Introduction to Mechanics, Heat, and Sound /FIC 318

Lecture III

- Motion in two dimensions
  - projectile motion
- The Laws of Motion
  - Forces, Newton's first law
  - Inertia, Newton's second law
  - Newton's third law
  - Application of Newton's laws

YOGYAKARTA STATE UNIVERSITY

Lightning Review

Last lecture:

1. Motion in one dimension: displacement, velocity, acceleration…
   - ✓ velocity (magnitude & direction) and speed (magnitude)
   - ✓ “velocity” means “instantaneous velocity”

Review Problem: You are throwing a ball straight up in the air. At the highest point, the ball’s

(1) velocity and acceleration are zero

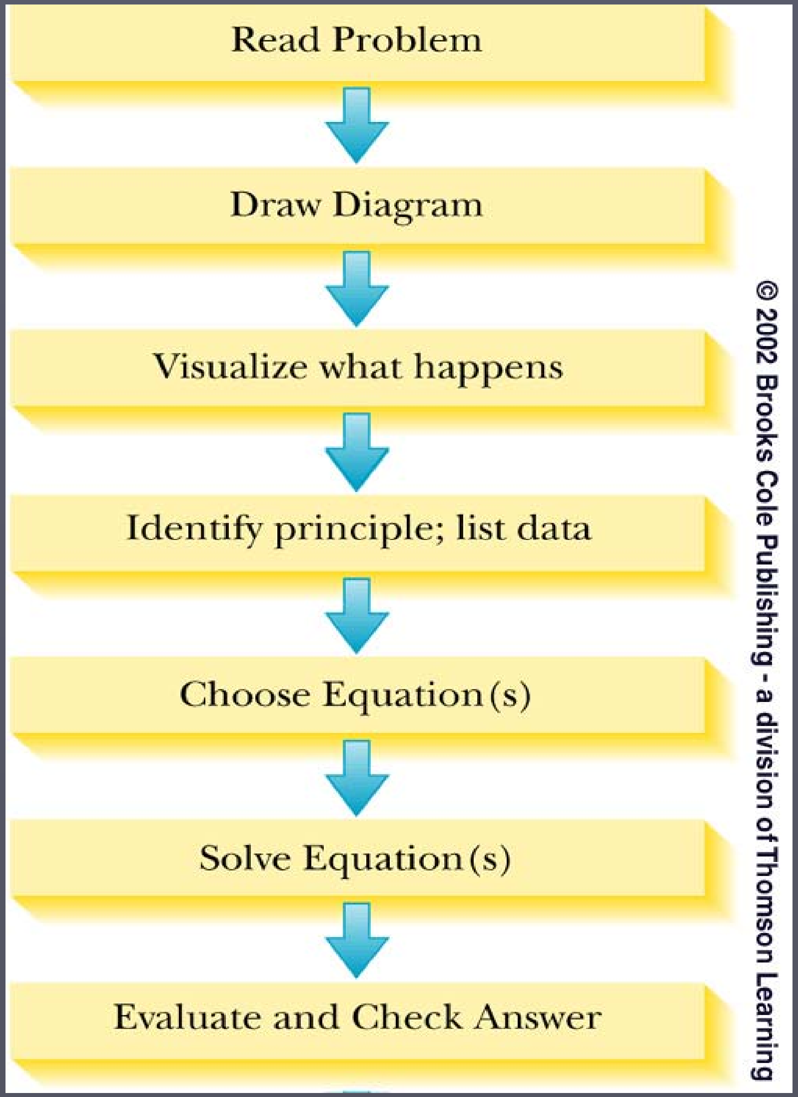
(2) velocity is nonzero but its acceleration is zero

(3) acceleration is nonzero, but its velocity is zero

(4) velocity and acceleration are both nonzero

Problem Solving Strategy

PROBLEM: A firefighter attempts to measure the height of the building by walking out a distance of 46.0 m from its base and shining a flashlight beam towards its top. He finds that when the beam is elevated at an angle of 39.0 , the beam just strikes the top of the building. Find the height of the building.



Problem Solving Strategy

Given:

angle: $\theta = 39.0^{\circ}$
distance: $d = 46.0m$

Find:

Height=?

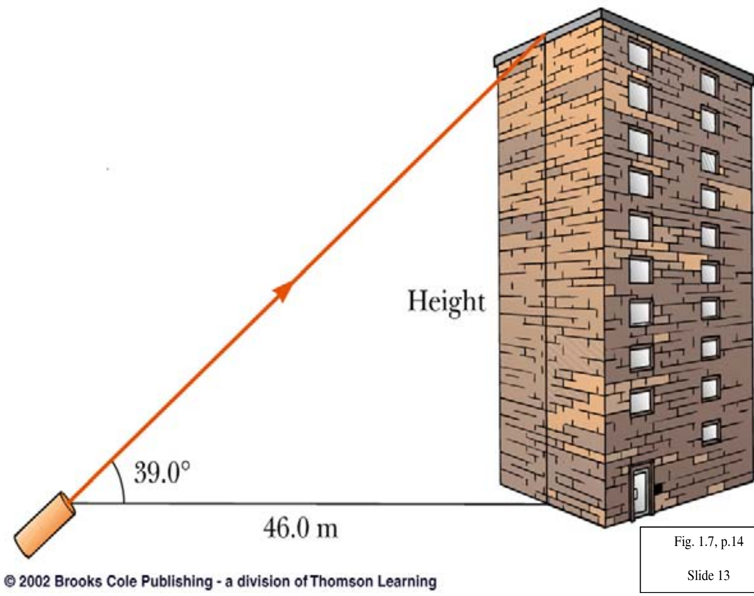


Fig. 1.7, p.14

Slide 13

Key idea: beam of light, building wall and distance from the building to the firefighter form a right triangle! Know: angle and one side, need to determine another side. NOTE: tangent is defined via two sides!

$$\tan\alpha = \frac{height\ of\ building}{dist.},$$

$$height = dist.\times\tan\alpha = (\tan 39.0^{\circ})(46.0\ m) = 37.3\ m$$

Evaluate answer: 1. Makes sense (a 37 m building is Ok)
2. Units are correct.

V. Motion in Two Dimensions

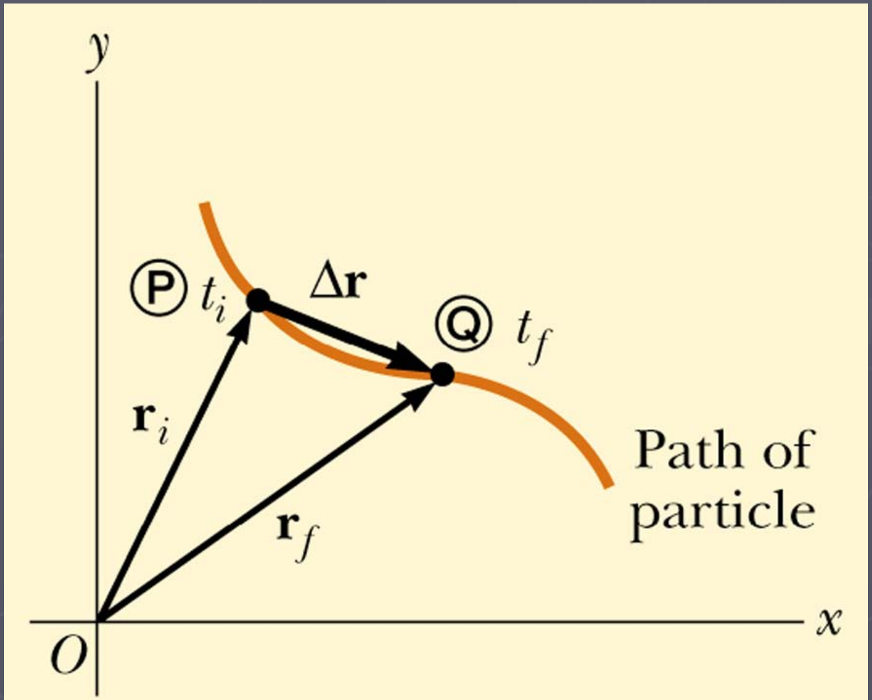
Motion in Two Dimensions

- Using + or – signs is not always sufficient to fully describe motion in more than one dimension
  - Vectors can be used to more fully describe motion
- Still interested in displacement, velocity, and acceleration

Displacement

- The position of an object is described by its position vector, **r**
- The **displacement** of the object is defined as the ***change in its position***

$$\Delta \mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$$



Velocity

- The average velocity is the ratio of the displacement to the time interval for the displacement

$$\bar{v} = \frac{\Delta \vec{r}}{\Delta t}$$

- The instantaneous velocity is the limit of the average velocity as Δt approaches zero
  - The direction of the instantaneous velocity is along a line that is tangent to the path of the particle and in the direction of motion

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

Acceleration

- The average acceleration is defined as the rate at which the velocity changes

$$\bar{a} = \frac{\Delta \vec{v}}{\Delta t}$$

- The instantaneous acceleration is the limit of the average acceleration as Δt approaches zero

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

Ways an Object Might Accelerate

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

- The magnitude of the velocity (the speed) can change
- The direction of the velocity can change
  - Even though the magnitude is constant
- Both the magnitude and the direction can change

Summary of kinematic equations from Lecture II (constant acceleration)

TABLE 2.3 **Equations for Motion in a Straight Line Under Constant Acceleration**

Equation	Information Given by Equation
$v = v_0 + at$	Velocity as a function of time
$\Delta x = \frac{1}{2}(v_0 + v)t$	Displacement as a function of velocity and time
$\Delta x = v_0 t + \frac{1}{2}at^2$	Displacement as a function of time
$v^2 = v_0^2 + 2a\,\Delta x$	Velocity as a function of displacement

Note: Motion is along the x axis. At $t = 0$, the velocity of the particle is v_0.

