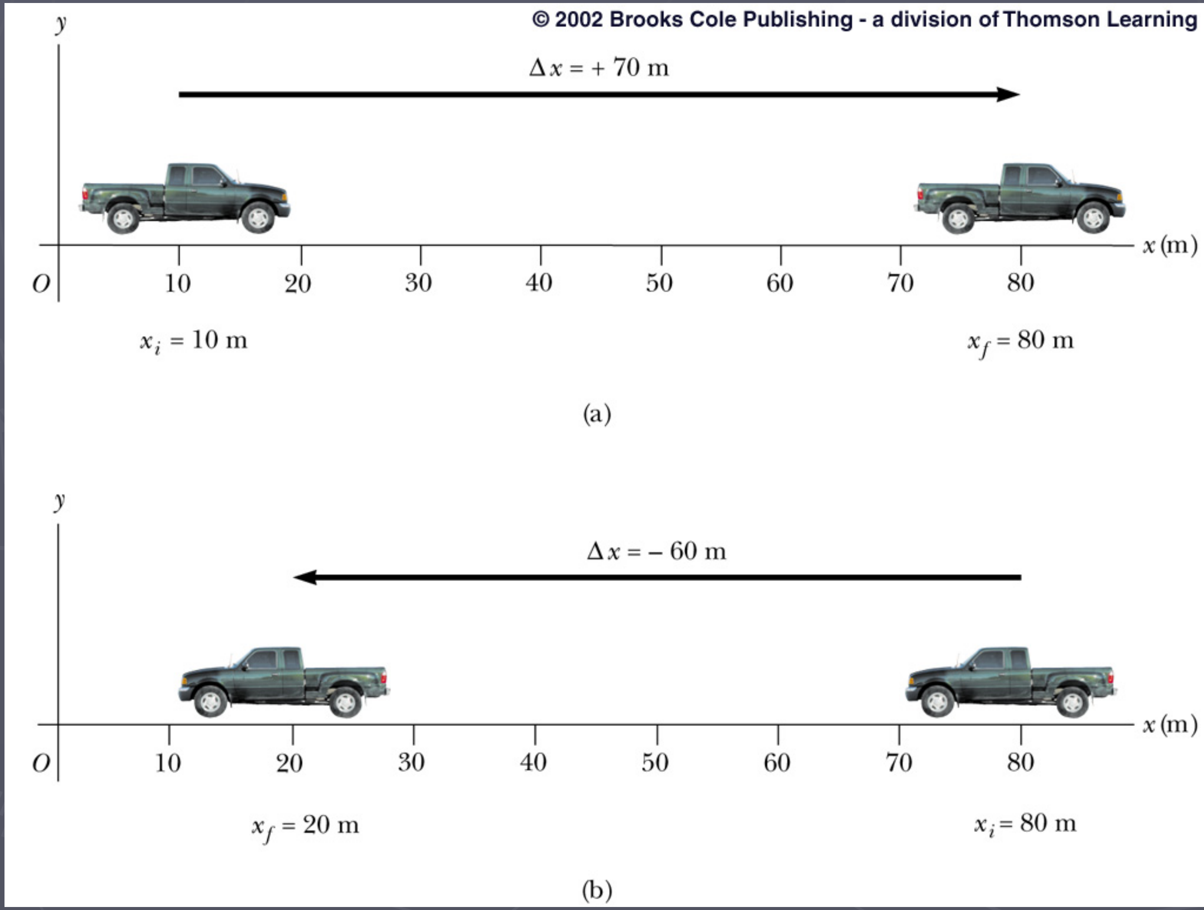
- Position is defined in terms of a **frame of reference**
  - One dimensional, so generally the x- or y-axis
- Displacement measures the change in position
  - Represented as Δx (if horizontal) or Δy (if vertical)
  - Vector quantity (i.e. needs directional information)
    - + or - is generally sufficient to indicate direction for one-dimensional motion



Units	
SI	Meters (m)
CGS	Centimeters (cm)
US Cust	Feet (ft)

