EP-307 Introduction to Quantum Mechanics

Reference Texts

Principles of Quantum Mechanics
R Shanker
Modern Quantum Mechanics
J.J. Sakurai

Method of Assessment

Four surprise quiz of 10 marks each

Midsemester examination 20 marks

End Semester 40 marks

Tone of the course

- How and why?
- The merging of two different disciplines
  - Linear Algebra
  - Physical world
- Course on modern logic
- Enjoy the course– Let it be like Movie, Music, Swimming --- Whatever you like

Contents of the course

- Couple of Lectures on why Quantum Mechanics?

  And then Just Shanker all the way

- Mathematical Introduction
  - Linear Vector Spaces
  - Dirac Notation
  - Linear Operators
  - Active and Passive transformations
  - Eigenvalue problem
  - Genralization to Infinite Dimensions

Contents of the Course (contd..)

- The Postulates of Quantum Mechanics
  - The Postulates
  - Definition of the postulates
  - The Schroedinger's Equation
- Simple Problems in One Dimension
  - The free particle
  - The particle in a box
  - Continuity Equation for probability
  - Single Step Potential – a problem in scattering

Contents of the Course (contd..)

- The Classical Limit
- Simple Harmonic Oscillator
  - Why?
  - Quantization
  - Oscillator in energy basis
  - Genralization of postulate II
  - Gauge Invariance and choice of phase for wavefunction

Contents of the Course (contd..)

- The Path Integral Formulation of Quantum theory
  - The Path Integral Recipe
  - An approximate U(t) for free particle
  - Path Integral evaluation for free particl
- Symmetries and their Consequences
  - Translational Invariance
  - Time Translational Invariance
  - Parity Invariance
  - Time-Reversal Invariance

?

Contents of the Course (contd..)

- Rotational Invariance & Angular Momentum
  - Translations in Two Dimensions
  - Rotations in Two Dimensions
  - The Eigenvalue Problem of L_z
  - Angular Momentum in 3 Dimensions
  - Eigenvalue Problem of L^2 & L_Z

- The Hydrogen Atom
  - The Eigenvalue Problem
  - The Degeneracy of the Hydrogen Spectrum

The Beginning

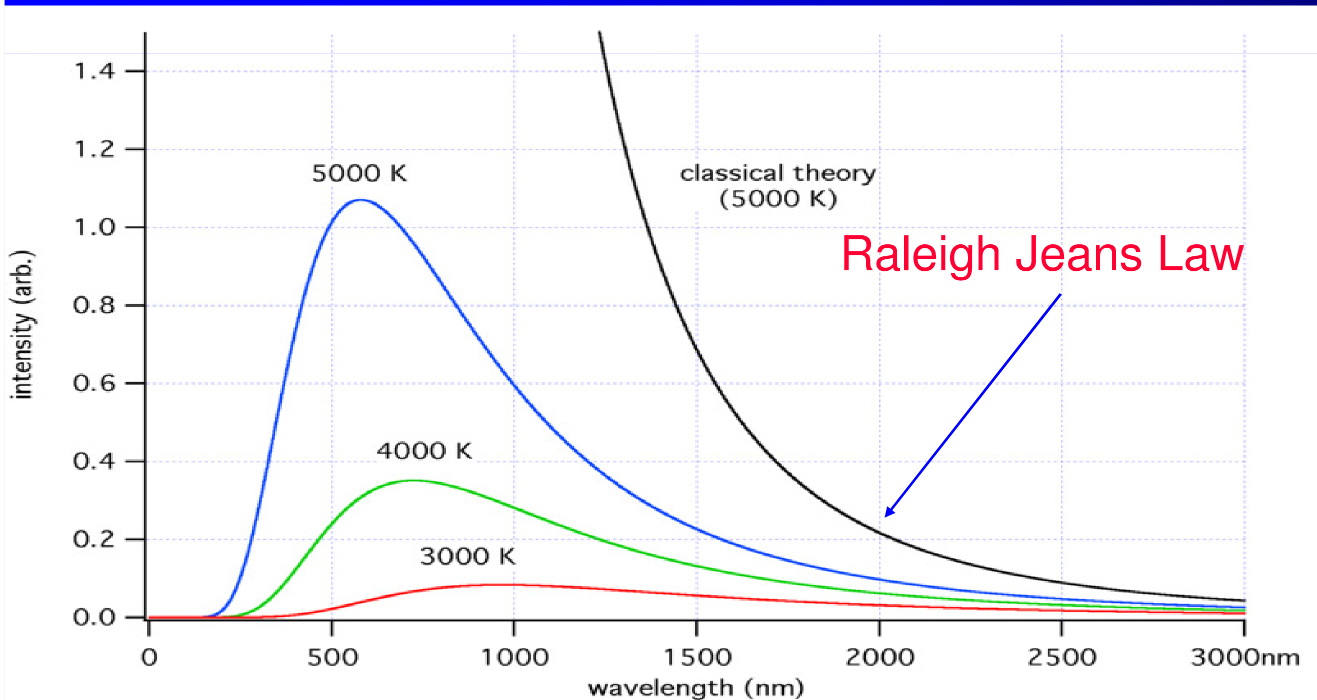
- What is a Physical Law?
  - A statement of nature which each experimenter must arrive at
  - Experiments done in different frames must yield same results
  - Describes the physical world
  - Description is dynamic…..
- Why should truth be a function of time?
  - Laws formulated with observations
  - Observations depends on accuracy of the instruments
  - Advancement of technology leads to better instrumentation
  - Laws that remain true gain in stature, those which don’t must be abandoned
  - Domain of physical law

