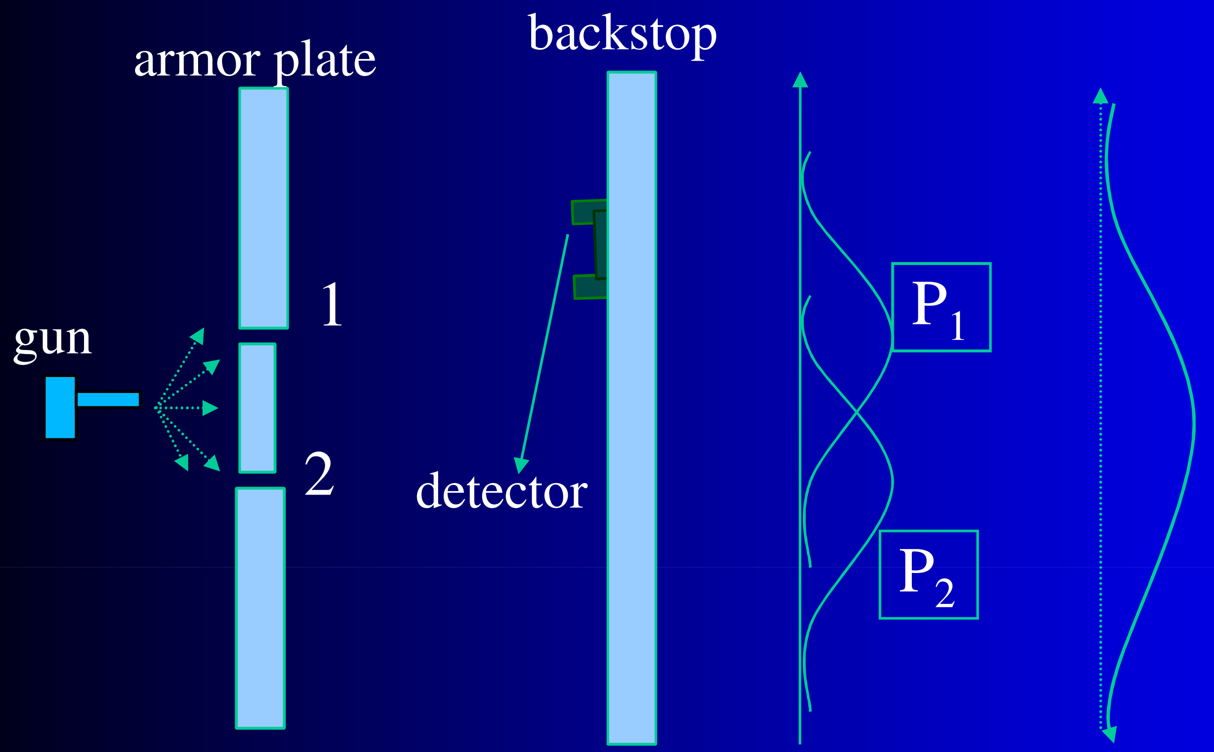
Lecture 4 - Feynmans thought experiments

- Things on a very small scale behave like nothing we have direct experience about.
- Even the experts do not uderstand it the way they would like to
- We know how large objects will act but things on small scale just do not act that way
- Act in sort of abstract or imaginative fashion. We can not connect it with our direct experience
- Examines phenomenon which is impossible to explain in any classical way and has in it heart of QUANTUM MECHANICS
- Contains the only mystery
- Cannot make the mystery go away by explaining how it works

Thought Experiment with Bullets



$$P_{12} = P_1 + P_2$$

Machine gun sprays bullets
Over a large angular spread

A wall made of armor plate with two holes in it to allow the bullets to pass through

A backstop made of wood & a detector which collects bullets and can be moved in x direction

What is the probability that that a bullet which passes thru the holes in the wall will arrive at the backstop at a distance x from the center?

