26/2/15 (A.)

We know that the solution of

$$y' - 2ty = 0 \quad \text{is} \quad y = c_1 e^{t^2}.$$

We will try to find the solution of

$$y' - 2ty = 1 \quad \text{by assuming} \quad y = c_1(t) \cdot e^{t^2}.$$

So we have

$$y' = c_1'(t) e^{t^2} + c_1(t) \cdot 2t \cdot e^{t^2}.$$

Substituting into $y' - 2ty = 1$. yield

$$c_1'(t) e^{t^2} + c_1(t) \cdot 2t \cdot e^{t^2} - 2t \cdot c_1(t) e^{t^2} = 1$$

$$c_1'(t) = \frac{1}{e^{t^2}} = e^{-t^2}$$

$$c_1(t) = \int_0^t e^{-s^2} ds + K$$

Thus $$y = \left(\int_0^t e^{-s^2} ds + K \right) \cdot e^{t^2}$$

$$y = \underbrace{e^{t^2} \int_0^t e^{-s^2} ds}_{\text{I}} + \underbrace{K e^{t^2}}_{\text{II}}$$

Term II (Ke^{t^2}) is general solution of $y' - 2ty = 0$.

Term I ($e^{t^2} \int_0^t e^{-s^2} ds$) is particular/special solution of $y' - 2ty = 1$. (check!)

26/2/15 (B)

Solve the diff eq $\frac{dy}{dt} + \frac{1}{2}y = \frac{1}{2}e^{t/3}$. --- (1)

First : we will find the solution of $\frac{dy}{dt} + \frac{1}{2}y = 0$.
(the homogeneous form)

$$\frac{dy}{dt} = -\frac{1}{2}y$$

$$\frac{dy}{dt}\Big/ y = -\frac{1}{2}$$

$$\ln y = -\frac{1}{2}t + c \longrightarrow |y| = e^{c} \cdot e^{-\frac{1}{2}t} \quad \text{~~atau~~ or}$$

$$y = K e^{-\frac{1}{2}t}.$$

Second : Assuming the solution of (1) has ~~in~~ form ~~of~~

$$y = K(t)\, e^{-\frac{1}{2}t}.$$

So $y' = K'(t)\, e^{-\frac{1}{2}t} - \frac{1}{2}K(t)\, e^{-\frac{1}{2}t}$.

Substituting in to (1) yields

$$K'(t)\, e^{-\frac{1}{2}t} - \frac{1}{2}K(t)\, e^{-\frac{1}{2}t} + \frac{1}{2}\cdot K(t)\, e^{-\frac{1}{2}t} = \frac{1}{2}e^{t/3}$$

$$K'(t)\, e^{-\frac{1}{2}t} = \frac{1}{2}e^{t/3}$$

$$K'(t) = \frac{1}{2}e^{5/6\, t}$$

$$K(t) = \frac{3}{5}e^{\frac{5}{6}t} + c_1.$$

Thus the solutions is $y = \left(\frac{3}{5}e^{\frac{5}{6}t} + c_1\right)\cdot e^{-\frac{1}{2}t}$

$$y = \underbrace{\frac{3}{5}e^{\frac{1}{3}t}} + \underbrace{c_1 e^{-\frac{1}{2}t}}$$

→ the general solution of $\frac{dy}{dt} + \frac{1}{2}y = 0$

particular/special solution of $\frac{dy}{dt} + \frac{1}{2}y = \frac{1}{2}e^{t/3}$.

note : This procedure called by VARIATIONS OF PARAMETERS

The general solution of

$$y' + 3y = t + e^{-2t}$$

is

$$y = \frac{\int e^{\int +3\,dt} \cdot (t + e^{-2t})\,dt + c}{e^{\int +3\,dt}}$$

$$= \frac{\int e^{+3t}(t + e^{-2t})\,dt + c}{e^{+3t}}$$

$$= \frac{\int te^{3t} + e^{t}\,dt + c}{e^{3t}}$$

$$= \frac{\int te^{3t}\,dt + e^{t} + c}{e^{3t}}$$

$$= \frac{\frac{1}{3}\int t\,de^{3t} + e^{t} + c}{e^{3t}}$$

$$= \frac{\frac{1}{3}t \cdot e^{3t} - \frac{1}{3}\int e^{3t}\,dt + e^{t} + c}{e^{3t}}$$

$$= \frac{\frac{1}{3}te^{3t} - \frac{1}{9}e^{3t} + e^{t} + c}{e^{3t}}$$

$$y = \frac{1}{3}t - \frac{1}{9} + e^{-2t} + ce^{-3t}$$

If the initial condition is $y(0) = 1$, then

we get $1 = 0 - \frac{1}{9} + 1 + c$

$$c = \frac{1}{9}$$

Thus the particular solution is $y = \frac{1}{3}t - \frac{1}{9} + e^{-2t} + \frac{1}{9}e^{-3t}$

(c)