Displacement

- Displacement measures the change in position
  - represented as Δx or Δy

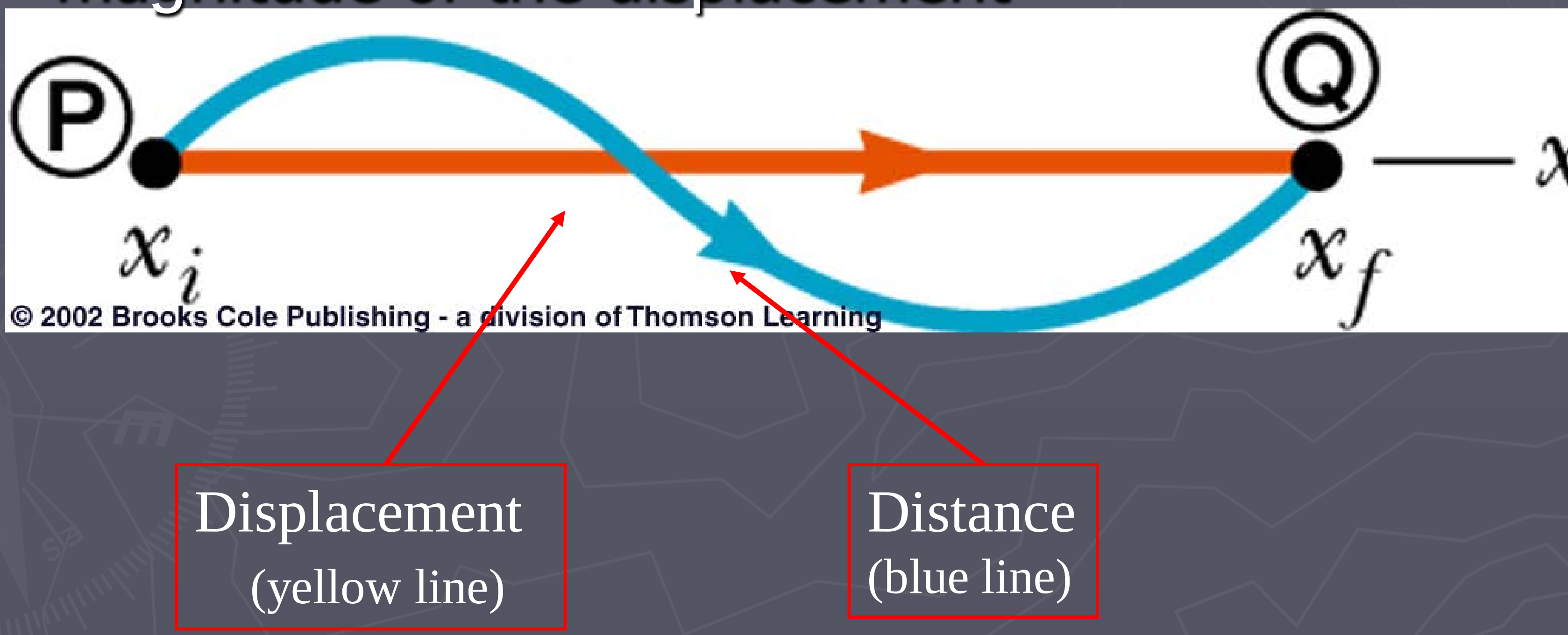


$$\Delta x_1 = x_f - x_i$$
$$= 80\,m - 10\,m$$
$$= \underline{+70\,m}$$ ✓

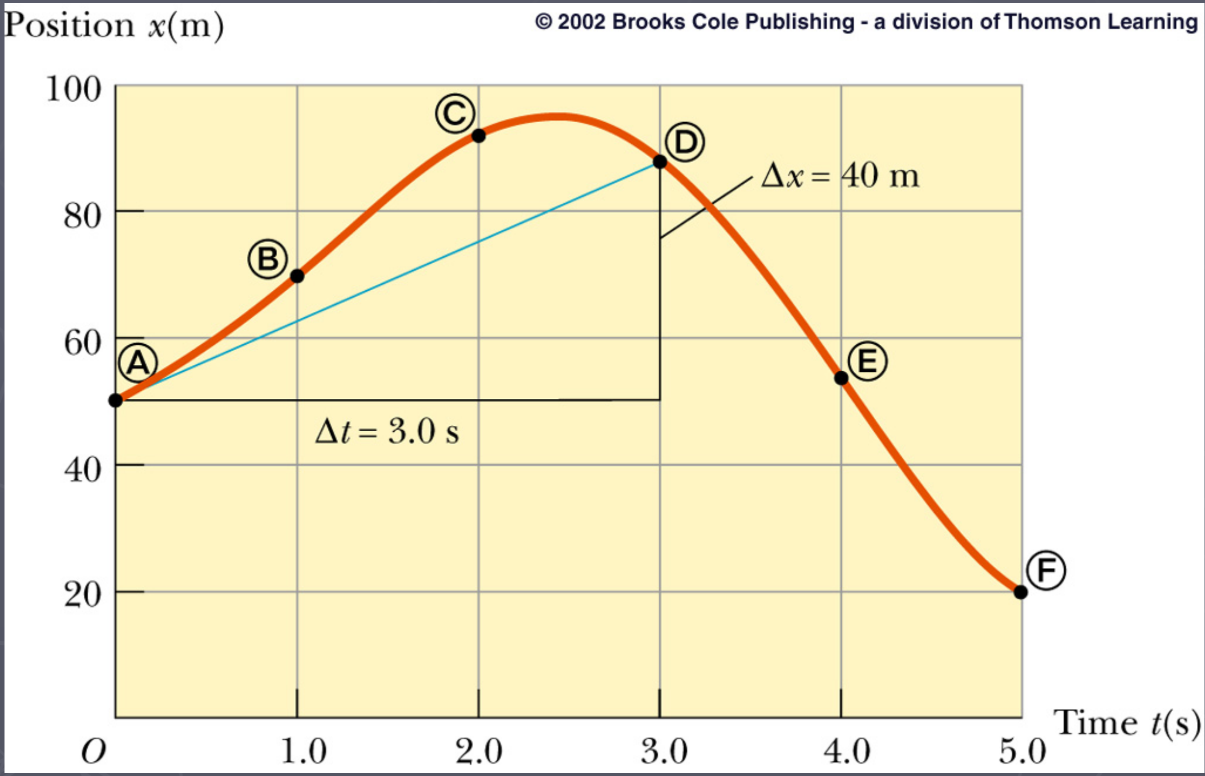
$$\Delta x_2 = x_f - x_i$$
$$= 20\,m - 80\,m$$
$$= \underline{-60\,m}$$ ✓

Distance or Displacement?

- Distance may be, but is not necessarily, the magnitude of the displacement



Position-time graphs



- Note: position-time graph is <u>not necessarily a straight line</u>, even though the motion is along x-direction

ConcepTest 1

An object (say, car) goes from one point in space to another. After it arrives to its destination, its displacement is

1. either greater than or equal to
2. always greater than
3. always equal to
4. either smaller or equal to
5. either smaller or larger

than the distance it traveled.

Please fill your answer as question 1 of General Purpose Answer Sheet

ConcepTest 1

An object (say, car) goes from one point in space to another. After it arrives to its destination, its displacement is

1. either greater than or equal to
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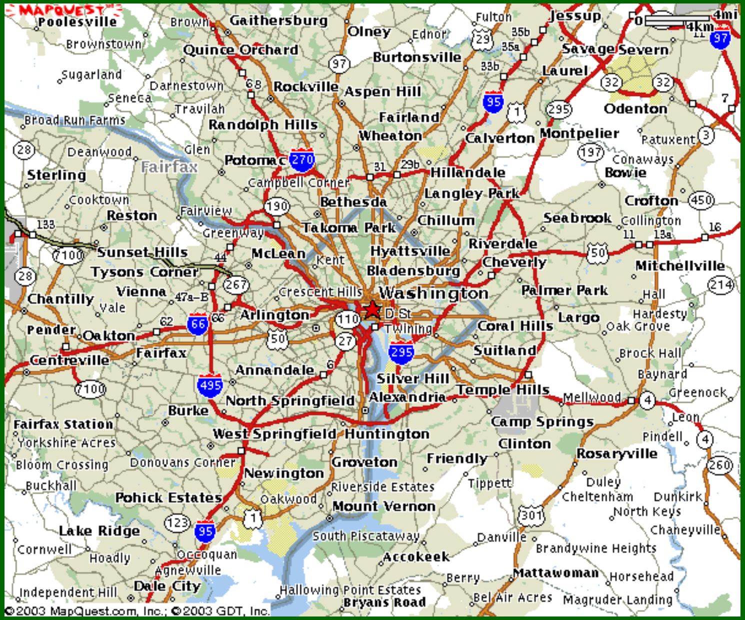
than the distance it traveled.

Please fill your answer as **question 2** of General Purpose Answer Sheet

ConcepTest 1 (answer)

An object (say, car) goes from one point in space to another. After it arrives to its destination, its displacement is



1. either greater than or equal to
2. always greater than
3. always equal to
4. either smaller or equal to ✓
5. either smaller or larger

than the distance it traveled.

Note: displacement is a vector from the final to initial points, distance is total path traversed

Average Velocity

- It takes time for an object to undergo a displacement
- The **average velocity** is rate at which the displacement occurs

$$\vec{v}_{average} = \frac{\Delta \vec{x}}{\Delta t} = \frac{\vec{x}_f - \vec{x}_i}{\Delta t}$$

- Direction will be the same as the direction of the displacement (Δt is always positive)

More About Average Velocity

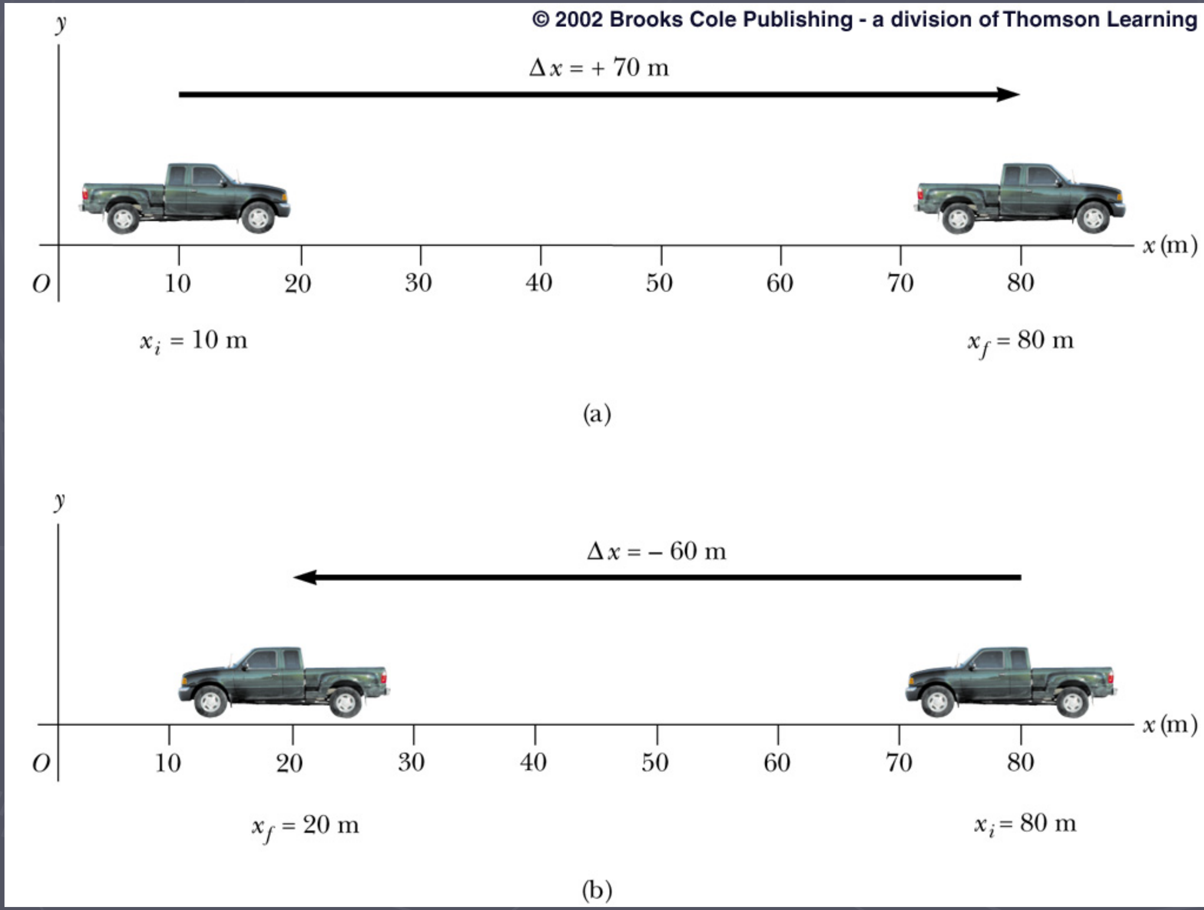
- Units of velocity:

Units	
SI	Meters per second (m/s)
CGS	Centimeters per second (cm/s)
US Customary	Feet per second (ft/s)

- Note: other units may be given in a problem, but generally will need to be converted to these

Example:

Suppose that in both cases truck covers the distance in 10 seconds:



$$\vec{v}_{1\,average} = \frac{\Delta \vec{x}_1}{\Delta t} = \frac{+70m}{10s}$$

$$= \underline{+7\,m/s}$$

$$\vec{v}_{2\,average} = \frac{\Delta \vec{x}_2}{\Delta t} = \frac{-60m}{10s}$$

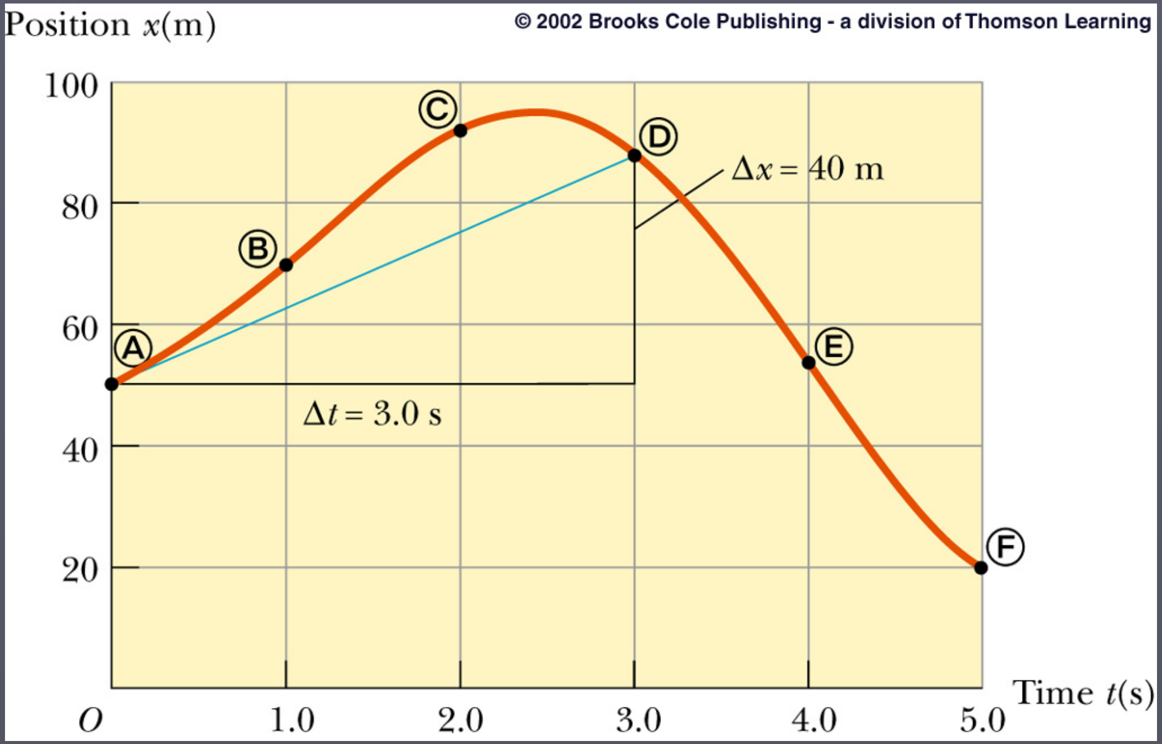
$$= \underline{-6\,m/s}$$

Speed

- Speed is a scalar quantity (no information about sign/direction is need)
  - same units as velocity
  - Average speed = total distance / total time
- Speed is the magnitude of the velocity

Graphical Interpretation of Average Velocity

- Velocity can be determined from a position-time graph



$$\vec{v}_{average} = \frac{\Delta \vec{x}}{\Delta t} = \frac{+40m}{3.0s}$$

$$= \underline{+13\, m/s}$$

- Average velocity equals the slope of the line joining the initial and final positions

Instantaneous Velocity

- Instantaneous velocity is defined as the limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero

$$\vec{v}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{x}_f - \vec{x}_i}{\Delta t}$$

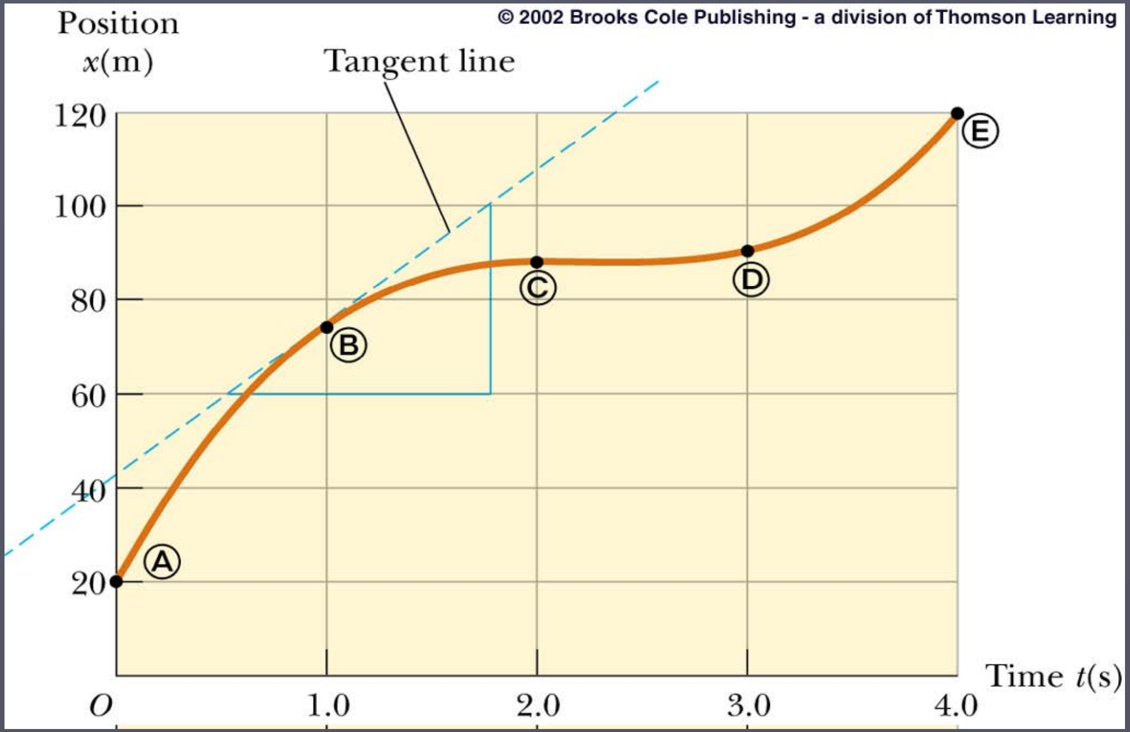
- The instantaneous velocity indicates what is happening at every point of time

Uniform Velocity

- Uniform velocity is constant velocity
- The instantaneous velocities are always the same
  - All the instantaneous velocities will also equal the average velocity

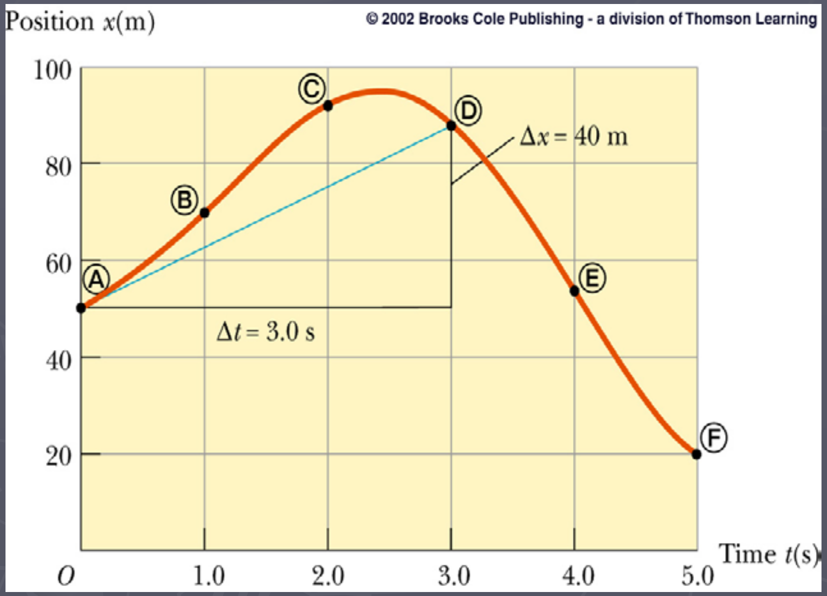
Graphical Interpretation of Instantaneous Velocity