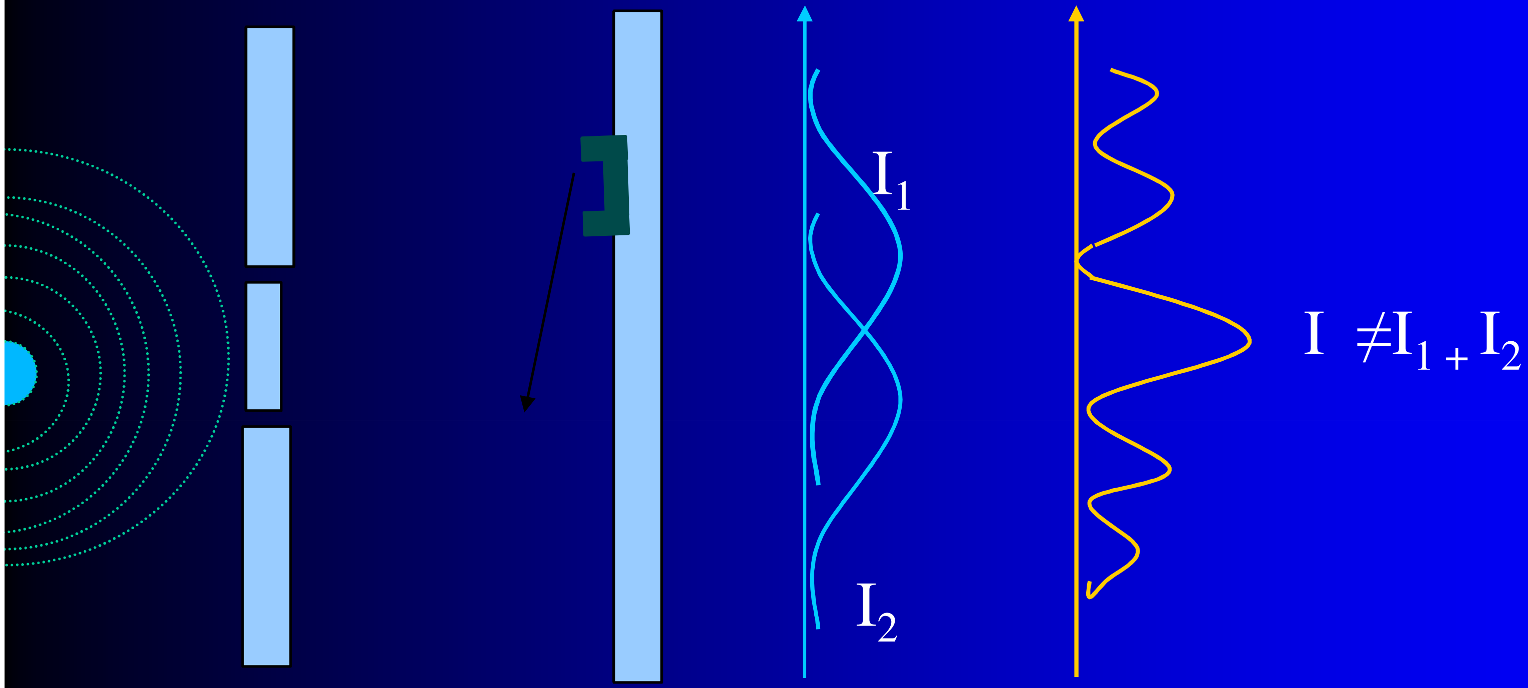
Bullet Experiment

- Why speak of probability?
- Probability –chance that the bullet will arrive at the detector
- Can measure by counting the number which arrive within certain time
- The ratio of this number to the total number of bullets detected is called probability
- Bullets arrive in lumps – one whole bullet
- Probability as P_{12} because bullets may have come from either hole 1 or hole 2.
- P_1 probability with only hole 1 open
- P_2 probability with only hole 2 open
- Probabilities just ADD

Thought Experiments with Water Waves

Shallow trough of water
small object labeled wave
source jiggled up and down

A detector –gadget which measures
The intensity of wave motion
Scale ∝ square of the height of wave motion



Right of the source
a wall with two holes

Beyond that a second
wall an absorber
so that no reflection

Thought Experiments with Water Waves (contd…)

- Intensity can have any value
- No lumpiness in the wave intensity
- We measure the wave intensity for various values of x and we get I_{12}
- Original wave is diffracted at the two holes
- New circular waves spread out interfering and giving us the pattern I_{12}
- I_1 is the intensity of the wave with hole 1 open hole 2 closed I_2 with hole 2 open and hole 1 closed

Thought Experiments with Water Waves (contd…)

- The Intensity I_{12} is certainly not the sum of I_1 & I_2.
- We say there is interference of two waves
- At some places where the waves are "in phase" & the wave peaks add together to give a large amplitude
- There is constructive inteference correspond to large values of I_{12}
- At places where the two waves arrive with a phase difference of π the waves will interfere destructively --low values of I_{12}
- Quantitative relationship between I_1 I_2 & I_{12}
- The instantaneous height of the water wave = $h_1 e^{i\omega t}$ where the amplitude h_1 in general is a complex number. Similarly for hole 2 it is $h_2 e^{i\omega t}$