Example: Projectile Motion

- An object may move in both the x and y directions simultaneously (i.e. in two dimensions)
- The form of two dimensional motion we will deal with is called **projectile motion**
- We may:
  - ignore air friction
  - ignore the rotation of the earth
- With these assumptions, an object in projectile motion will follow a parabolic path

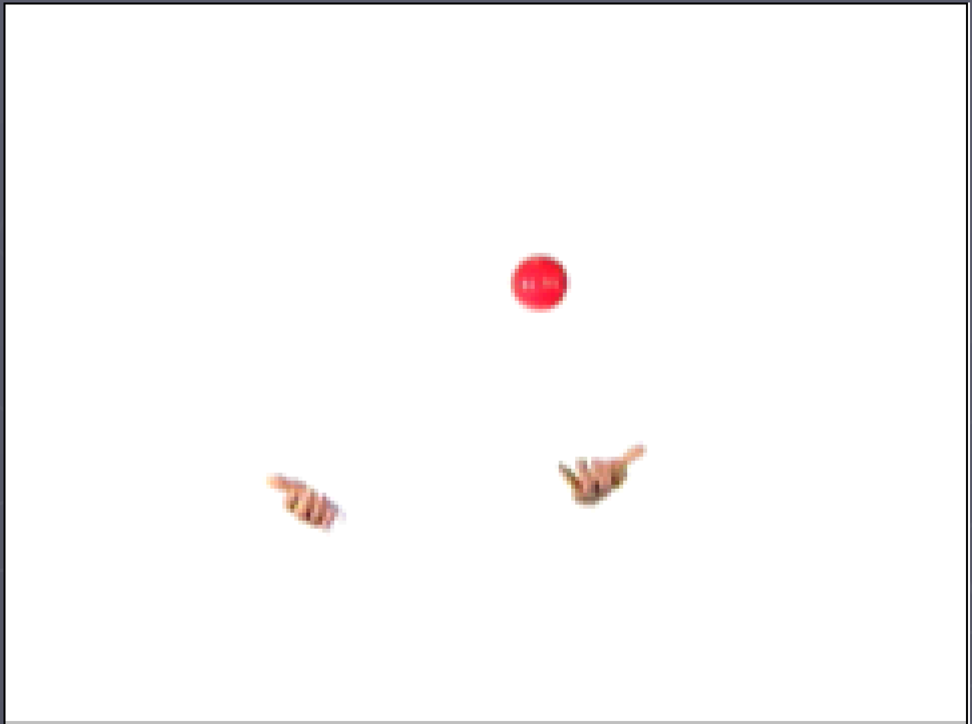
Notes on Projectile Motion:

- once released, only gravity pulls on the object, just like in up-and-down motion

- since gravity pulls on the object downwards:

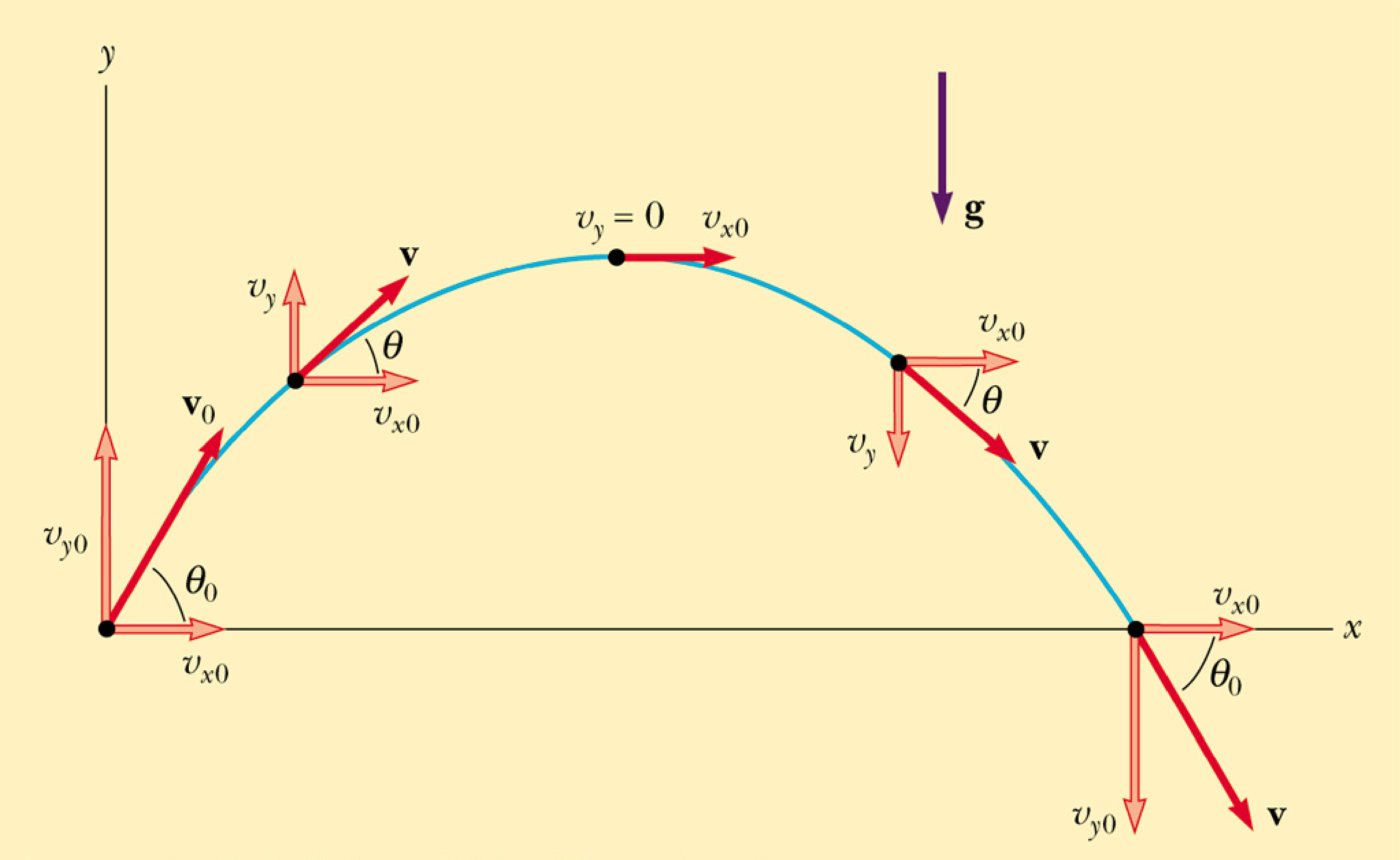
  - ✓ vertical acceleration downwards
  - ✓ NO acceleration in horizontal direction

Let's watch the movie!



projmtn.mov

Projectile Motion



Rules of Projectile Motion

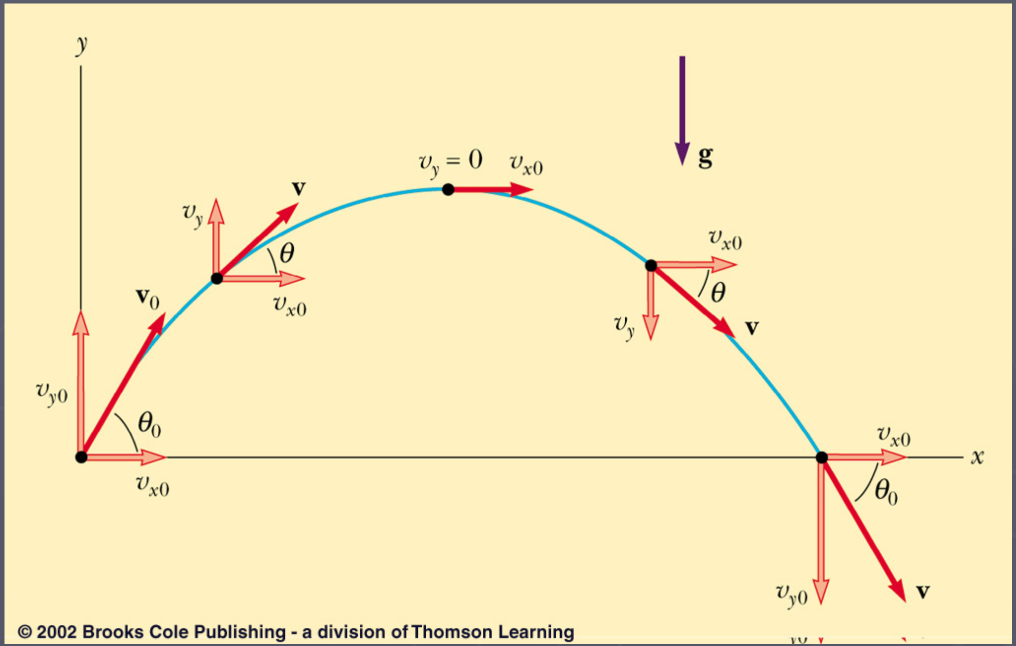
- Introduce coordinate frame: y is up
- The x- and y-components of motion can be treated independently
- Velocities (incl. initial velocity) can be broken down into its x- and y-components
- The x-direction is uniform motion

$$a_x = 0$$

- The y-direction is free fall

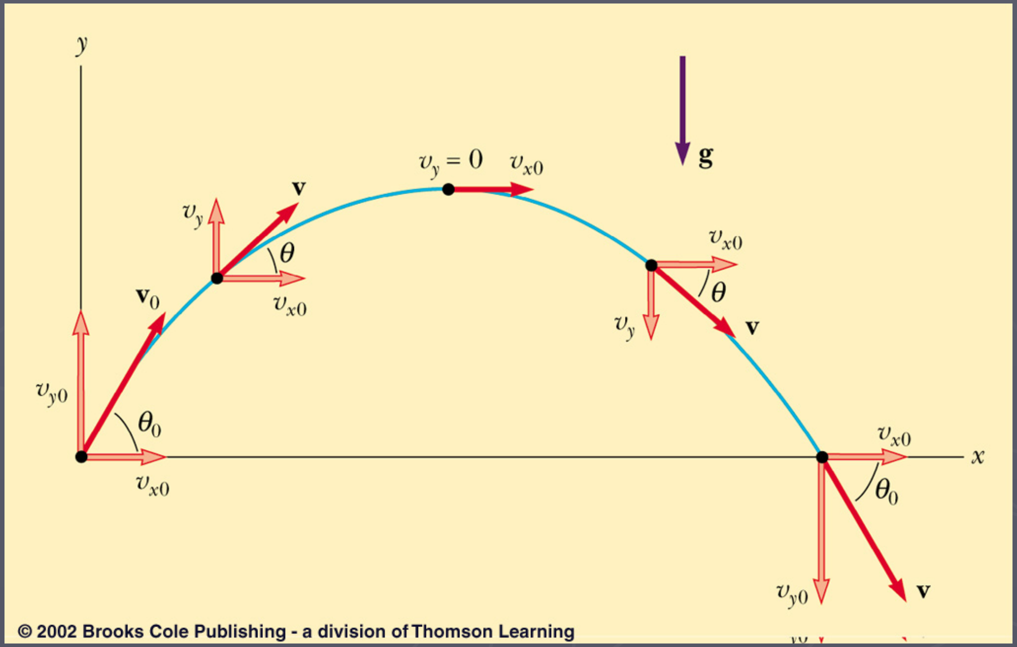
$$|a_y| = g$$

Some Details About the Rules



- x-direction
  - $a_x = 0$
  - $v_{xo} = v_o \cos\theta_o = v_x = \text{constant}$
  - $x = v_{xo}t$
    - This is the only operative equation in the x-direction since there is uniform velocity in that direction