Matter & Radiation

- Classical Mechanics
  - Formulated by Galileo, Newton, Euler, Lagrange Hamilton
  - Remained unaltered for three centuries
- Some History
  - Beginning of last century two entities---
    - Matter & Radiation
  - Matter described by Newtons laws
  - Radiation by Maxwell's equation
  - It was thought that we now understood all…
  - First breakthru came with radiations emitted by a black body

The Black Body Radiation

Why Black Body?



What was the observation

$\lambda_m T$ = Constant

Total Power radiated $\propto T^4$

Raleigh Jeans Law

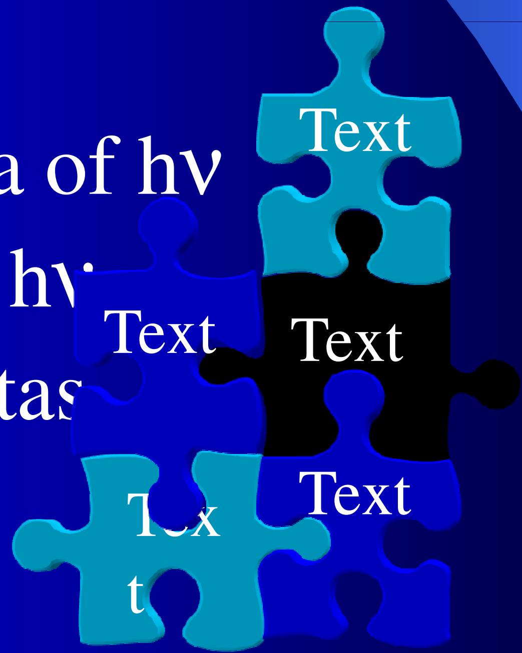
Black Body continues

- At a more basic level why should there be three laws which apparently have no concern with each other describe one physical phenomenon?
- Why is it a physical phenomenon?
- Planck solved the mystery by enunciating that it emitted radiation in quantas of $h\nu$

Enter Einstein !

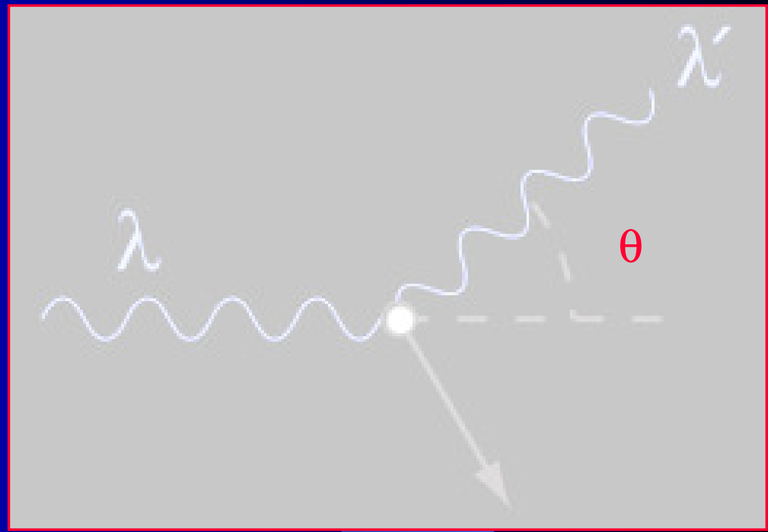
- If radiation is emitted in quantas they should also be absorbed in quatas
- He could explain photo electric effect using this…
- If light is absorbed in quanta of $h\nu$
- If it is emitted in quantas of $h\nu$