- Instantaneous velocity is the slope of the tangent to the curve at the time of interest

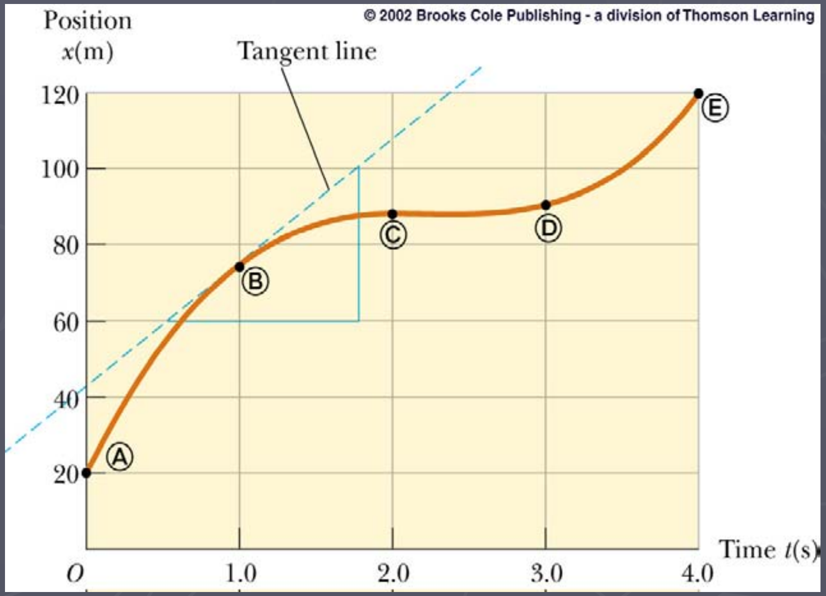


- The instantaneous speed is the magnitude of the instantaneous velocity

Average vs Instantaneous Velocity

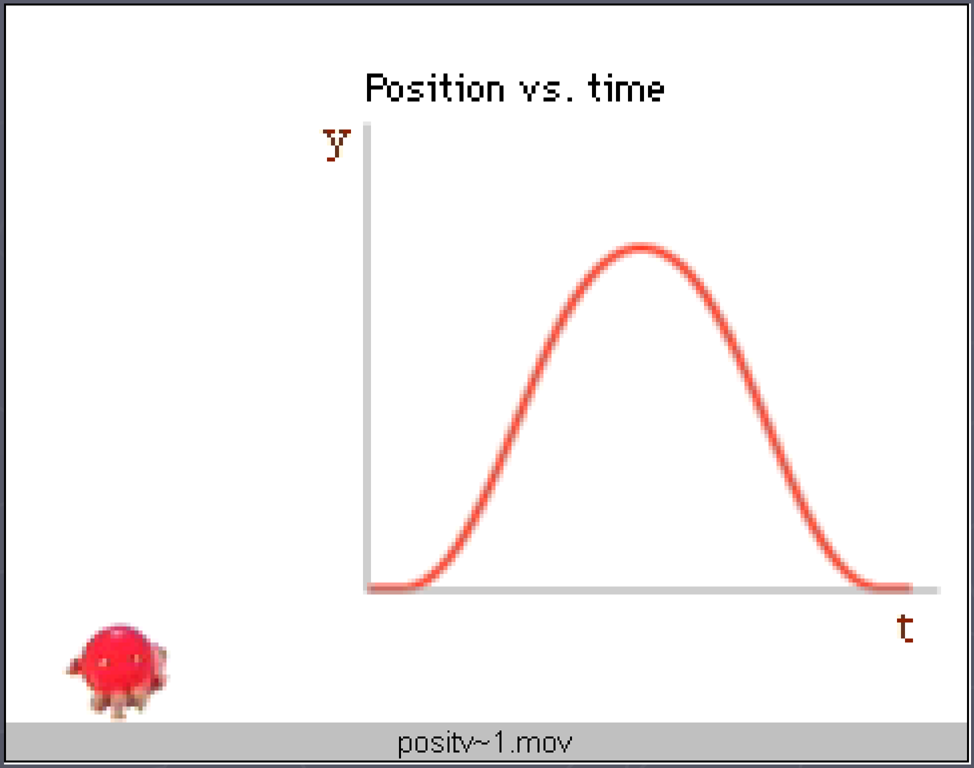


Average velocity



Instantaneous velocity

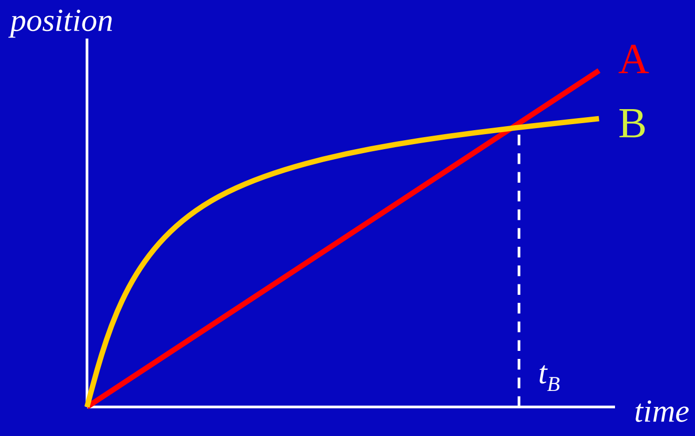
Let's watch the movie!



ConcepTest 2

The graph shows position as a function of time for two trains running on parallel tracks. Which of the following is true:

1. at time t_B both trains have the same velocity
2. both trains speed up all the time
3. both trains have the same velocity at some time before t_B
4. train A is longer than train B
5. all of the above statements are true



Please fill your answer as question 3 of General Purpose Answer Sheet

ConcepTest 2

The graph shows position as a function of time for two trains running on parallel tracks. Which of the following is true:

1. at time t_B both trains have the same velocity
2. both trains speed up all the time
3. both trains have the same velocity at some time before t_B
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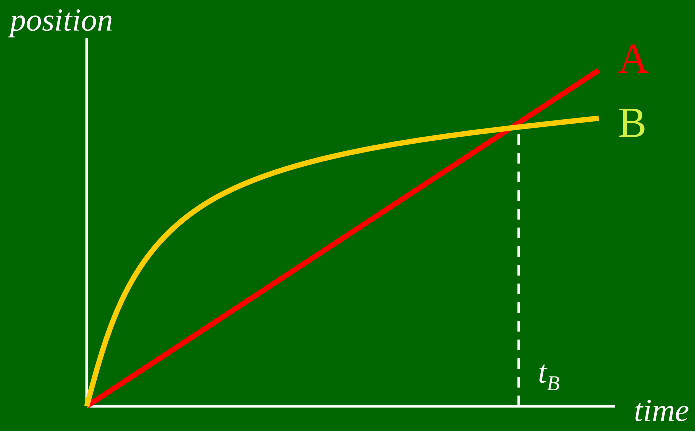
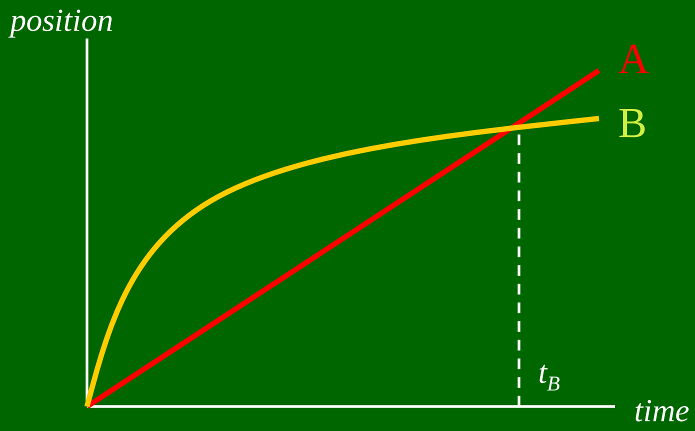