More Details About the Rules



- y-direction
  - $v_{yo} = v_o \sin\theta_o$
  - take the positive direction as upward
  - then: free fall problem
    - only then: $a_y = -g$ (in general, $|a_y| = g$)
  - uniformly accelerated motion, so the motion equations all hold

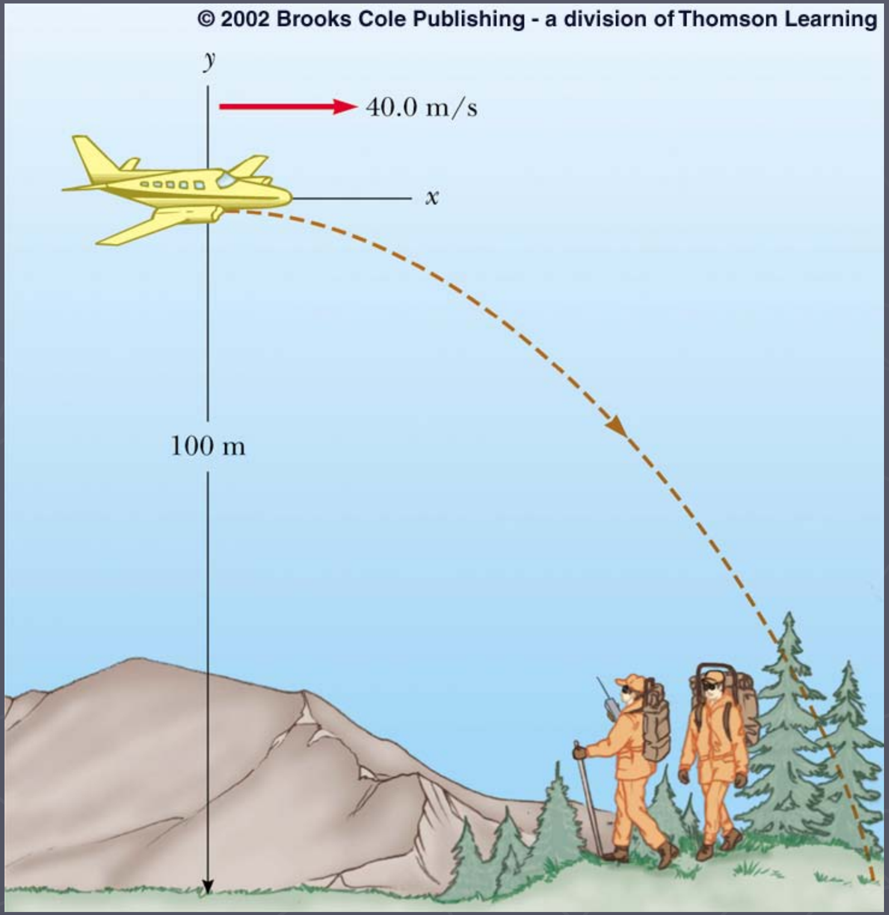
Velocity of the Projectile

- The velocity of the projectile at any point of its motion is the vector sum of its x and y components at that point

$$v = \sqrt{v_x^2 + v_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{v_y}{v_x}$$

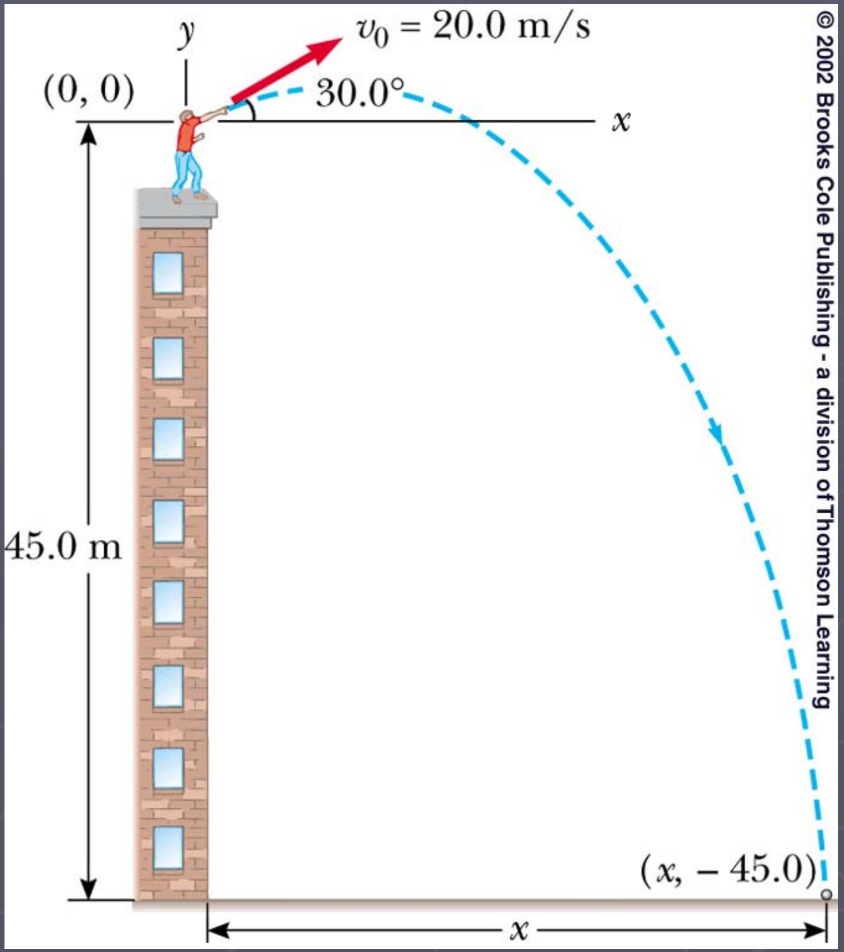
Examples of Projectile Motion:

- An object may be fired horizontally
- The initial velocity is all in the x-direction
  - $v_o = v_x$ and $v_y = 0$
- All the general rules of projectile motion apply



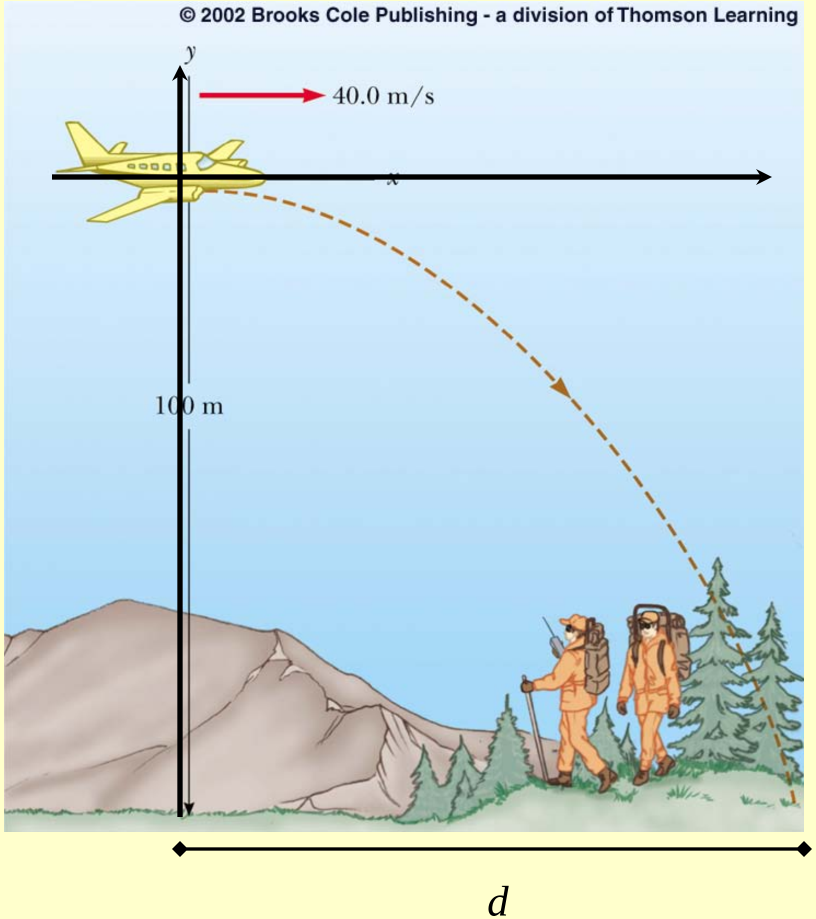
Non-Symmetrical Projectile Motion

- Follow the general rules for projectile motion
- Break the y-direction into parts
  - up and down
  - symmetrical back to initial height and then the rest of the height



Example problem:

An Alaskan rescue plane drops a package of emergency rations to a stranded party of explorers. The plane is traveling horizontally at 40.0 m/s at a height of 100 m above the ground.
Where does the package strike the ground relative to the point at which it was released?



Given:

velocity: v=40.0 m/s
height: h=100 m

Find:

Distance d=?