  $\Rightarrow$ Must consist of quantas



Enter Compton

$$\lambda - \lambda' = \frac{h}{m_e c^2}(1 - Cos\,\theta)$$



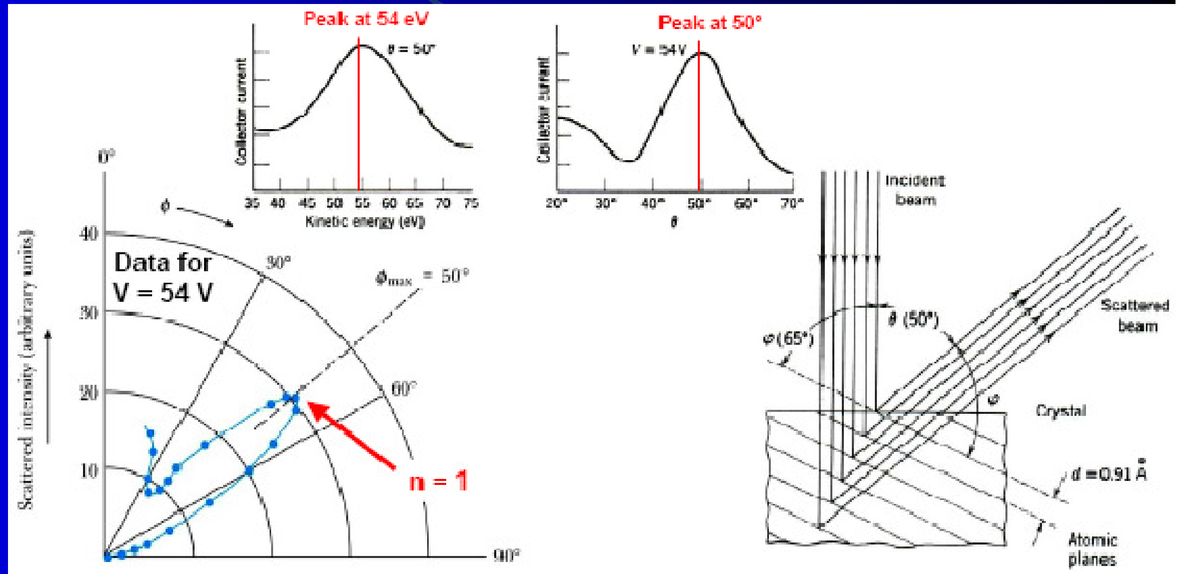
- What was the importance of Compton effect?
- Collision between two particles
  - Energy-momentum must both be conserved simultaneously
- Light consist of particles called photons
- What about phenomenon of Interference & diffraction?
- Logical tight rope of Feyman
- Light behaves sometimes as particles sometimes as waves

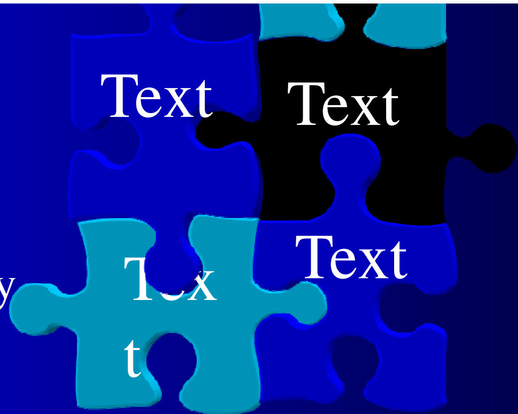
Enter de Broglie !

- Radiation behaved sometimes as particles Sometimes as waves
- What about Matter?

- De Broglie's hypothesis
- Several questions cropped up!
  - ⇒ What is it?
    - --Particle or Waves
  - ⇒What about earlier results?
- What is a good theory?
  - ⇒ Need not tell you whether an electron is a wave or a particle
  - ⇒if you do an experiment it should tell you whether it will behave as a wave or particle.

