Designing Fuzzy Time Series Model and Its Application to Forecasting Inflation Rate

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Abstract. *Fuzzy time series is a dynamic process with linguistic values as its observations. Modelling fuzzy time series developed by some researchers used the discrete membership functions and table lookup scheme from training data. Table lookup scheme is a simple method that can be used to overcome the conflicting rule by determining each rule degree. The weakness of fuzzy time series model based on table look up scheme is that the fuzzy relations may not be complete so the fuzzy relations can not cover all values in the domain. This paper presents new method to modelling fuzzy time series combining table lookup scheme and singular value decomposition methods which use continuous membership function. Table lookup scheme is used to construct fuzzy relation from training data and then singular value decomposition of firing strength matrix is used to reduce fuzzy relations. Furthermore, this method is applied to forecast inflation rate in Indonesia based on six-factors one-order fuzzy time series. This result is compared with neural network method and the proposed method gets a higher forecasting accuracy rate than the neural network method.*

Key words. *fuzzy time series, fuzzy rule, table lookup scheme, firing strength matrix, singular value decomposition, inflation rate.*

1. Introduction

Fuzzy time series is a dynamic process with linguistic values as its observations. In recently, fuzzy time series model was developed by some researchers. Song, Q and Chissom, B.S (1993a) developed fuzzy time series by fuzzy relational equation using Mamdani's method. In this modeling, determining the fuzzy relation need large computation. Then Song, Q and Chissom, B.S (1993b, 1994) constructed first order fuzzy time series for time invariant and time variant cases. This model needs complexity computation for fuzzy relational equation. Furthermore, to overcome the weakness of the model, Chen, S.M. (1996) designed fuzzy time series model by clustering of fuzzy relations.

Hwang (1998) constructed fuzzy time series model to forecast the enrollment in Alabama University. Fuzzy time series model based on heuristic model gives more accuracy than its model designed by some previous researchers (Huarng, 2001). Then, forecasting for enrollment in Alabama University based on high order fuzzy time series resulted more accuracy prediction (Chen, S.M., 2002). First order fuzzy time series model is also developed by Sah, M. and Degtiarev, K.Y. (2004), and Chen, S.M. and Hsu, C.C. (2004). The weakness of the fuzzy relations based on table look up scheme method is that the fuzzy relations may not be complete so the fuzzy relations can not cover all values in the domain.

Forecasting the inflation rate in Indonesia by fuzzy model resulted more accuracy than that by regress method (Abadi, *et al*, 2006). Then, Abadi, *et al* (2007) constructed fuzzy time series model using table lookup scheme to forecast Bank Indonesia certificate and the result give high accuracy. Then, Abadi, *et* *al*(2008) shown that forecasting inflation rate using singular value method have a higher accuracy than that using Wang's method.

The all above research use a discrete membership function. There are interesting topics in modeling fuzzy time series especially in determining model that gives good prediction accuracy. In this paper, we will design fuzzy time series model combining table lookup scheme and singular value decomposition using continuous membership function to improve the prediction accuracy. Then, its result is used to forecast inflation rate in Indonesia. The proposed method has a higher prediction accuracy than Wang's method and neural network method in application to forecasting the inflation rate.

The rest of this paper is organized as follows. In section 2, we introduce the *QR*-factorization and singular value decomposition of matrix and its properties. In section 3, we briefly review the definition of fuzzy time series and its properties. In section 4, we present a new method to construct fuzzy time series model based on training data. In section 5, we apply the proposed method to forecasting the inflation rate. We also compare the proposed method with the Wang's method and neural network method in the forecasting inflation rate. Finally, some conclusions are discussed in section 6.

2. *QR* factorization and singular value decomposition

In this section, we will introduce *QR* factorization and singular value decomposition of matrix and its properties referred from Scheick, J.T. (1997).