1. Introduce coordinate frame:
   Oy: y is directed up
   Ox: x is directed right

2. Note: $v_{ox} = v = +40$ m/s
   $v_{oy} = 0$ m/s

$$\underline{Oy}: y = \frac{1}{2} g t^2, \; so \quad t = \sqrt{\frac{2y}{g}}$$

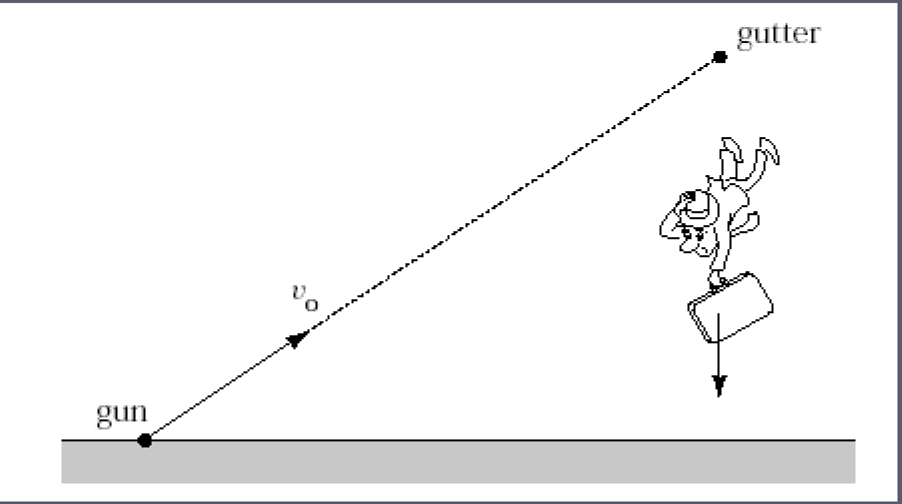
$$or: \quad t = \sqrt{\frac{2(-100\,m)}{-9.8\,m/s^2}} = 4.51\,s$$

$$\underline{Ox}: x = v_{x0} t, \; so \quad x = (40\,m/s)(4.51\,s) = \underline{180\,m}$$

✓

ConcepTest 1

Consider the situation depicted here. A gun is accurately aimed at a dangerous criminal hanging from the gutter of a building. The target is well within the gun's range, but the instant the gun is fired and the bullet moves with a speed v_o, the criminal lets go and drops to the ground. What happens? The bullet



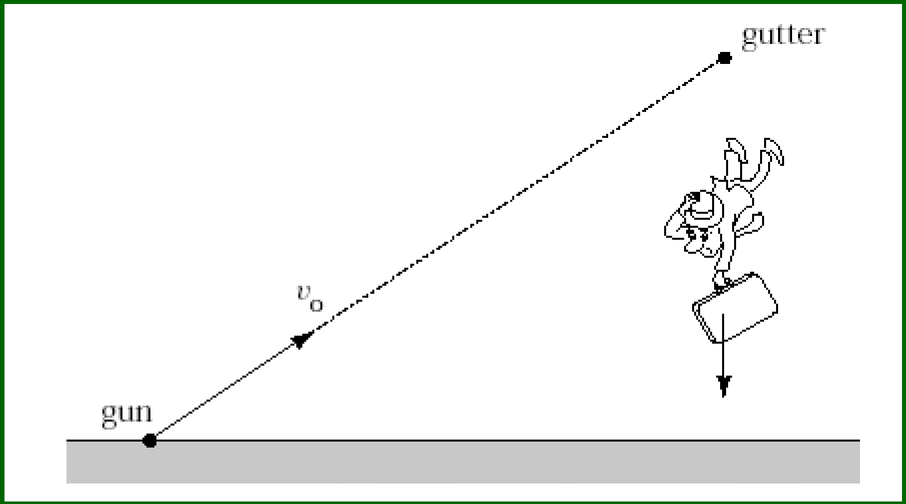
1. hits the criminal regardless of the value of v_o.
2. hits the criminal only if v_o is large enough.
3. misses the criminal.

Please fill your answer as question 7 of General Purpose Answer Sheet

ConcepTest 1

Convince your neighbor!

Consider the situation depicted here. A gun is accurately aimed at a dangerous criminal hanging from the gutter of a building. The target is well within the gun's range, but the instant the gun is fired and the bullet moves with a speed v_o, the criminal lets go and drops to the ground. What happens? The bullet

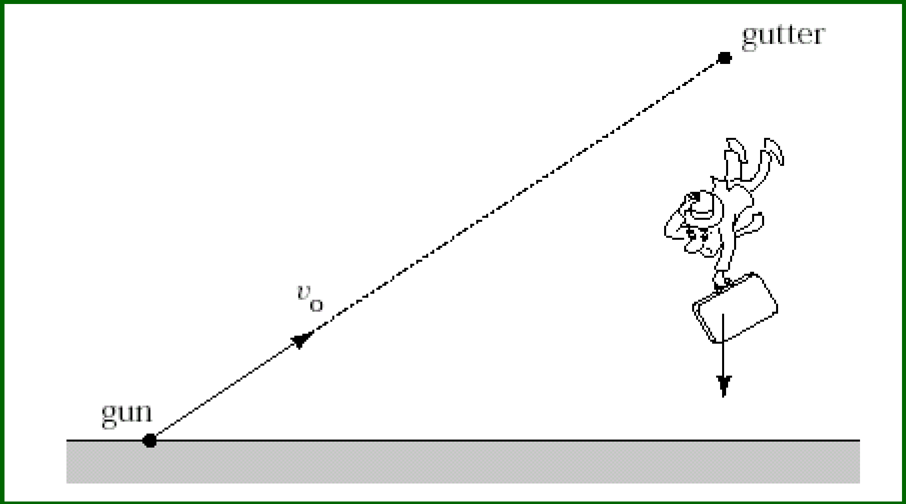


1. hits the criminal regardless of the value of v_o.
2. hits the criminal only if v_o is large enough.
3. misses the criminal.

Please fill your answer as **question 8** of General Purpose Answer Sheet

ConcepTest 1

Consider the situation depicted here. A gun is accurately aimed at a dangerous criminal hanging from the gutter of a building. The target is well within the gun's range, but the instant the gun is fired and the bullet moves with a speed v_o, the criminal lets go and drops to the ground. What happens? The bullet



1. hits the criminal regardless of the value of v_o. ✓
2. hits the criminal only if v_o is large enough.
3. misses the criminal.

Note: The downward acceleration of the bullet and the criminal are identical, so the bullet will hit the target – they both "fall" the same distance!

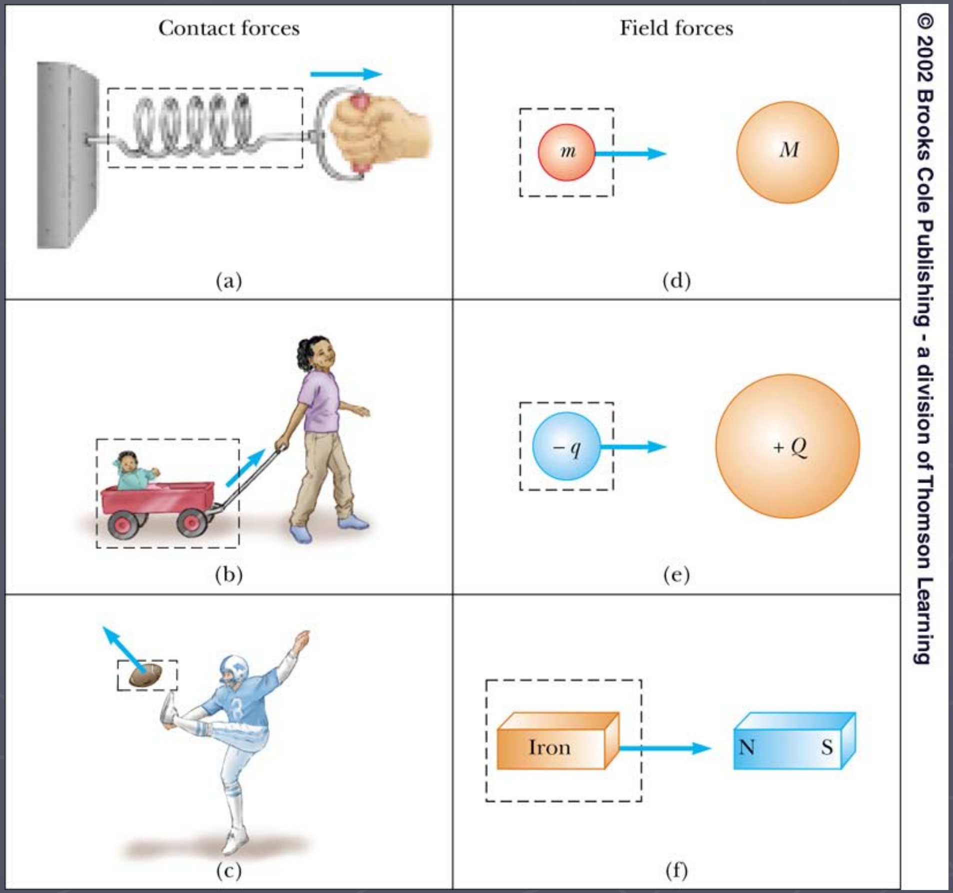
VI. The Laws of Motion

Classical Mechanics

- Describes the relationship between the motion of objects in our everyday world and the forces acting on them
- Conditions when Classical Mechanics does not apply
  - very tiny objects (< atomic sizes)
  - objects moving near the speed of light

Forces

- Usually think of a force as a push or pull
- Vector quantity
- May be contact
  or
  field force



Fundamental Forces

- Types
  - Strong nuclear force
  - Electromagnetic force
  - Weak nuclear force
  - Gravity
- Characteristics
  - All field forces
  - Listed in order of decreasing strength
  - Only gravity and electromagnetic in mechanics

Newton's First Law

- If no forces act on an object, it continues in its original state of motion; that is, unless something exerts an external force on it, an object at rest remains at rest and an object moving with some velocity continues with that same velocity.

Newton’s First Law, cont.

- External force
  - any force that results from the interaction between the object and its environment
- Alternative statement of Newton’s 1st Law
  - When there are no external forces acting on an object, the acceleration of the object is zero.

Let's watch the movie!

