3rd meeting:

Divisibility and The Greatest Common Divisor

1. Divisibility

Definition: If a and b are integers with $a \neq 0$, we say that a divides b if there is an integer c such that b = ac. If a divides b, we say that a is factor of b.

Theorem (Division algorithm): If a and b are integers such that b >0, then there is unique integer q and r such that a = b q + r with $0 \leq r < b$.

2. The greatest common divisor (gcd):

Let a and b be given integers that are not both zero. The greatest common divisor of a nd b , denoted by gcd(a,b), is positive integer d satisfying the following:
(i). $d \mid a$ and $d \mid b$
(ii). If $c \mid a$ and $c \mid b$, then $c \leq d$.

Theorem: If a and b are integers that are not both zero, then there exist integers x and y such that ax + by = gcd(a,b).

Two integer a and b, not both of those are zero, are said to relatively prime if gcd(a,b) =1.

Theorem: Two integer a and b, not both of those are zero, are relatively prime if and only if there exist integer x and y such that ax + by = 1.

Discussion:

1. Prove that $a \mid b \text{ and } a \mid c \Rightarrow a^2 \mid bc$.
2. Prove that if $a \mid b$ and $c \mid d$, then $ac \mid bd$.
3. Is it true that if $a \mid (b-c)$, then $a \mid b$ or $a \mid c$? Explain your answer.
4. Is it true that if $a \mid bc$, then $a \mid b$ or $a \mid c$? Explain your answer.
5. Is it true that if there exist x and y such that ax + by = d, then d = gcd(a,b)? Explain your answer.
6. Is it true that if $(a,b) = d$, then $d \mid ax + by, \forall x, y \in Z$. Explain your answer.
7. Find integers m and n such that 45m + 75n = gcd(45, 75).
8. Find integers x and y such that 102x + 222y = gcd(102, 222).
9. Find integers k and t such that 51k + 87t = gcd(51, 87).
10. Find integers s and x such that 981s + 1234 x = gcd(981, 1234).