Thought Experiments with Water Waves (contd…)

- The mathematics of the whole process becomes very simple
- If hole 1 is open then intensity I_1 is given by $= |h_1|^2$
- If hole 2 is open then intensity I_2 is given by $= |h_2|^2$
- If both the holes are open the wave heights add to give the height $(h_1 + h_2)e^{i\omega t}$
- $I_{12} = |h_1 + h_2|^2$

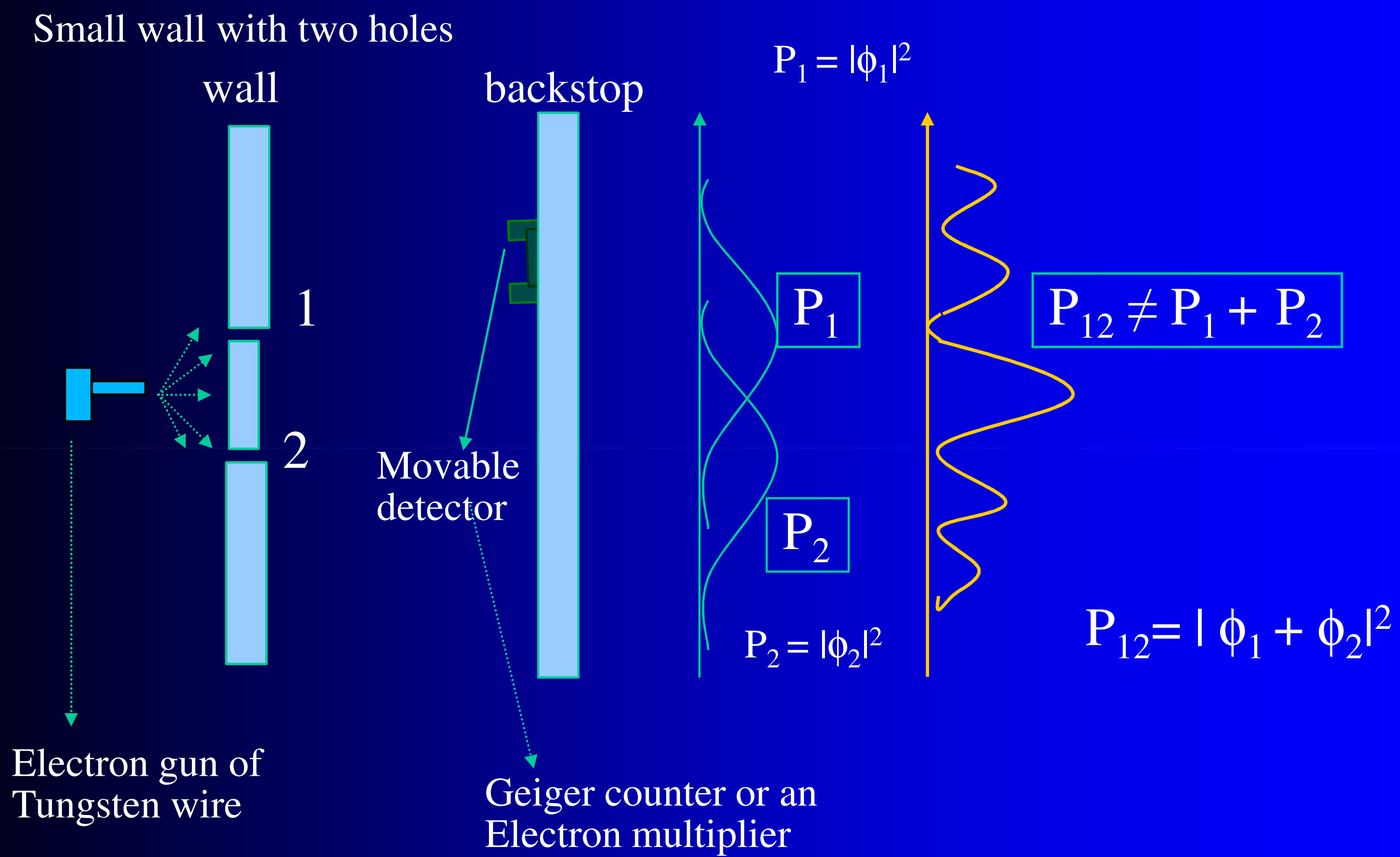
  $= |h_1|^2 + |h_2|^2 + 2\,|h_1|\,|h_2| \cos\delta$

  Where δ is the phase difference between h_1 & h_2

In terms of intensities we have $I_{12} = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\cos\delta$