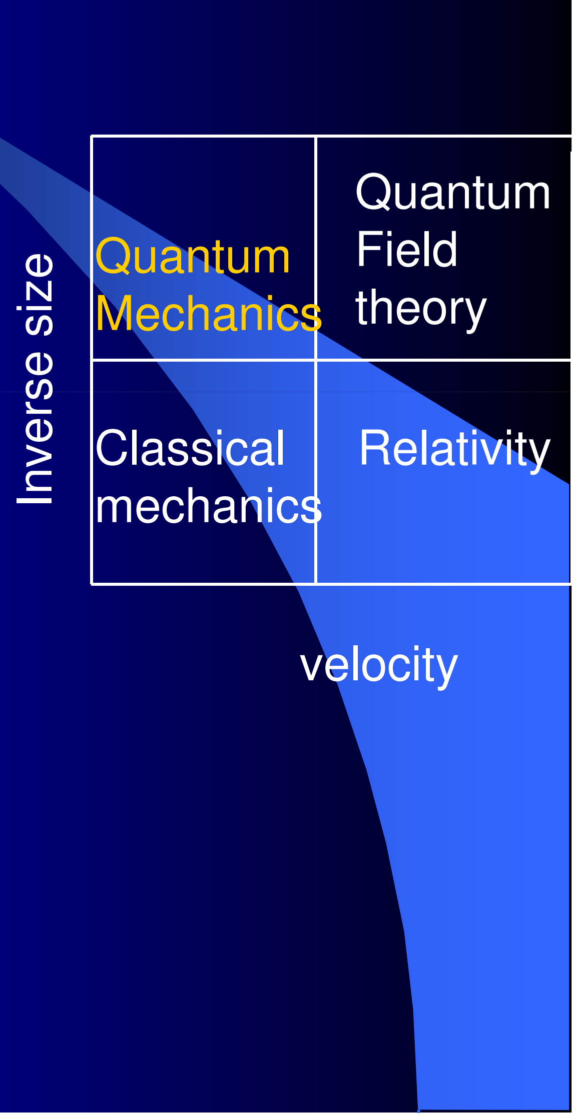
Second Question brings us to the domain of the theory





Domain of a theory

- Domain D_n of the phenomenon described by the new theory
- Subdomain D_0 where the old theory is reliable.
- Within the sub domain D_0 either theory may be used to make quantitative predictions
- It may be easier and faster to apply the old theory
- New theory brings in not only numerical changes but also radical conceptual changes
- These will have bearing on all of D_n



Lecture 2

- Thought Experiments
- Stern-Gerlach Experiments
- Analogy with mathematics of light
- Feynman’s double slit thought experiment