Please fill your answer as **question 4** of General Purpose Answer Sheet

ConcepTest 2 (answer)

The graph shows position as a function of time for two trains running on parallel tracks. Which of the following is true:

1. at time t_B both trains have the same velocity
2. both trains speed up all the time
3. both trains have the same velocity at some time before t_B
4. train A is longer than train B
5. all of the above statements are true



Note: the slope of curve B is parallel to line A at some point $t < t_B$

Average Acceleration

- Changing velocity (non-uniform) means an acceleration is present
- Average acceleration is the rate of change of the velocity

$$\vec{a}_{average} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

- Average acceleration is a vector quantity (i.e. described by both magnitude and direction)

Average Acceleration

- When the sign of the velocity and the acceleration are the same (either positive or negative), then the speed is increasing
- When the sign of the velocity and the acceleration are opposite, the speed is decreasing

Units	
SI	Meters per second squared (m/s^2)
CGS	Centimeters per second squared (cm/s^2)
US Customary	Feet per second squared (ft/s^2)

Instantaneous and Uniform Acceleration

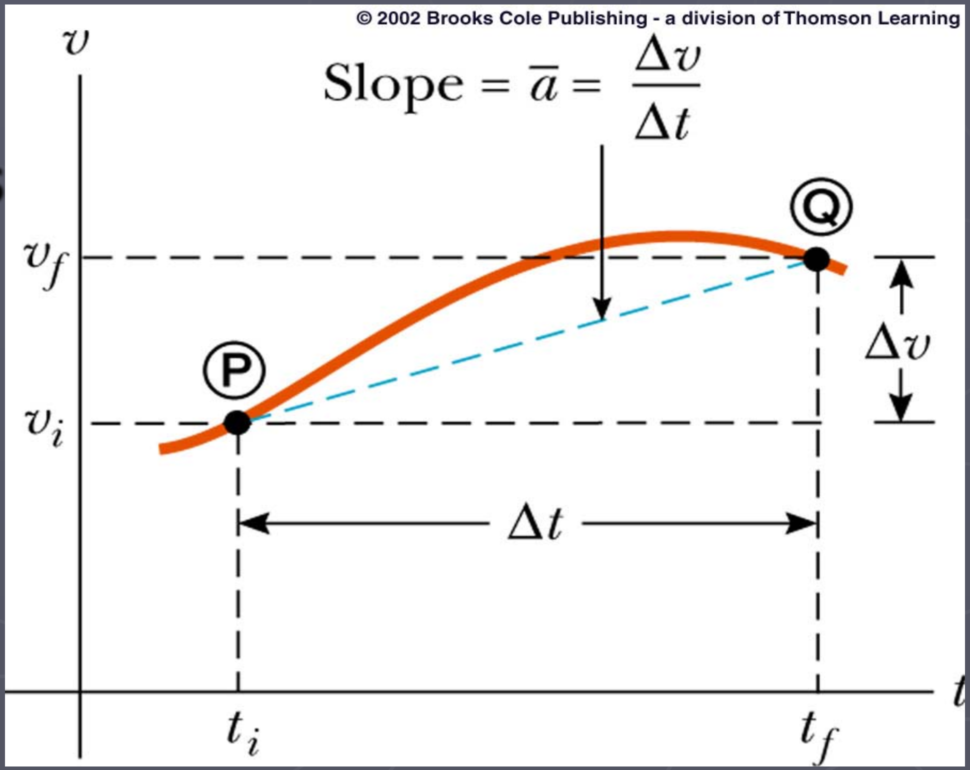
- Instantaneous acceleration is the limit of the average acceleration as the time interval goes to zero

$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

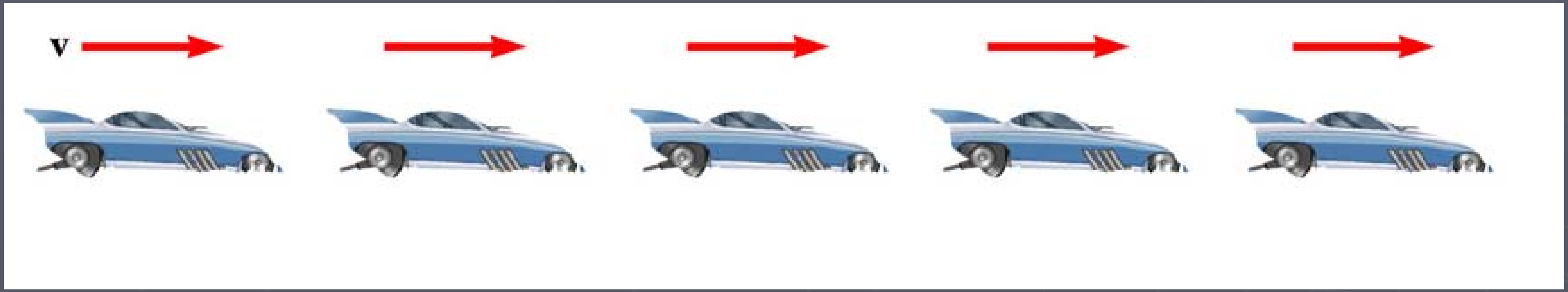
- When the instantaneous accelerations are always the same, the acceleration will be uniform
  - The instantaneous accelerations will all be equal to the average acceleration

Graphical Interpretation of Acceleration

- Average acceleration is the slope of the line connecting the initial and final velocities on a velocity-time graph
- Instantaneous acceleration is the slope of the tangent to the curve of the velocity-time graph

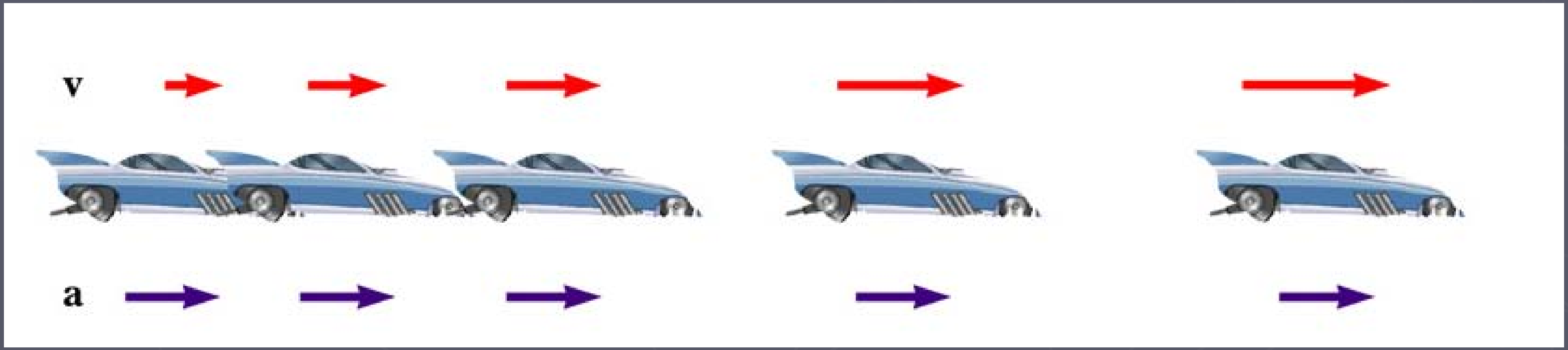


Example 1: Motion Diagrams



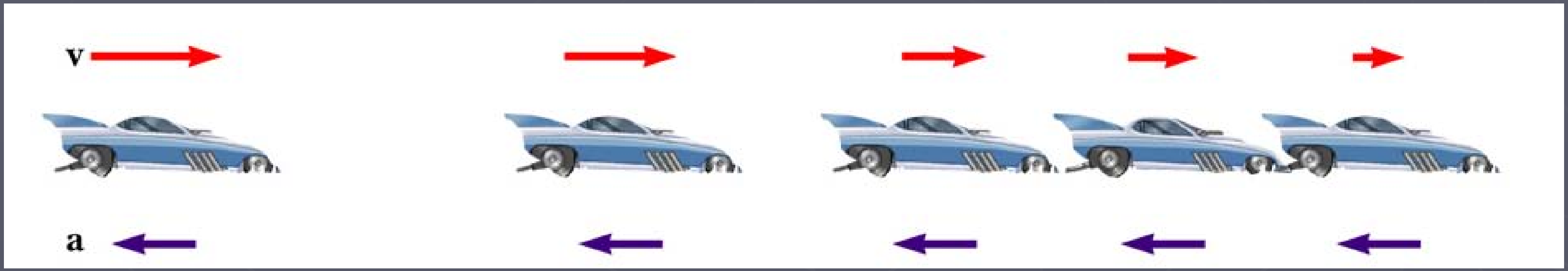
- Uniform velocity (shown by red arrows maintaining the same size)
- Acceleration equals zero