Inertia and Mass

- Inertia is the tendency of an object to continue in its original motion
- Mass is a measure of the inertia, i.e resistance of an object to changes in its motion due to a force
- Recall: mass is a scalar quantity

Units of mass	
SI	kilograms (kg)
CGS	grams (g)
US Customary	slug (slug)

Inertia and Mass: Two Examples

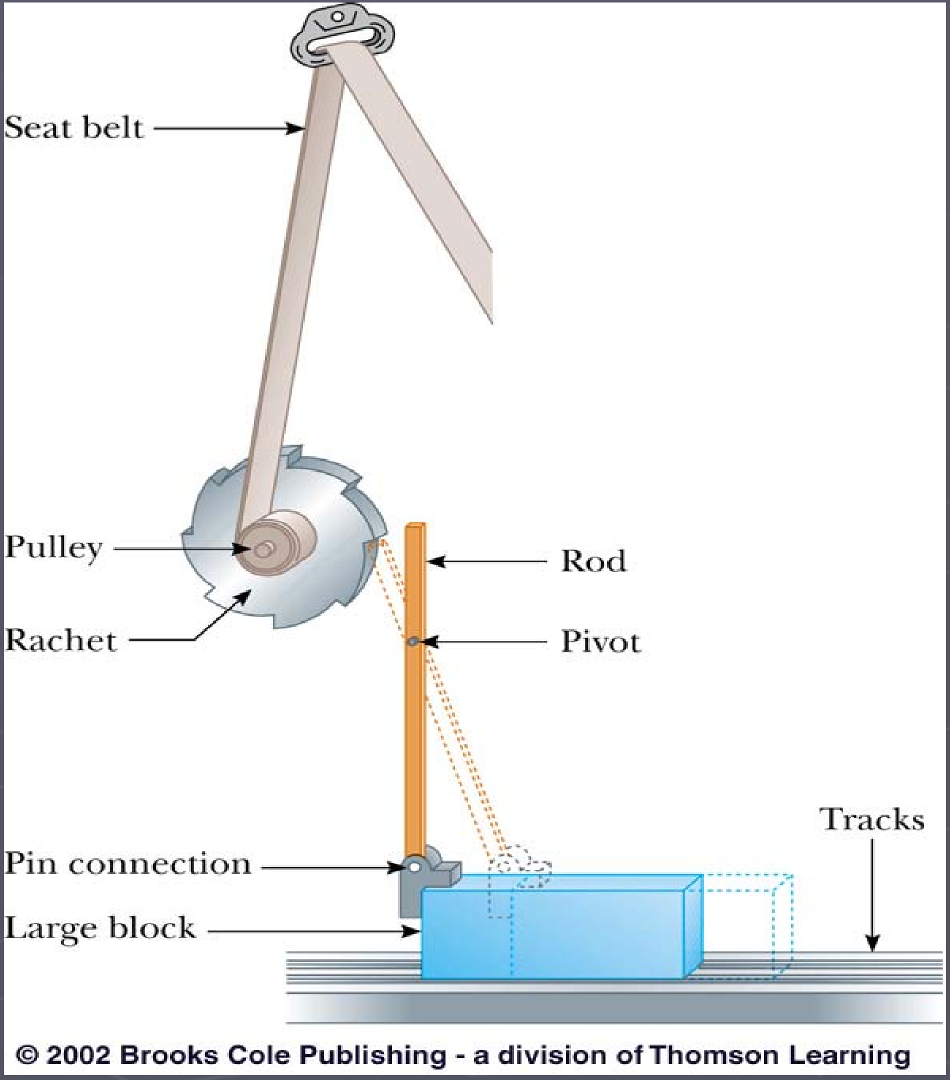
Runaway train

- Inertia is the tendency of an object to continue in its original motion
- Mass is a measure of the inertia, i.e resistance of an object to changes in its motion due to a force

Inertia and Mass: Two Examples

Seatbelt

- Inertia is the tendency of an object to continue in its original motion
- Mass is a measure of the inertia, i.e resistance of an object to changes in its motion due to a force



Newton's Second Law

- The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

$$\vec{a} \propto \frac{\sum \vec{F}}{m} \quad or \quad \sum \vec{F} = m\vec{a}$$

  - **F** and **a** are both vectors
- Can also be applied three-dimensionally
  - acceleration can also be caused by change of the direction of velocity

Newton's Second Law

- Note: $\sum \vec{F}$ represents the vector sum of all external forces acting on the object.
- Since N2ndL is a vector equation, we can always write it in terms of components:

$$\sum \vec{F} = m\vec{a}: \quad \begin{cases} F_x = ma_x \\ F_y = ma_y \\ F_z = ma_z \end{cases}$$

Units of Force

- SI unit of force is a Newton (N)

$$1\,\text{N} \equiv 1\,\frac{\text{kg m}}{\text{s}^2}$$

Units of force	
SI	Newton ($N = kg\ m/s^2$)
CGS	Dyne ($dyne = g\ cm/s^2$)
US Customary	Pound ($lb = slug\ ft/s^2$)

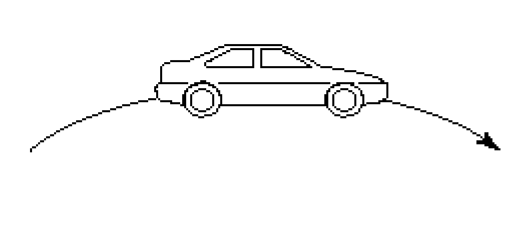
- $1\ N = 10^5\ dyne = 0.225\ lb$

Example: force on the bullet

A 5.0-g bullet leaves the muzzle of a rifle with a speed of 320 m/s. What total force (assumed constant) is exerted on the bullet while it is traveling down the 1-m-long barrel of the rifle?

ConcepTest 2

A car rounds a curve while maintaining a constant speed. Is there a net force on the car as it rounds the curve?

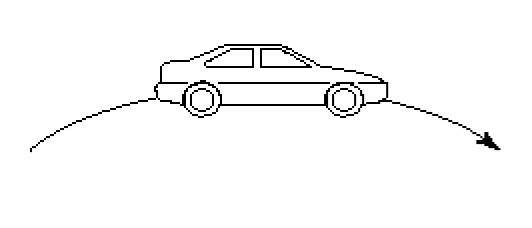


1. No—its speed is constant.
2. Yes.
3. It depends on the sharpness of the curve and the speed of the car.
4. It depends on the driving experience of the driver.

Please fill your answer as question 9 of General Purpose Answer Sheet

ConcepTest 2

A car rounds a curve while maintaining a constant speed. Is there a net force on the car as it rounds the curve?

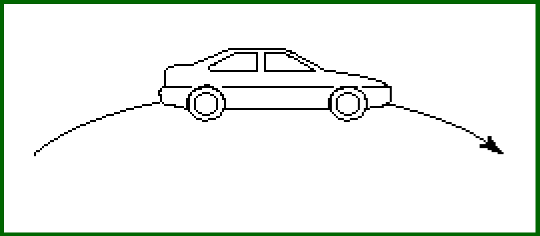


1. No—its speed is constant.
2. Yes.
3. It depends on the sharpness of the curve and the speed of the car.
4. It depends on the driving experience of the driver.

Please fill your answer as **question 10** of General Purpose Answer Sheet

ConcepTest 2

A car rounds a curve while maintaining a constant speed. Is there a net force on the car as it rounds the curve?



1. No—its speed is constant.
2. Yes. ✓
3. It depends on the sharpness of the curve and the speed of the car.
4. It depends on the driving experience of the driver.

Note: Acceleration is a change in the speed and/or direction of an object. Thus, because its direction has changed, the car has accelerated and a force must have been exerted on it.

Gravitational Force

- Mutual force of attraction between any two objects
- Expressed by Newton's Law of Universal Gravitation:

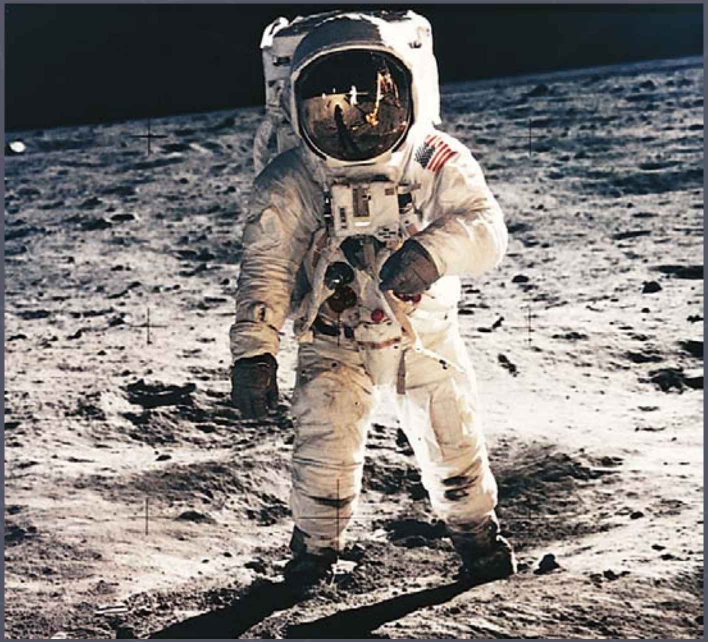
$$F_g = G\frac{m_1 m_2}{r^2}$$

Weight

- The magnitude of the gravitational force acting on an object of mass *m* near the Earth's surface is called the weight *w* of the object
  - $w = m\,g$ is a special case of Newton's Second Law
- *g* can also be found from the Law of Universal Gravitation

More about weight

- Weight is **not** an inherent property of an object
  - mass **is** an inherent property
- Weight depends upon location



Let's watch two movies!

astron~1.mov

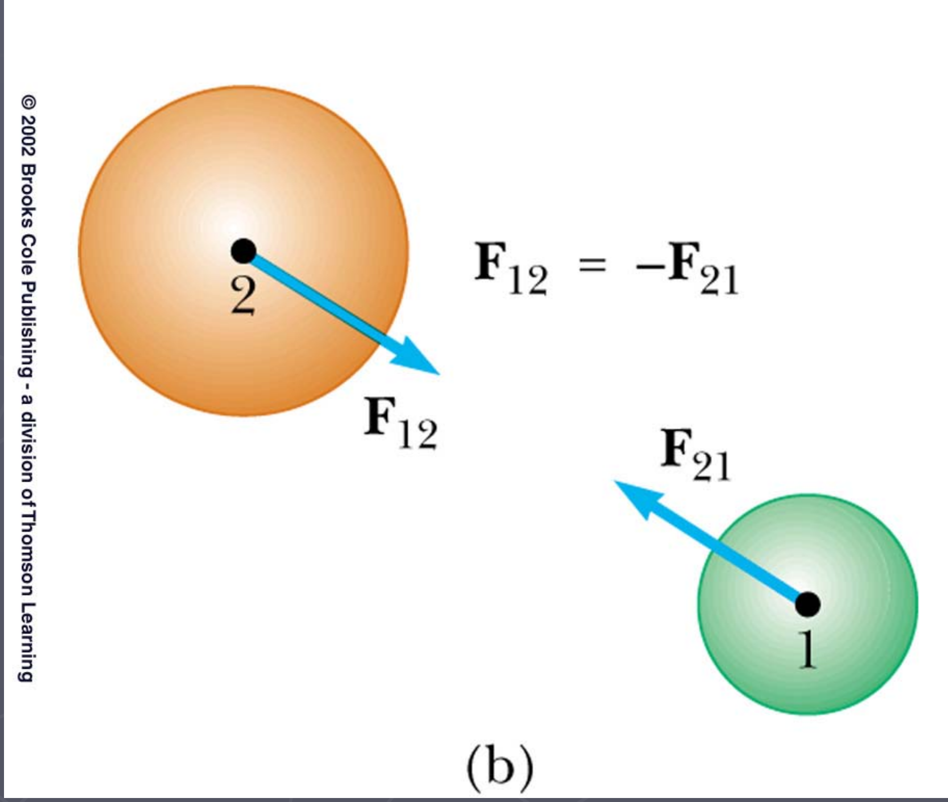
weight~1.mov

Newton's Third Law

- If two objects interact, the force F_{12} exerted by object 1 on object 2 is equal in magnitude but opposite in direction to the force F_{21} exerted by object 2 on object 1.
  - Equivalent to saying a single isolated force cannot exist