- If amplitudes add and intensity can have any value

Experiment with Electrons



Originally when Feynman's book came out probably he was unaware and wrote "We should say right away that you should not try to set this experiment. The experiment would Have to be impossibly small

Experiment with Electrons (contd…)

- We hear clicks from the detector –all clicks same. There are no half clicks
- Clicks come erratically – statistical process
- As the detector is moved around the rate at which the clicks appear is faster or slower but the size of each click is same
- If we put two separate detectors at the backstop one or the other would click
- Electrons arrive in identical lumps
- What is the relative probability that an electron lump will arrive at a distance of x from the center
- This is the interesting curve P_{12}

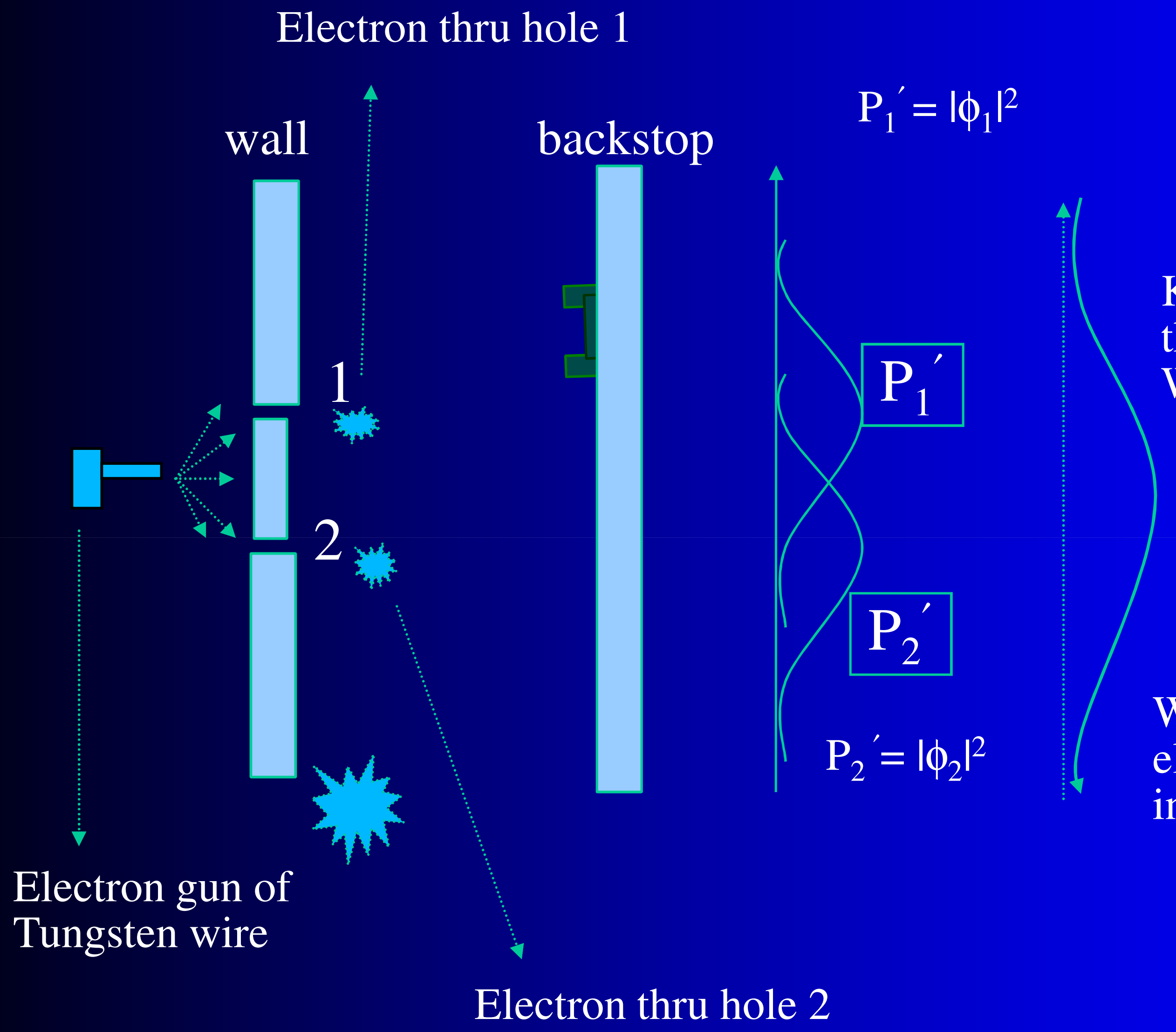
The Interference of electrons (understanding their behaviour)