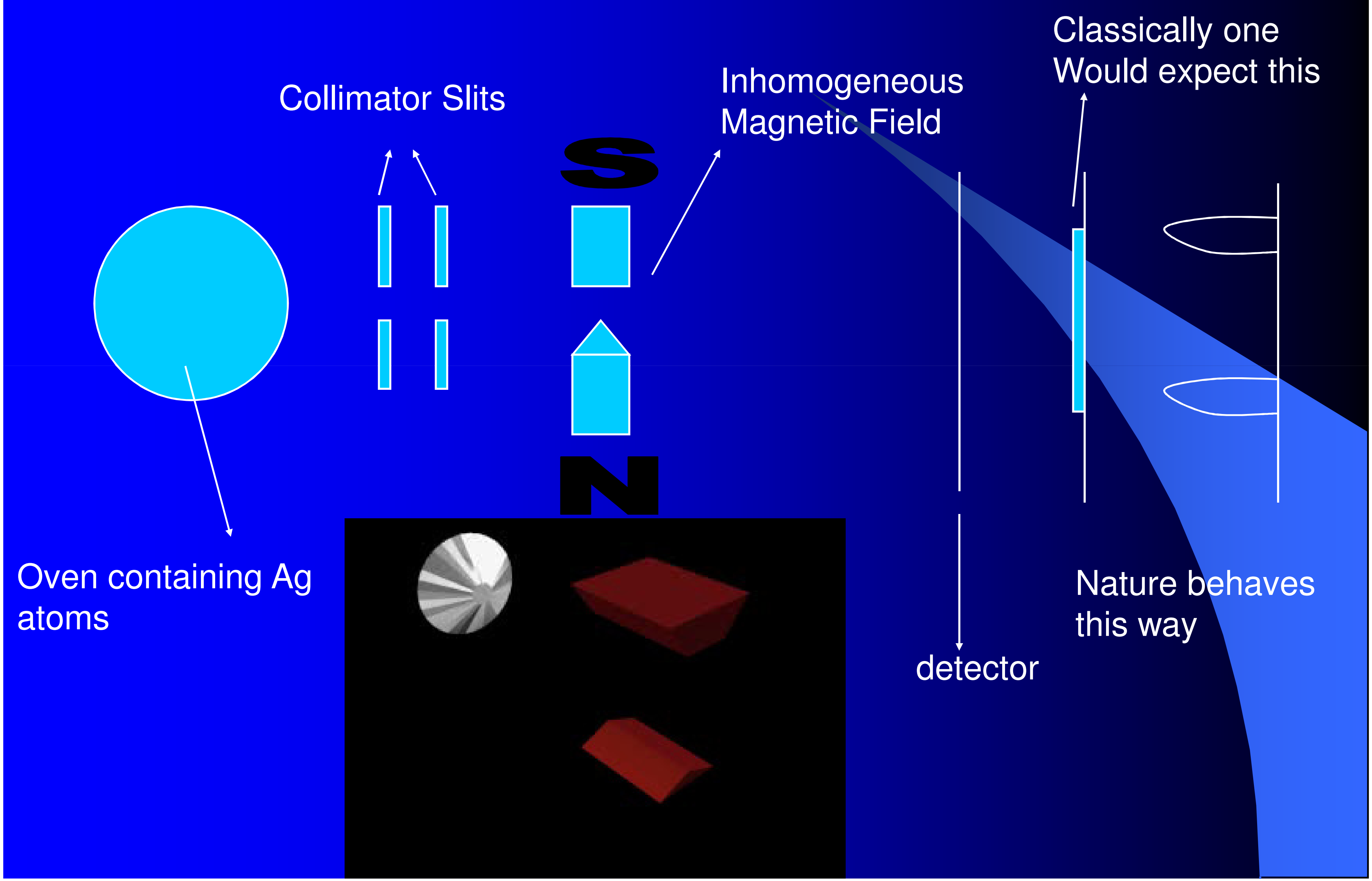
Thought Experiments

- We are formulating a new theory!
- Why are we formulating a new theory?
- In the last lecture tried to motivate you why we need a new theory?
- How?
- Radiation sometimes behaves as
  - Particles
  - Waves
- Same is true for Matter

Thought Experiments

- We must walk on a logical tight rope
- What is Feyman's logical tightrope?
- We have given up asking whether the electron is a particle or a wave
- What we demand from our theory is that given an experiment we must be able to tell whether it will behave as a particle or a wave.
- We need to develop a language for this new theory.
- We need to develop the Mathematics which the language of TRUTH which we all seek
- What Kind of Language we seek is the motivation for next few lectures.

Stern-Gerlach Experiment



Stern Gerlach Experiment unplugged

- Silver atom has 47 electrons where 46 electrons form a symmetrical electron cloud with no net angular momentum
- Neglect nuclear spin
- Atom has angular momentum –solely due to the intrinsic spin of the 47th electron
- Magnetic moment μ of the atom is proportional to electron spin
- If $\mu_z < 0$ (then $S_z > 0$) atom experiences an upward force & vice versa
- Beam will split according to the value of μ_z

$$\mu = \frac{e}{m_e c} S$$

$$Energy = -\vec{\mu}.\vec{B}$$

$$F_z = \mu_z \frac{\partial B}{\partial z}$$

Stern-Gerlach Experiment (contd)

- One can say it is an apparatus which measures the z component of $\mu \Rightarrow S_z$
- If atoms randomly oriented
  - No preferred direction for the orientation of μ
  - Classically spinning object $\Rightarrow$ μz will take all possible values between μ & $-\mu$
- Experimentally we observe two distinct blobs
- Original silver beam into 2 distinct component
- Experiment was designed to test quantisation of space
  - Remember Bohr-Sommerfeld quantisation experiment