Let *B* be *m* x *n* matrix and suppose $m \le n$. The *QR* factorization of *B* is given by $B = QR$, where Q is *m* x *m* orthogonal matrix and *m* x *n* matrix $R = [R_{11}\ R_{12}]$ with R_{11} is *m* x *m* upper triangular matrix. The *QR* factorization of matrix *B* always exists and can be computed by Gram-Schmidt orthogonalization. Any *m* x *n* matrix *A* can be expressed as

$$A = USV^T \qquad (1)$$

where *U* and *V* are orthogonal matrices of dimensions *m* x *m*, *n* x *n* respectively and *S* is *m* x *n* matrix whose entries are 0 except $s_{ii} = \sigma_i \quad i = 1, 2, ..., r$ with $\sigma_1 \ge \sigma_2 \ge ... \ge \sigma_r \ge 0$, $r \le \min(m, n)$. Equation (1) is called a singular value decomposition (*SVD*) of *A* and the numbers σ_i are called singular values of *A*. If U_i , V_i are columns of *U* and *V* respectively, then equation (1) can be written as $A = \sum_{i=1}^{r} \sigma_i U_i V_i^T$.

Let $\| A \|_F^2 = \sum_{i,j} a_{ij}^2$ be the Frobenius norm of *A*. Because *U* and *V* are orthogonal matrices, then $\| U_i \| = 1$ and $\| V_i \| = 1$. Hence $\| A \|_F^2 = \left\| \sum_{i=1}^{r} \sigma_i U_i V_i^T \right\|_F^2 = \sum_{i=1}^{r} \sigma_i^2$. Let $A = USV^T$ be *SVD* of *A*. For given $p \le r$, the optimal rank *p* approximation of *A* is given by $A_p = \sum_{i=1}^{p} \sigma_i U_i V_i^T$.

Then $\| A - A_p \|_F^2 = \left\| \sum_{i=1}^{r} \sigma_i U_i V_i^T - \sum_{i=1}^{p} \sigma_i U_i V_i^T \right\|_F^2 = \left\| \sum_{i=p+1}^{r} \sigma_i U_i V_i^T \right\| = \sum_{i=p+1}^{r} \sigma_i^2$. This means that A_p is the best rank *p* approximation of *A* and the approximation error depend only on the summation of the square of the rest singular values.

3. Fuzzy time series

In this section, we introduce the following definitions and properties of fuzzy time series referred from Song, Q and Chissom, B.S. (1993).

Definition 1. Let $Y(t) \subset \mathbf{R}$, *t* = ..., 0, 1, 2, ..., **,** be the universe of discourse on which fuzzy sets $f_i(t)$ (*i* = 1, 2, 3,...) are defined and $F(t)$ is the collection of $f_i(t)$, *i* = 1, 2, 3,...,then $F(t)$ is called **fuzzy time series** on $Y(t)$, *t* = ..., 0, 1, 2, 3, ...

In the Definition 1, $F(t)$ can be considered as a linguistic variable and $f_i(t)$ as the possible linguistic values of $F(t)$. The value of $F(t)$ can be different depending on time *t* so $F(t)$ is function of time *t*. The following procedure gives how to construct fuzzy time series model based on fuzzy relational equation.

Definition 2. Let *I* and *J* be indices sets for $F(t-1)$ and $F(t)$ respectively. If for any $f_j(t) \in F(t)$, $j \in J$, there exists $f_i(t-1) \in F(t-1)$, $i \in I$ such that there exists a fuzzy relation $R_{ij}(t, t-1)$ and

$f_j(t) = f_i(t-1) \circ R_{ij}(t,t-1)$, let $R(t,t-1) = \bigcup_{i,j} R_{ij}(t,t-1)$ where $\cup$ is union operator, then $R(t,t-1)$ is called fuzzy relation between $F(t)$ and $F(t-1)$. This fuzzy relation can be written as

$$F(t) = F(t-1) \circ R(t,t-1). \tag{2}$$

where $\circ$ is max-min composition.

In equation (2), we must compute all values of fuzzy relations $R_{ij}(t,t-1)$ to determine value of $F(t)$. Based on above definitions, concept for first order and *m*-order of fuzzy time series can be defined.

Definition 3. If $F(t)$ is caused by $F(t-1)$ only or by $F(t-1)$ or $F(t-2)$ or … or $F(t-m)$, then the fuzzy relational equation

$$F(t) = F(t-1) \circ R\ (t,t-1) \text{ or}$$

$$F(t) = (F(t-1) \cup F(t-2) \cup ... \cup F(t-m)) \circ R_0(t,t-m) \tag{3}$$

is called first order model of $F(t)$.