Example 2:



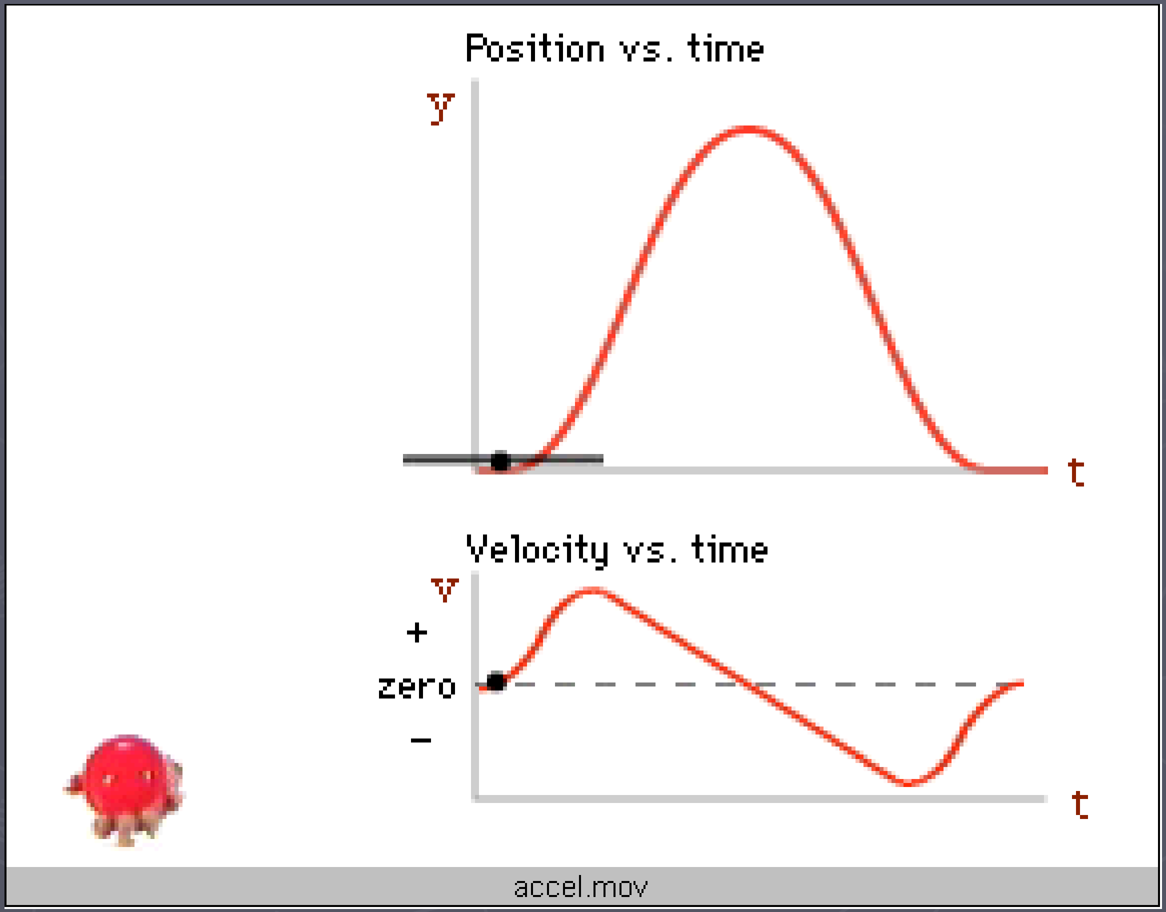
- Velocity and acceleration are in the same direction
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is increasing (red arrows are getting longer)

Example 3:



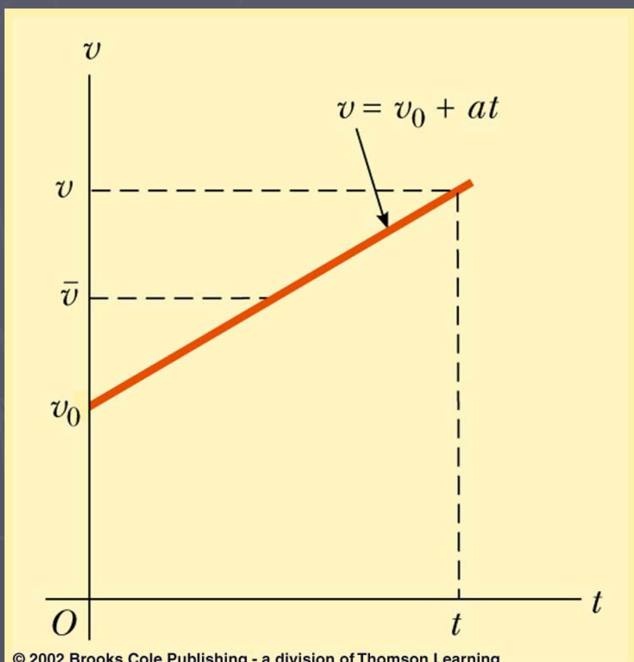
- Acceleration and velocity are in opposite directions
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is decreasing (red arrows are getting shorter)

Let’s watch the movie!



One-dimensional Motion With Constant Acceleration

- If acceleration is uniform (i.e. $\bar{a} = a$):



$$a = \frac{v_f - v_o}{t_f - t_0} = \frac{v_f - v_o}{t}$$

thus:

$$v_f = v_o + at$$

- Shows velocity as a function of acceleration and time

One-dimensional Motion With Constant Acceleration

- Used in situations with uniform acceleration

$$v_f = v_o + at$$

$$\Delta x = v_{average} t = \left(\frac{v_o + v_f}{2} \right) t$$

Velocity changes uniformly!!!

$$\Delta x = v_o t + \frac{1}{2} a t^2$$

$$v_f^2 = v_o^2 + 2a\Delta x$$

Notes on the equations

$$\Delta x = v_{average}\, t = \left(\frac{v_o + v_f}{2}\right) t$$

- Gives displacement as a function of velocity and time

$$\Delta x = v_o t + \frac{1}{2} a t^2$$

- Gives displacement as a function of time, velocity and acceleration

$$v_f^2 = v_o^2 + 2a\Delta x$$

- Gives velocity as a function of acceleration and displacement

Summary of kinematic equations

TABLE 2.3 Equations for Motion in a Straight Line Under Constant Acceleration

Equation	Information Given by Equation
$v = v_0 + at$	Velocity as a function of time
$\Delta x = \frac{1}{2}(v_0 + v)\,t$	Displacement as a function of velocity and time
$\Delta x = v_0 t + \frac{1}{2}at^2$	Displacement as a function of time
$v^2 = v_0{}^2 + 2a\,\Delta x$	Velocity as a function of displacement

Note: Motion is along the x axis. At $t = 0$, the velocity of the particle is v_0.