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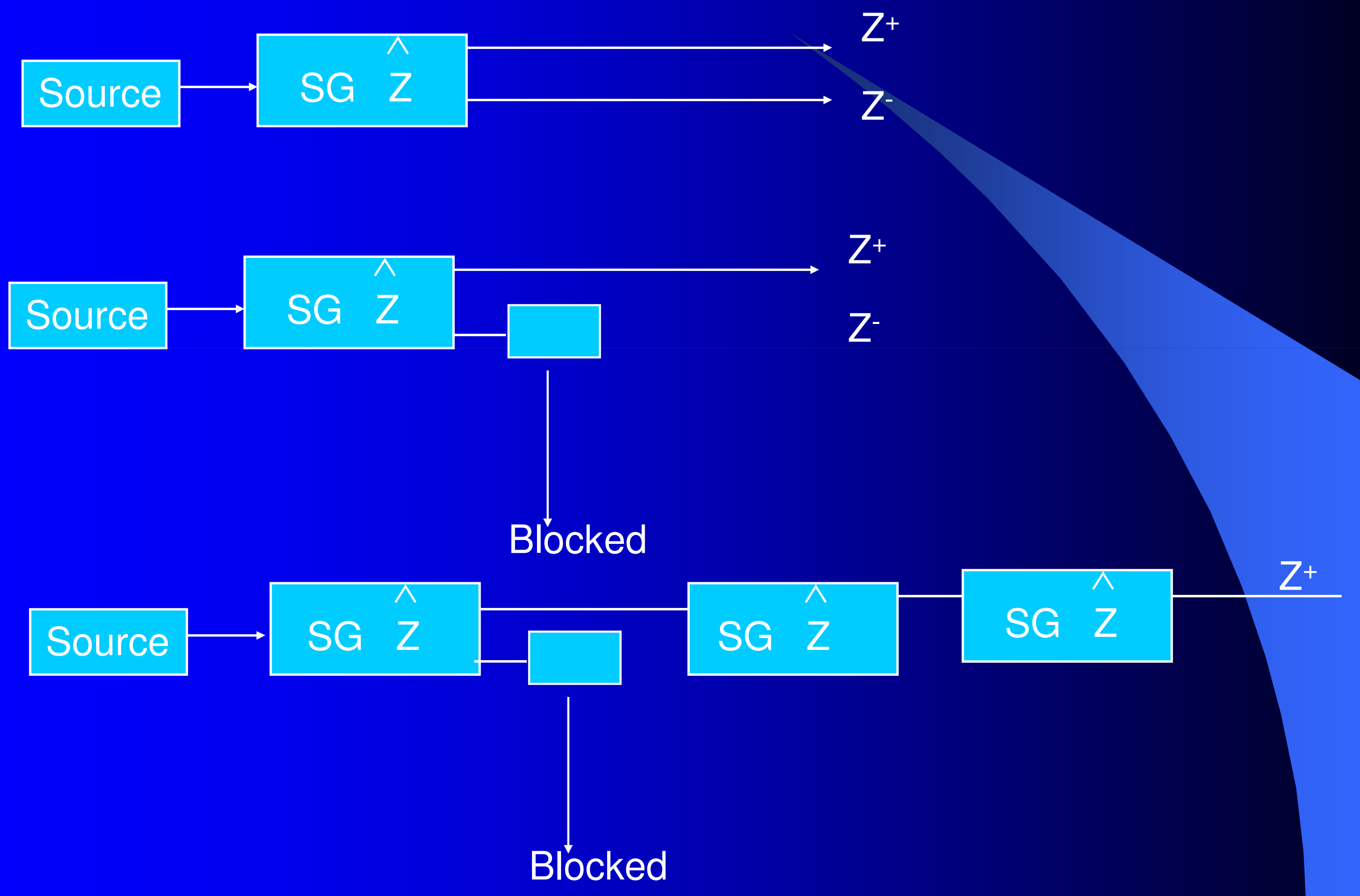
What have we learnt from the experiment

- Two possible values of the Z component of S observed S_Z^{UP} & S_Z^{down}
- Refer to them as S_Z^+ & S_Z^- $\Rightarrow$ Multiples of some fundamental constants, turns out to be $+\frac{\hbar}{2}$ & $-\frac{\hbar}{2}$
- Spin is quantised
- Nothing is sacred about the z direction, if our apparatus was in x direction we would have observed S_x^+ & S_x^- instead

SG $\hat{Z}$ ⟶ This box is the Stern Gerlach Apparatus with magnetic Field in the z direction