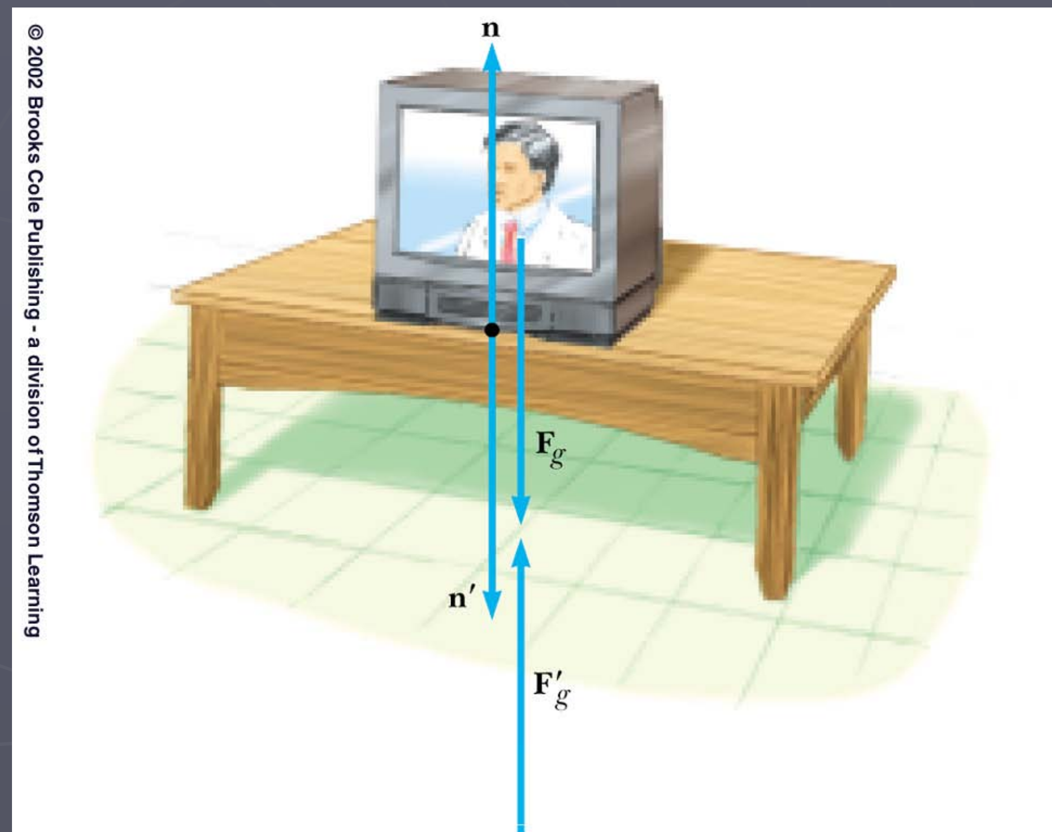
Example: Newton's Third Law

- Consider collision of two spheres
- F_{12} may be called the *action* force and F_{21} the *reaction* force
  - Actually, either force can be the action or the reaction force
- The action and reaction forces act on **different** objects



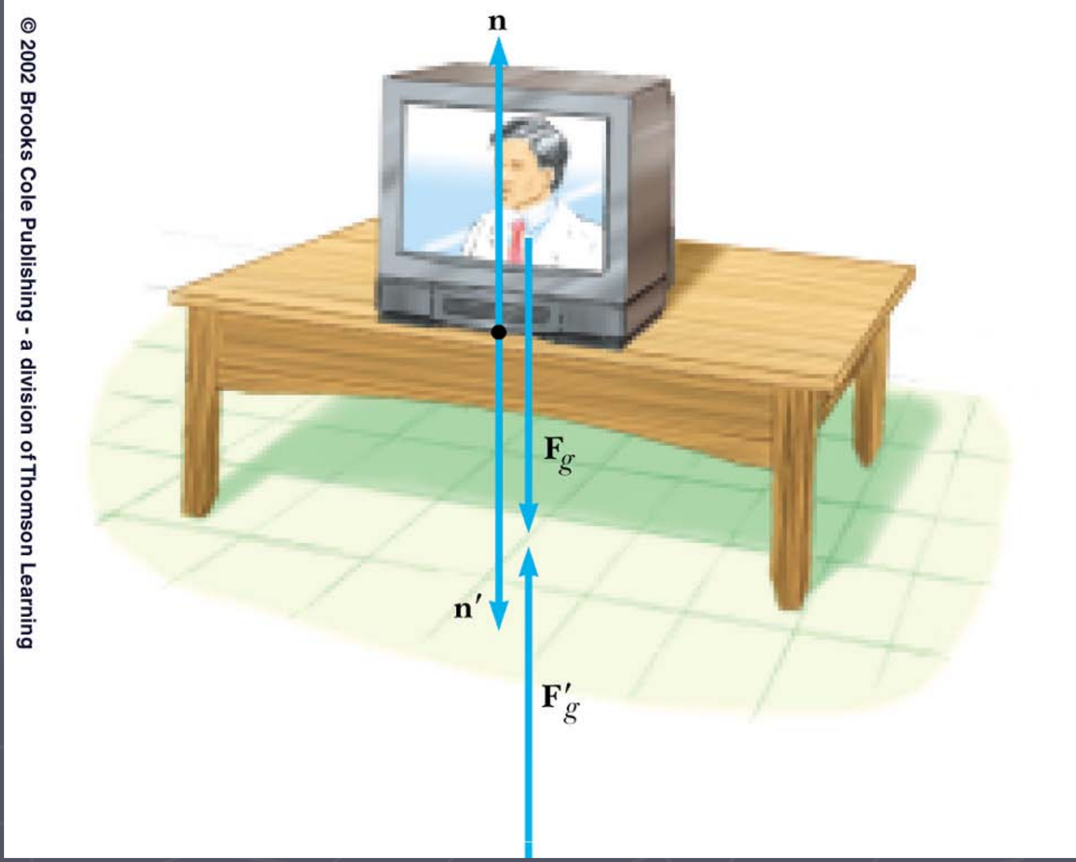
Example 1: Action-Reaction Pairs

- n and n’
  - n is the *normal* force, the force the table exerts on the TV
  - n is always perpendicular to the surface
  - n’ is the reaction – the TV on the table
  - n = - n’



Example 2: Action-Reaction pairs

- F_g and F_g'
  - F_g is the force the Earth exerts on the object
  - F_g' is the force the object exerts on the earth
  - $F_g = -F_g$'



Forces Acting on an Object

- Newton's Law uses the forces acting *on* an object
- n and F_g are acting on the object
- n' and F_g' are acting on other objects

ConcepTest 3

Consider a person standing in an elevator that is accelerating upward. The upward normal force *N* exerted by the elevator floor on the person is

1. larger than
2. identical to
3. smaller than
4. equal to zero, i.e. irrelevant to

the downward weight *W* of the person.

Please fill your answer as question 11 of General Purpose Answer Sheet

ConcepTest 3

Consider a person standing in an elevator that is accelerating upward. The upward normal force *N* exerted by the elevator floor on the person is

1. larger than
2. identical to
3. smaller than
4. equal to zero, i.e. irrelevant to

the downward weight *W* of the person.

Please fill your answer as **question 12** of General Purpose Answer Sheet

ConcepTest 3

Consider a person standing in an elevator that is accelerating upward. The upward normal force *N* exerted by the elevator floor on the person is

1. larger than ✓
2. identical to
3. smaller than
4. equal to zero, i.e. irrelevant to

the downward weight *W* of the person.

Note: In order for the person to be accelerated upward, the normal force exerted by the elevator floor on her must exceed her weight.

Applying Newton's Laws

- Assumptions
  - Objects behave as particles
    - can ignore rotational motion (for now)
  - Masses of strings or ropes are negligible
  - Interested only in the forces acting on the object
    - can neglect reaction forces

Free Body Diagram

- Must identify all the forces acting on the object of interest
- Choose an appropriate coordinate system
- If the free body diagram is incorrect, the solution will likely be incorrect

Applying Newton's Laws

- Make a sketch of the situation described in the problem, introduce a coordinate frame
- Draw a free body diagram for the isolated object under consideration and label all the forces acting on it
- Resolve the forces into x- and y-components, using a convenient coordinate system
- Apply equations, keeping track of signs
- Solve the resulting equations

Equilibrium

- An object either at rest or moving with a constant velocity is said to be in *equilibrium*
- The net force acting on the object is zero

$$\sum \vec{F} = 0 \quad \rightarrow \quad \sum F_x = 0, \quad \sum F_y = 0$$

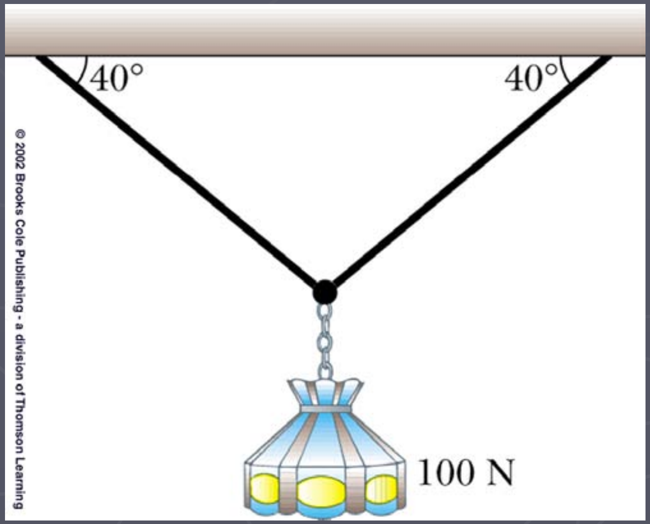
- Easier to work with the equation in terms of its components

Solving Equilibrium Problems

- Make a sketch of the situation described in the problem
- Draw a free body diagram for the isolated object under consideration and label all the forces acting on it
- Resolve the forces into x- and y-components, using a convenient coordinate system
- Apply equations, keeping track of signs
- Solve the resulting equations

Example 1. Equilibrium problems

Find the tension in the two wires that support the 100-N light fixture in figure below