Free Fall

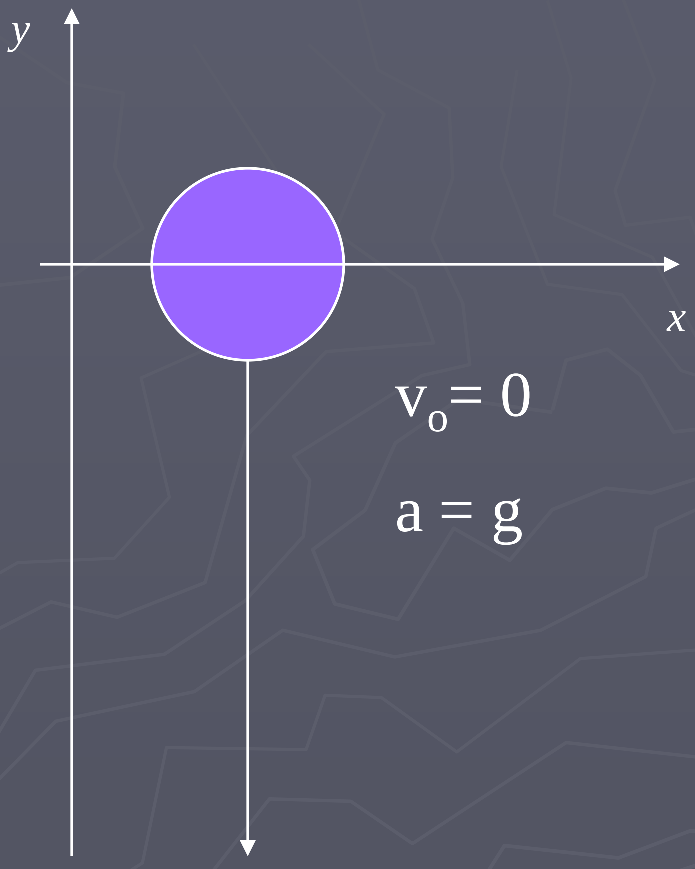
- All objects moving under the influence of only gravity are said to be in free fall
- All objects falling near the earth's surface fall with a constant acceleration
- This acceleration is called the acceleration due to gravity, and indicated by g

Acceleration due to Gravity

- Symbolized by g
- $g = 9.8\ m/s^2$ (can use $g = 10\ m/s^2$ for estimates)
- g is always directed downward
  - toward the center of the earth

Free Fall -- an Object Dropped

- Initial velocity is zero
- Frame: let up be positive
- Use the kinematic equations
  - Generally use y instead of x since vertical

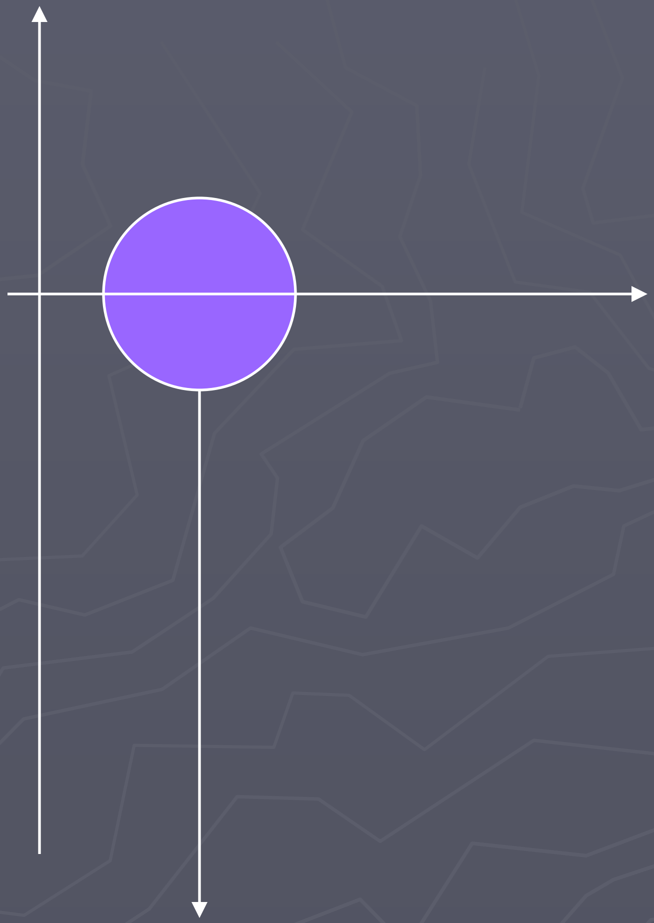


$$\Delta y = \frac{1}{2} a t^2$$

$$a = -9.8\, m/s^2$$

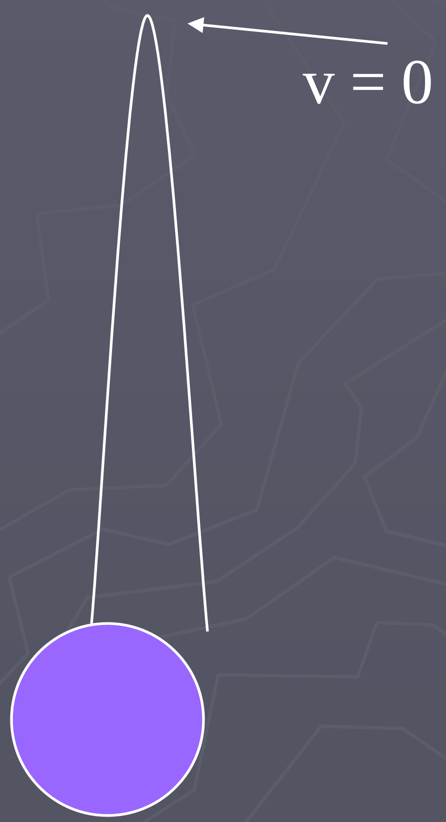
Free Fall -- an Object Thrown Downward

- $a = g$
  - With upward being positive, acceleration will be negative, $g = -9.8\ m/s^2$
- Initial velocity $\neq 0$
  - With upward being positive, initial velocity will be negative



Free Fall -- object thrown upward

- Initial velocity is upward, so positive
- The instantaneous velocity at the maximum height is zero
- a = g everywhere in the motion
  - g is always downward, negative