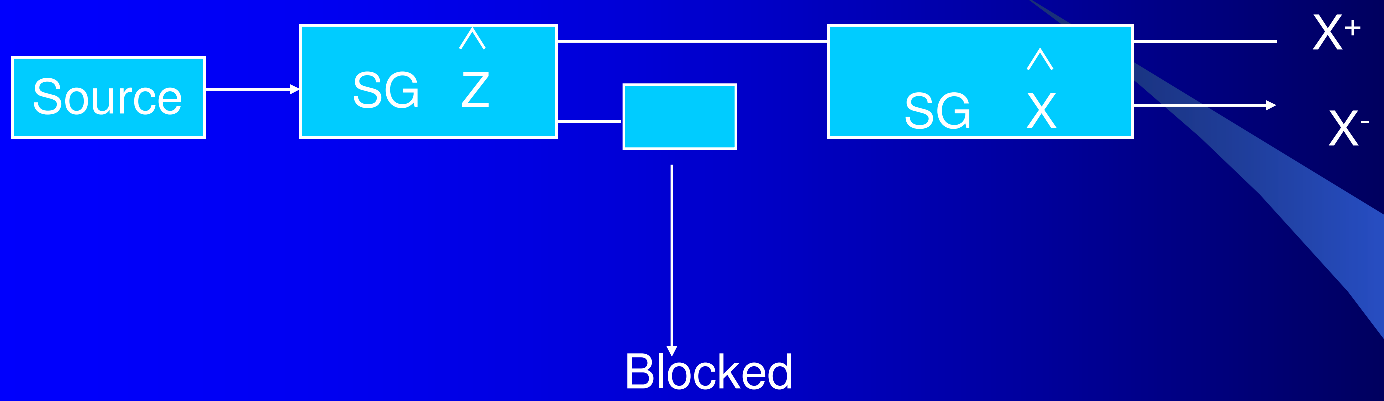
Thought Experiments start



Thought Experiment continues

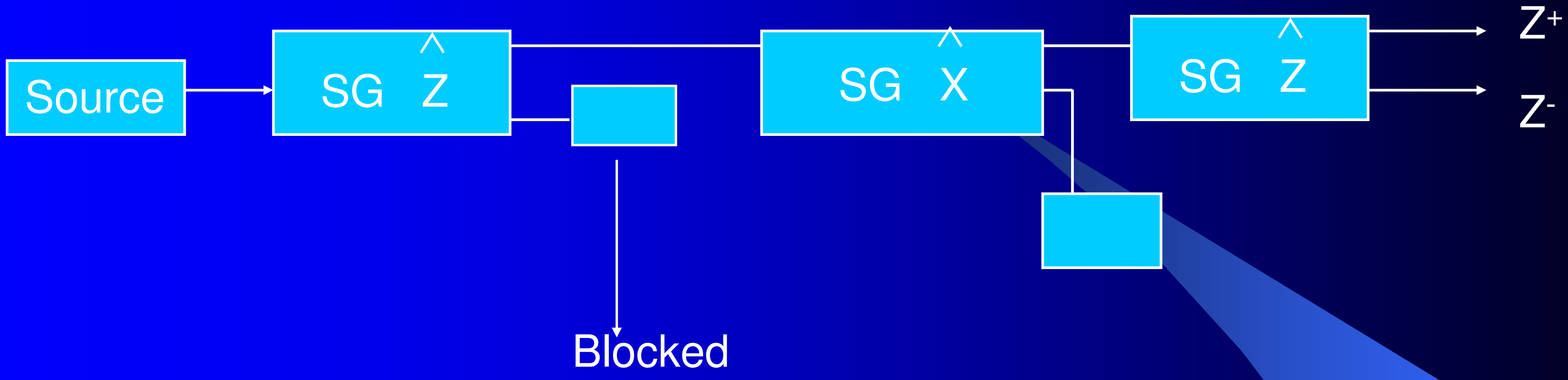
- No matter how many SG in z direction we put, there is only one beam coming out
- Silver atoms were oriented in all possible directions
- The Stern-Gerlach Apparatus which is a measuring device puts those atoms which were in all possible states in either one of the two states specific to the Apparatus
- Once the SG App. put it into one of the states repeated measurements did not disturb the system

Another thought experiment



Does It mean that 50% of the atoms in the S_z^+ beam coming out Of the first apparatus are made of atoms characterized by S_x^+ & 50% of the time by S_x^-

Testing the hypothesis



We Observe that from the final SG Z there are two beams Emerging

No way to explain as S_z^- was blocked

Only conclusion we can draw is that the second Measurement disturbed the first measurement

The Second measurement put the system in states specific To it. The third measurement which was different from 2nd

Conclusions from our experiment

- Measurements disturb a quantum system in an essential way
- The boxes are nothing but measurements
- Measurements put the QM System in one of the special states
- Any further measurement of the same variable does not change the state of the system
- Measurement of another variable may disturb the system and put it in one of its special states.