- Since they come in lumps – electron has either come thru hole 1 or hole 2.
- Assuming the above electrons can be divided into two classes (1) Those that come thru hole 1 (2) Those that come thru hole 2 .
- With both holes open, the effect should be sum of the effects of one hole open
- Make measurements with hole 1 and 2 open and we get P_1 & P_2
- The result P_{12} with both holes open is clearly not the sum of P_1 & P_2
- In analogy with water waves we say there is interference.
- How is it possible? – Electrons either go thru hole 1 or hole 2 is not correct. They split in half… But no they arrived in lump.
- They went thru complicated path– hole 1 and then went around and then went thru hole 2

The Interference of electrons (understanding their behaviour)

- There are some points that had very few electrons when both holes are open but which receive many electrons when we close one hole. Closing one hole increased the number passing thru other.
- On the otherhand at the centre of the pattern P_{12} is more than twice as large $P_1 + P_2$. It is as if closing one hole decreased the probability thru other also
- Very mysterious… Yet the mathematics is very simple.. For P_{12} is quite similar to I_{12}. What is going on at the backstop can be described by two complex numbers ϕ_1 & ϕ_2
- The electrons arrive in lumps like particles and the probability of arrival of these lumps is distributed like the distribution of intensity of a wave. It is in this sense that electron behaves like particle as well as waves.
- For classical waves, intensity was defined as the mean over time of the square of the wave amplitude and used complex numbers as mathematical trick to simplify analysis.
- Quantum mechanics also we must use complex numbers.
- There are a large number of subtleties
- Finally since the number of electrons that arrive at a point is not the sum of electrons that go thru 1 and 2 hence the proposition that electrons go thru either hole 1 or hole 2 is false.

Watching the electrons



Keep track of whether we saw the flash near hole 1 or hole 2
We do find that

$$\boxed{P_{12}' = P_1' + P_2'}$$

We succeeded in watching the electrons but we destroyed the interference

Every time you hear a click you see a flash either near hole 1 Or hole 2.
Experimentally then the proposition that electron goes thru either hole 1 or hole 2 is necessarily correct.

Watching the electrons

- When we look at electrons, their distribution on the screen is different than we do not look
- We turn off the light source and P_{12}' becomes identical to P_{12} --- interference is restored…
- Turning on the light source disturbed the electrons – light gives a jolt to the electrons
- Turn down the brightness – Sometimes we hear the click but do not see the flash. Flash is always of the same size. Light is also like electron. Electrons were not seen as photon was not around.
- Make three columns – Column 1 – flash near hole 1 Column 2 – flash near hole 2 Column 3 – clicks heard but no flash. The one where you do not see the flash shows interference

Watching the electrons

- Those seen near hole 1 have probability distribution P_1' & near hole 2 have distribution P_2'
- Those in column 3 show interference pattern. If the electrons are not seen we observe interference pattern
- When we see a photon disturbs it and if we don't the photon does not disturb it.
- Reduce the momentum -- Increase the wavelength
- Remember two objects can be resolved if the distance between them is of the order of $\lambda/2$. As we increase the wavelength and it becomes greater than the distance between the two holes. We see one big flash but cannot tell thru which hole the electron went.
- It is at that time the interference pattern appears

Heisenberg's Uncertainty Principle

- Impossible to arrange light in such a way that one can tell thru which hole the electron passed and still see interference pattern.
- The new laws of nature that we are seeing would only be consistent if there was basic limitation on our experimental capabilities not previously recognised
- Complete theory of quantum mechanics rests on this principle
- What about the proposition that electrons go thru either hole 1 or hole 2– If one has a piece of apparatus which is capable of determining thru which hole the electron went then one can say it went thru either hole 1 or hole 2. When in the apparatus there is nothing to disturb the electron then we may not say that electron goes thru either hole 1 or hole 2
- This is the logical tightrope we must walk to describe nature successfully

What have we learnt

- The probability of an event in an ideal experiment is given by the square of the absolute value of a complex number ϕ which is called the probability amplitude:
  - P = Probability
  - ϕ = Probability amplitude
  - $P = |\phi|^2$

- When an event can occur in several alternative ways , then the probability amplitude for the event is the sum of the probability amplitude of each of the ways in which it can occur
  - $\phi = \phi_1 + \phi_2$
  - $P = |\phi_1 + \phi_2|^2$

- If the experiment determines which alternative is taken then probabilities add. The interference is lost…
  - $P = P_1 + P_2$