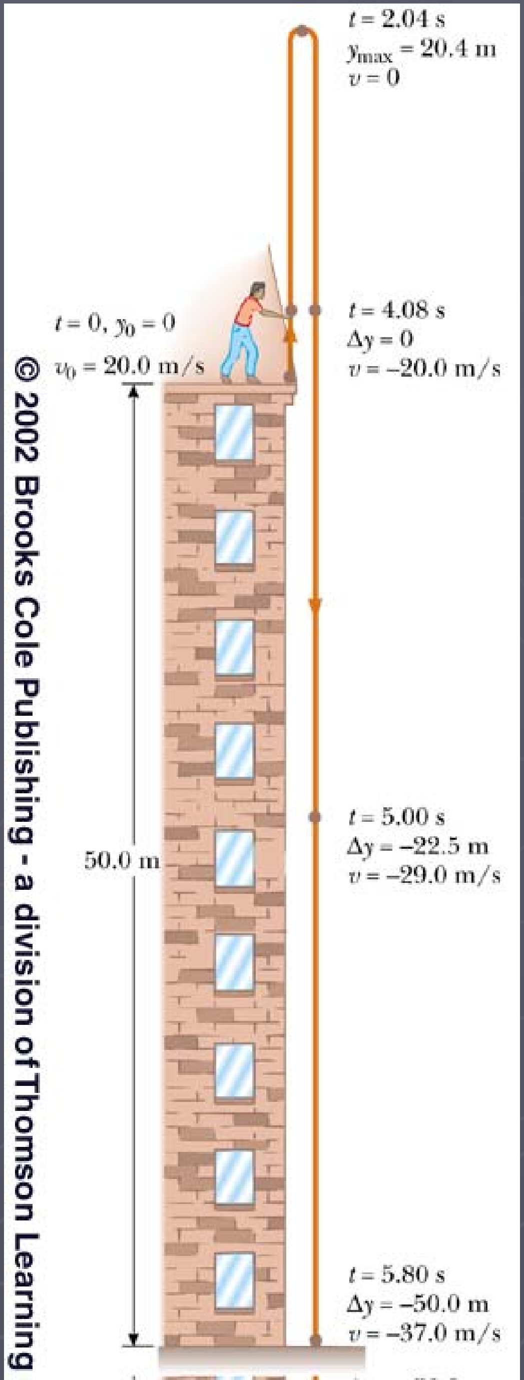


Thrown upward

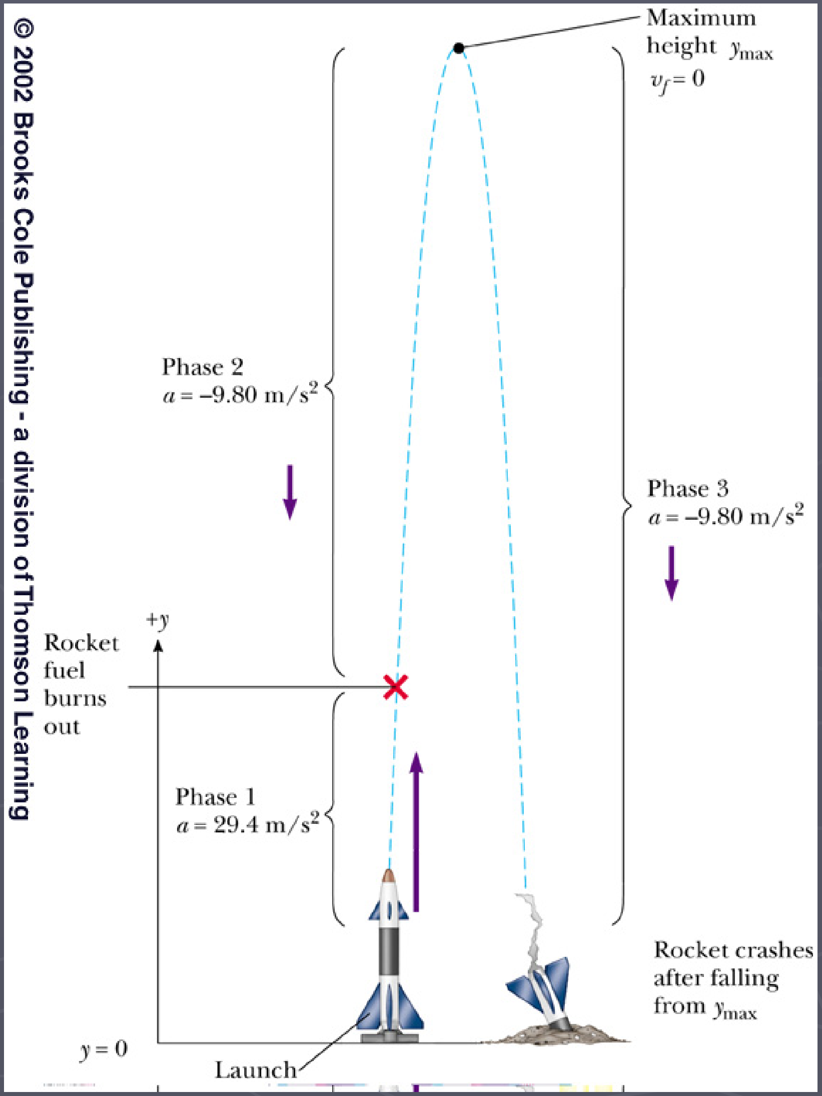
- The motion may be symmetrical
  - then $t_{up} = t_{down}$
  - then $v_f = -v_o$
- The motion may not be symmetrical
  - Break the motion into various parts
    - generally up and down

Non-symmetrical Free Fall

- Need to divide the motion into segments
- Possibilities include
  - Upward and downward portions
  - The symmetrical portion back to the release point and then the non-symmetrical portion



Combination Motions



ConcepTest 3

A person standing at the edge of a cliff throws one ball straight up and another ball straight down at the same initial speed. Neglecting air resistance, the ball to hit ground below the cliff with greater speed is the one initially thrown

1. upward
2. downward
3. neither – they both hit at the same speed

Please fill your answer as question 5 of General Purpose Answer Sheet

ConcepTest 3

A person standing at the edge of a cliff throws one ball straight up and another ball straight down at the same initial speed. Neglecting air resistance, the ball to hit ground below the cliff with greater speed is the one initially thrown

1. upward
2. downward
3. neither – they both hit at the same speed

Please fill your answer as **question 6** of General Purpose Answer Sheet

ConcepTest 3 (answer)

A person standing at the edge of a cliff throws one ball straight up and another ball straight down at the same initial speed. Neglecting air resistance, the ball to hit ground below the cliff with greater speed is the one initially thrown

1. upward
2. downward
3. neither – they both hit at the same speed

Note: upon the descent, the velocity of an object thrown straight up with an initial velocity v is exactly $-v$ when it passes the point at which it was first released.

ConcepTest 3 (answer)

A person standing at the edge of a cliff throws one ball straight up and another ball straight down at the same initial speed. Neglecting air resistance, the ball to hit ground below the cliff with greater speed is the one initially thrown

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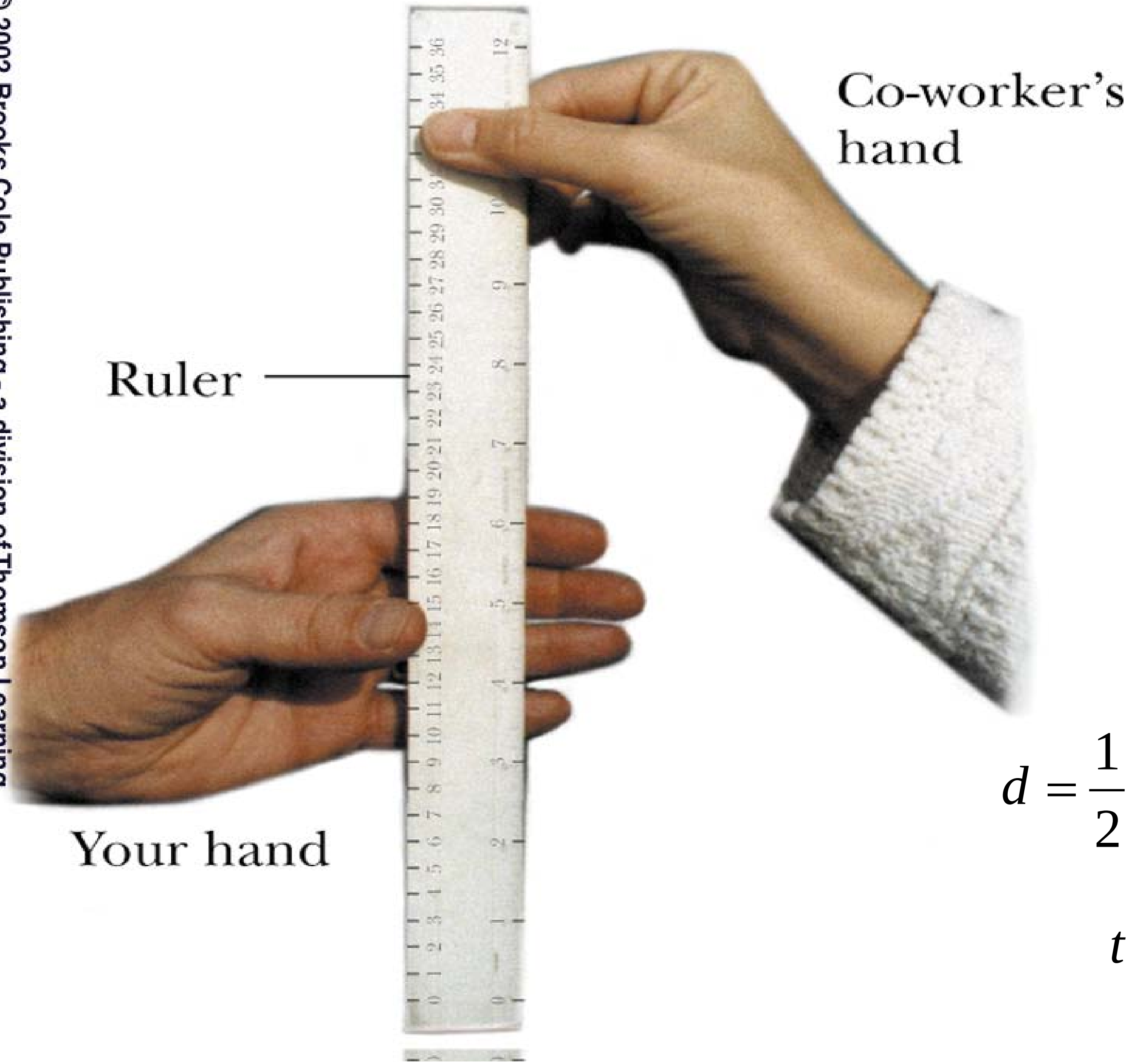
Note: upon the descent, the velocity of an object thrown straight up with an initial velocity v is exactly $-v$ when it passes the point at which it was first released.

Fun QuickLab: Reaction time



$$d = \frac{1}{2} g t^2, \; g = 9.8\, m/s^2$$

$$t = \sqrt{\frac{2d}{g}}$$