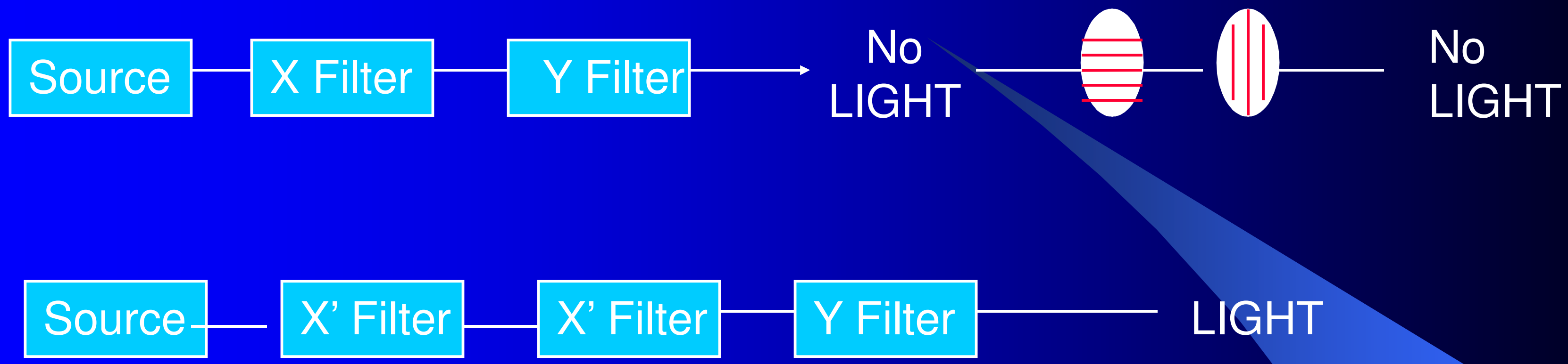
Complete Departure from Classical Physics

- Measurement of S_x destroys the information about S_z
  - We can never measure S_x & S_z together
    - Incompatible measurements
- How do you measure angular momentum of a spinning top, $L = I\omega$
  - Measure ω_x , ω_y , ω_z
  - No difficulty in specifying L_x L_y L_z

Analogy

- Consider a monochromatic light wave propagating in Z direction & it is polarised in x direction $E = E_0 \hat{x} Cos(kz - \varpi t)$
- Similarly linearly polarised light in y direction is represented by $E = E_0 \hat{y} Cos(kz - \varpi t)$
- A filter which polarises light in the x direction is called an X filter and one which polarises light in y direction is called a y filter
- An X filter becomes a Y filter when rotated by 90°

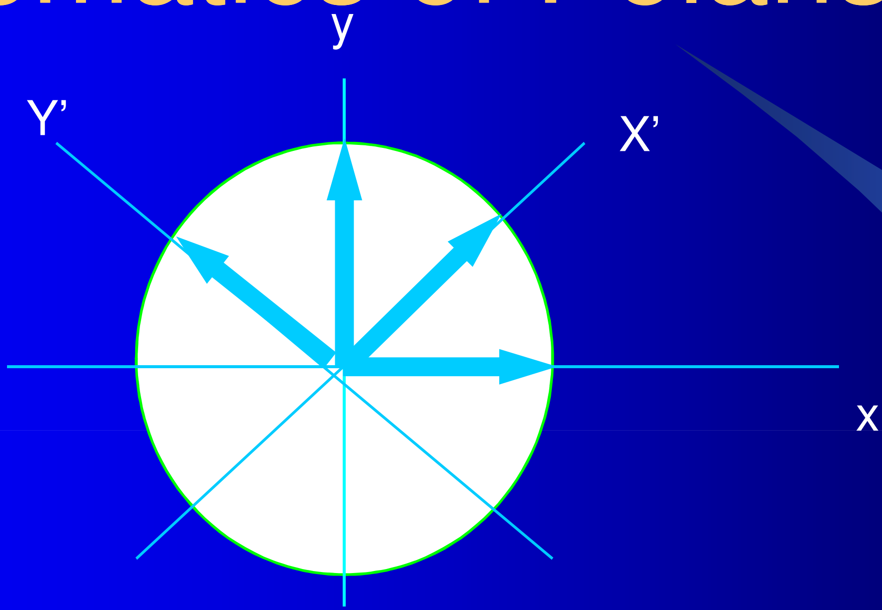
An Experiment with Light



- The selection of x` filter destroyed the information about the previous state of polarisation of light
- Quite analogous to situation earlier
- Carry the analogy further

  – $S_z \pm$ x & y polarised light
  – $S_x \pm$ x` & y` polarised light

Mathematics of Polarisation



$$E_0 \hat{x}' Cos(kz - \omega t) = E_0 \left[\frac{1}{\sqrt{2}} \hat{x} Cos(kz - \omega t) + \frac{1}{\sqrt{2}} \hat{y} Sin(kz - \omega t) \right]$$

$$E_0 \hat{y}' Cos(kz - \omega t) = E_0 \left[-\frac{1}{\sqrt{2}} \hat{x} Cos(kz - \omega t) + \frac{1}{\sqrt{2}} \hat{y} Sin(kz - \omega t) \right]$$

Where to Get More Information

- Other training sessions
- List books, articles, electronic sources
- Consulting services, other sources