Newton’s Second Law Problems

- Similar to equilibrium except

$$\sum \vec{F} = m\vec{a}$$

- Use components

$$\sum F_x = ma_x \qquad \sum F_y = ma_y$$

- a_x or a_y may be zero

Solving Newton's Second Law Problems

- Make a sketch of the situation described in the problem
- Draw a free body diagram for the isolated object under consideration and label all the forces acting on it
  - If more than one object is present, draw free body diagram for each object
- Resolve the forces into x- and y-components, using a convenient coordinate system
- Apply equations, keeping track of signs
- Solve the resulting equations

Forces of Friction

- When an object is in motion on a surface or through a viscous medium, there will be a resistance to the motion
  - This is due to the interactions between the object and its environment
- This is resistance is called the *force of friction*

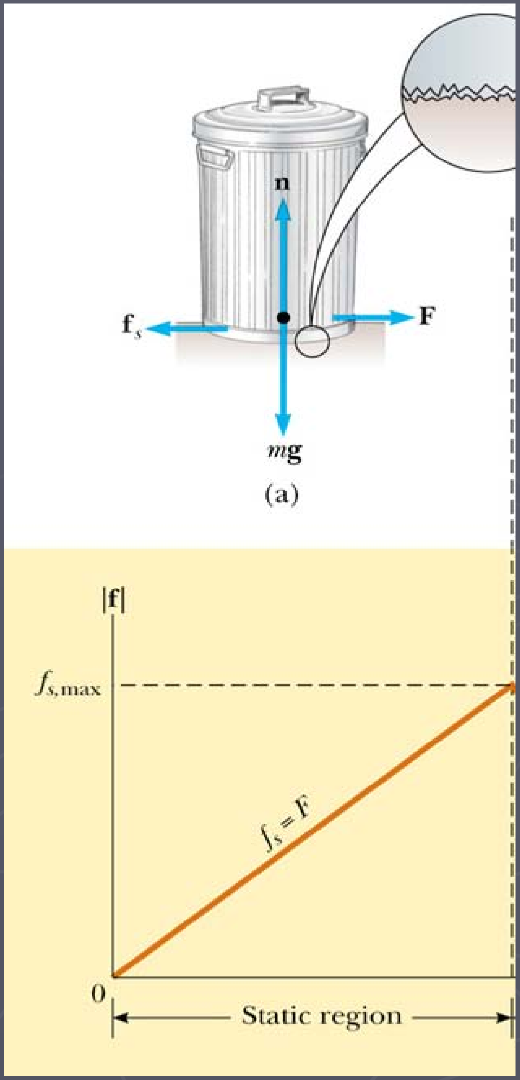
More About Friction

- Friction is proportional to the normal force
- The force of static friction is generally greater than the force of kinetic friction
- The coefficient of friction (μ) depends on the surfaces in contact
- The direction of the frictional force is opposite the direction of motion
- The coefficients of friction are nearly independent of the area of contact

Static Friction, f_s

- Static friction acts to keep the object from moving
- If F increases, so does f_s
- If F decreases, so does f_s

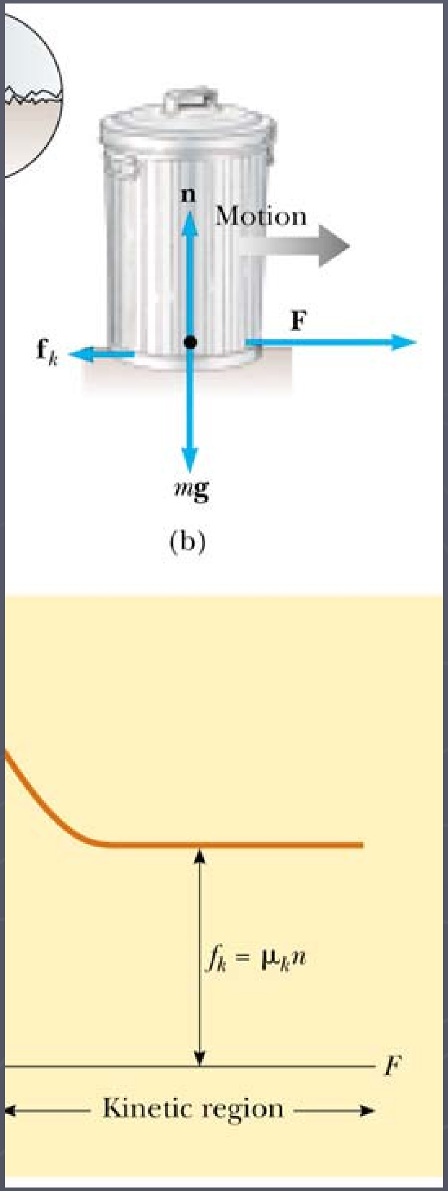
$$f_s \le \mu n$$



Kinetic Friction

- The force of kinetic friction acts when the object is in motion

$$f_k = \mu\ n$$



ConcepTest 4

You are pushing a wooden crate across the floor at constant speed.You decide to turn the crate on end, reducing by half the surface area in contact with the floor. In the new orientation, to push the same crate across the same floor with the same speed, the force that you apply must be about

1. four times as great
2. twice as great
3. equally great
4. half as great
5. one-fourth as great

as the force required before you changed the crate's orientation.

Please fill your answer as question 13 of General Purpose Answer Sheet

ConcepTest 4

You are pushing a wooden crate across the floor at constant speed.You decide to turn the crate on end, reducing by half the surface area in contact with the floor. In the new orientation, to push the same crate across the same floor with the same speed, the force that you apply must be about

1. four times as great
2. twice as great
3. equally great
4. half as great
5. one-fourth as great

as the force required before you changed the crate's orientation.

Please fill your answer as **question 14** of General Purpose Answer Sheet

ConcepTest 4

You are pushing a wooden crate across the floor at constant speed.You decide to turn the crate on end, reducing by half the surface area in contact with the floor. In the new orientation, to push the same crate across the same floor with the same speed, the force that you apply must be about

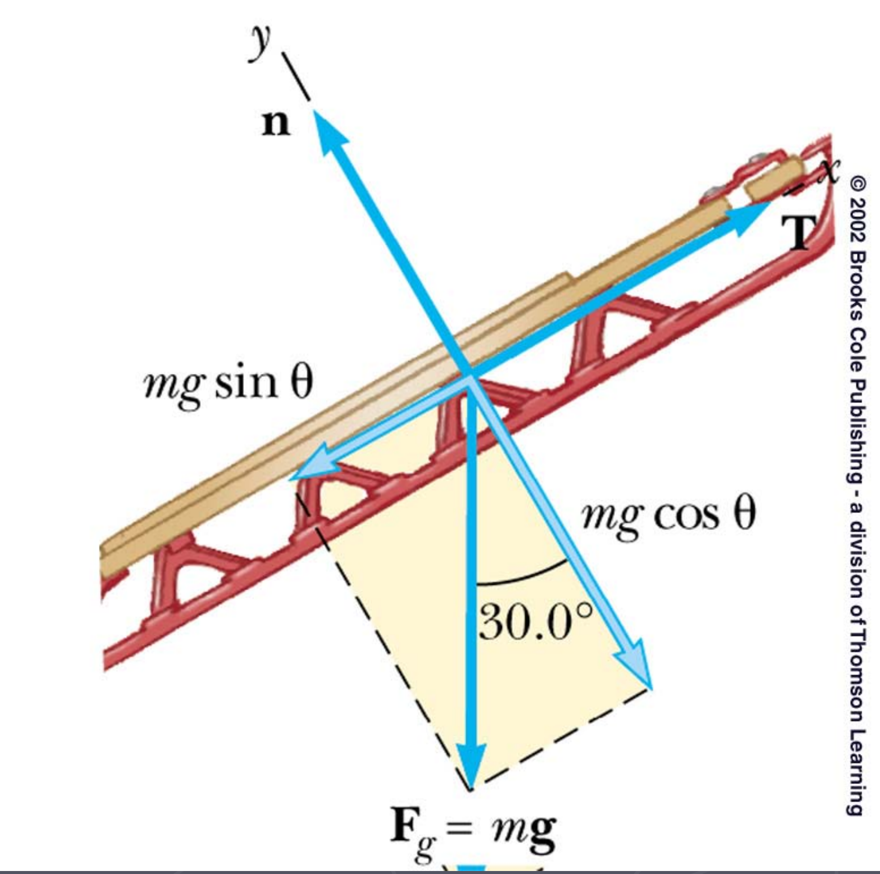
1. four times as great
2. twice as great
3. equally great ✓
4. half as great
5. one-fourth as great

as the force required before you changed the crate's orientation.

Note: The force is proportional to the coefficient of kinetic friction and the weight of the crate. Neither depends on the size of the surface in contact with the floor.

Inclined Planes

- Choose the coordinate system with x along the incline and y perpendicular to the incline
- Replace the force of gravity with its components



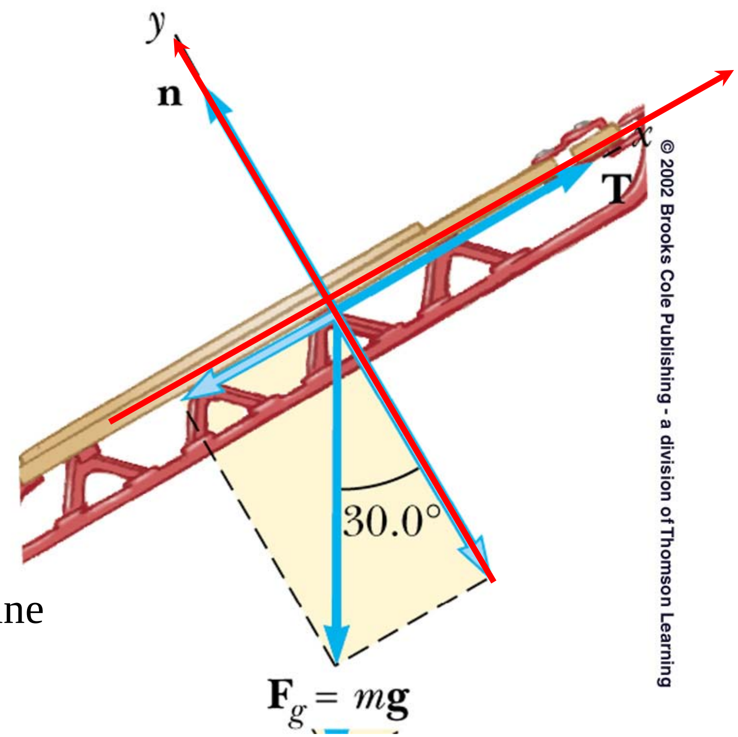
Example: Inclined Planes

Problem:

A child holds a sled at rest on frictionless, snow-covered hill, as shown in figure. If the sled weights 77.0 N, find the force **T** exerted by the rope on the sled and the force **n** exerted by the hill on the sled.

Example: Inclined Planes

- Choose the coordinate system with x along the incline and y perpendicular to the incline
- Replace the force of gravity with its components



<u>Given:</u>

angle: α=30

weight: w=77.0 N

<u>Find</u>:

Tension **T**=?

Normal **n**=?

1. Introduce <u>coordinate frame</u>:

Oy: y is directed perp. to incline

Ox: x is directed right, along incline

$$\underline{Note}: \sum \vec{F} = 0$$

$$Ox: \sum F_x = T - mg\sin\alpha = 0,$$

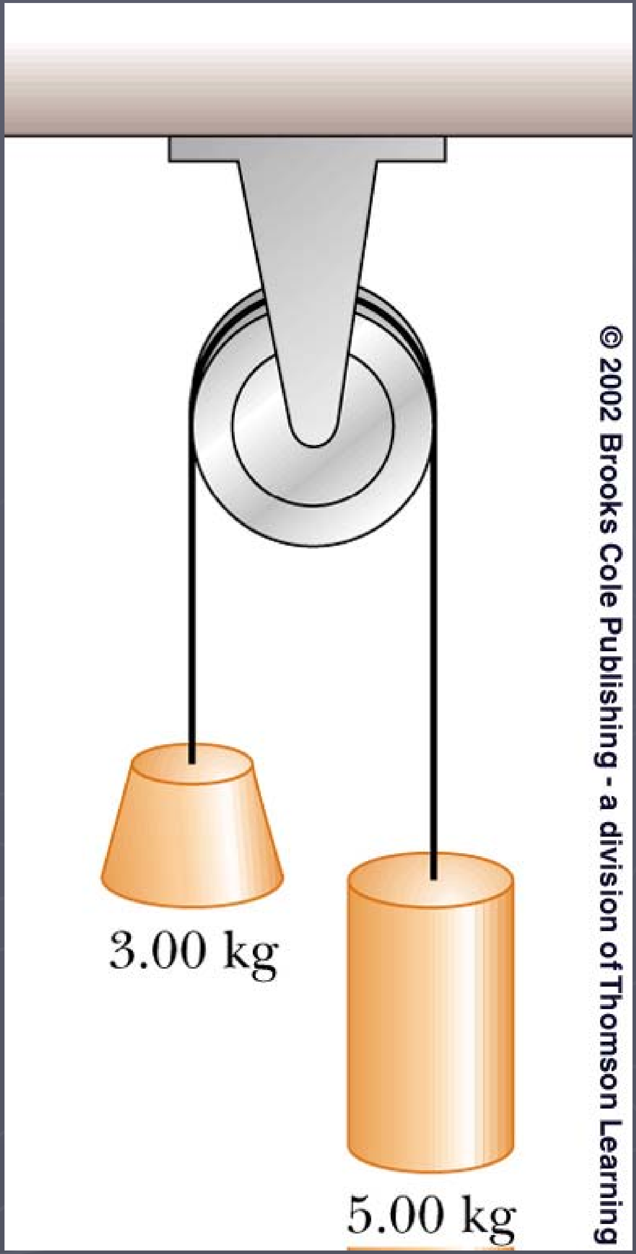
$$T = mg(\sin 30^\circ) = 77.0N(\sin 30^\circ) = \underline{38.5N}$$ ✓

$$Oy: \sum F_y = n - mg\cos\alpha = 0,$$

$$T = mg(\cos 30^\circ) = 77.0N(\cos 30^\circ) = \underline{66.7N}$$ ✓

Connected Objects

- Apply Newton's Laws separately to each object
- The acceleration of both objects will be the same
- The tension is the same in each diagram
- Solve the simultaneous equations



More About Connected Objects

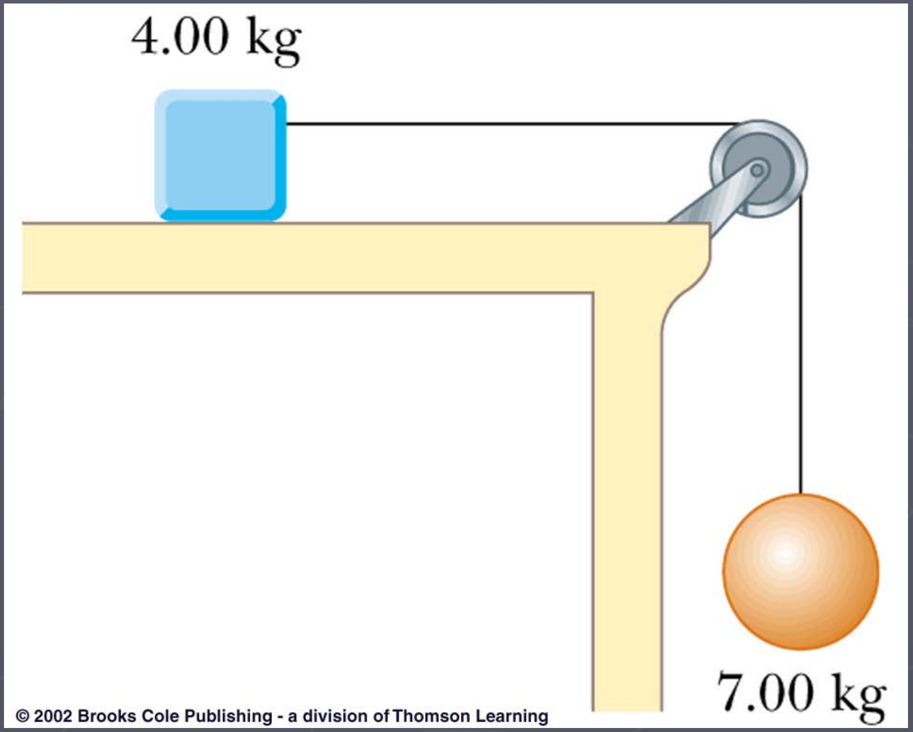
- Treating the system as one object allows an alternative method or a check
  - Use only external forces
    - Not the tension – it's internal
  - The mass is the mass of the system
- Doesn't tell you anything about any internal forces

Example: Connected Objects

Problem:

Two objects m_1=4.00 kg and m_2=7.00 kg are connected by a light string that passes over a frictionless pulley. The coefficient of sliding friction between the 4.00 kg object an the surface is 0.300. Find the acceleration of the two objects and the tension of the string.

- Apply Newton's Laws separately to each object
- The acceleration of both objects will be the same
- The tension is the same in each diagram
- Solve the simultaneous equations



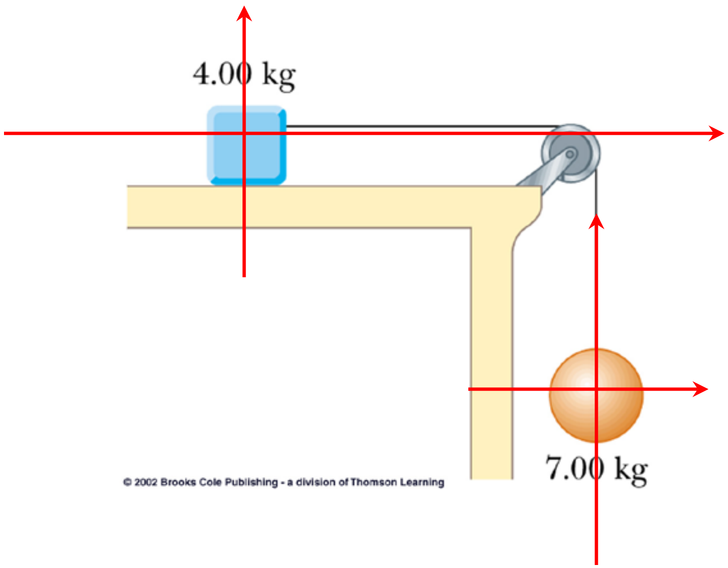
Example: Connected Objects

Given:

mass1: m_1=4.00 kg
mass2: m_2=7.00 kg
friction: μ=0.300

Find:

Tensions **T**=?
Acceleration **a**=?



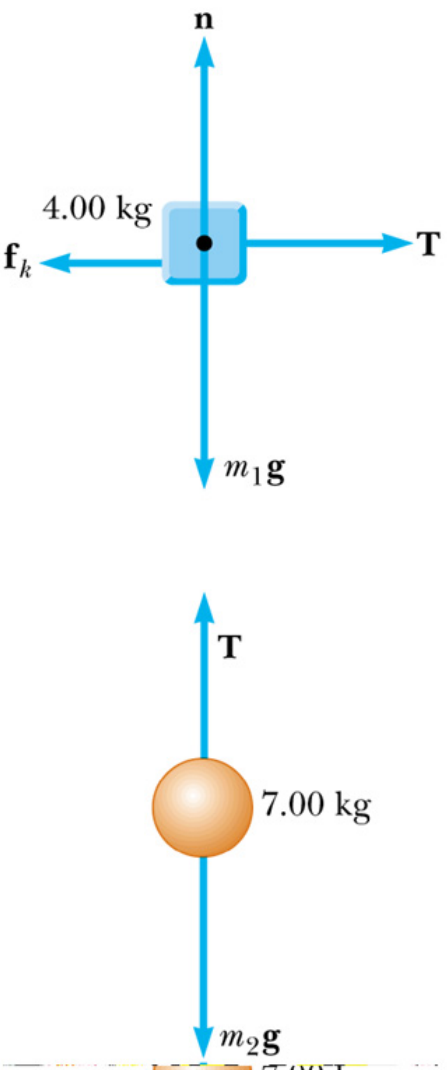
1. Introduce <u>two coordinate frames</u>:
   Oy: y's are directed up
   Ox: x's are directed right

$$\underline{Note}: \quad \sum \vec{F} = m\vec{a}, \quad and \quad f_k = \mu n$$

$$Mass1: Ox_1: \sum F_x = T - f_k = m_1 a,$$
$$Oy_1: \sum F_y = n - m_1 g = 0.$$
$$Mass2: Oy_2: \sum F_y = m_2 g - T = m_2 a.$$

Solving those equations:

a = 5.16 m/s^2
T = 32.4 N ✓



Terminal Speed

- Another type of friction is air resistance
- Air resistance is proportional to the speed of the object
- When the upward force of air resistance equals the downward force of gravity, the net force on the object is zero
- The constant speed of the object is the *terminal speed*