Definition 4. If $F(t)$ is caused by $F(t-1)$, $F(t-2)$, ... and $F(t-m)$ simultaneously, then the fuzzy relational equation

$$F(t) = (F(t-1) \times F(t-2) \times ... \times F(t-m)) \circ R_a(t,t-m) \tag{4}$$

is called *m*-order model of $F(t)$.

From equations (3) and (4), the fuzzy relations $R(t,t-1)$, $R_a(t,t-m)$, $R_o(t,t-m)$ are important factors to design fuzzy time series model. Furthermore for the first order model of $F(t)$, for any $f_j(t) \in F(t)$, $j \in J$, there exists $f_i(t-1) \in F(t-1)$, $i \in I$ such that there exists fuzzy relations $R_{ij}(t,t-1)$ and $f_j(t) = f_i(t-1) \circ R_{ij}(t,t-1)$. This is equivalent to "if $f_i(t-1)$, then $f_j(t)$", and then we have the fuzzy relation $R_{ij}(t,t-1) = f_i(t-1) \times f_j(t)$. Because of $R(t,t-1) = \bigcup_{i,j} R_{ij}(t,t-1)$, then

$$R(t,t-1) = \text{maks}_{i,j}\{\min(f_j(t), f_i(t-1))\}. \tag{5}$$

For the relation $R_o(t,t-m)$ of the first order model, we get

$$R_o(t,t-m) = \underset{p}{\text{maks}}\{\ \underset{k}{\text{maks}}\{\underset{ik,j}{\min}(f_{ik}(t-k), f_j(t))\}\ \}. \tag{6}$$

Based on *m*-order model of $F(t)$, we have

$$R_a(t,t-m) = \underset{p}{\text{maks}}\{\ \underset{j,i1,i2,...,im}{\min}(\ f_{i1}(t-1) \times f_{i2}(t-2) \times ... \times f_{im}(t-m) \times f_j(t)\ )\} \tag{7}$$

From equations (5), (6) and (7), we can compute the fuzzy relations using max-min composition.

Definition 5. If for $t_1 \neq t_2$, $R(t_1,t_1-1) = R(t_2,t_2-1)$ or $R_a(t_1,t_1-m) = R_a(t_2,t_2-m)$ or $R_o(t_1,t_1-m) = R_o(t_2,t_2-m)$, then $F(t)$ is called time-invariant fuzzy time series. Otherwise it is called time-variant fuzzy time series.

Time-invariant fuzzy time series models are independent of time *t*, those imply that in applications, the time-invariant fuzzy time series models are simpler than the time-variant fuzzy time series models. Therefore it is necessary to derive properties of time-invariant fuzzy time series models.

Theorem 1. If $F(t)$ is fuzzy time series and for any *t*, $F(t)$ has only finite elements $f_i(t)$, *i* = 1, 2, 3, ..., n, and $F(t) = F(t-1)$, then $F(t)$ is a time-invariant fuzzy time series.

Theorem 2. If $F(t)$ is a time-invariant fuzzy time series, then

$$R(t,t-1) = ... \cup f_{i1}(t-1) \times f_{j0}(t) \cup f_{i2}(t-2) \times f_{j1}(t-1) \cup ... \cup f_{im}(t-m) \times f_{jm-1}(t-m+1) \cup ...$$

where m is a positive integer and each pair of fuzzy sets is different.

Based on the Theorem 2, we should not calculate fuzzy relation for all possible pairs. We need only to use one possible pair of the element of $F(t)$ and $F(t-1)$ with all possible *t*'s. This implies that to construct time-invariant fuzzy time series model, we need only one observation for every *t* and we set fuzzy relations for every pair of observations in the different of time *t*. Then union of the fuzzy relations results a fuzzy relation for the model. Theorem 2 is very useful because we sometime have only one observation in every time *t*.

Let $F_1(t)$ be *fuzzy time series* on $Y(t)$, *t* = ..., 0, 1, 2, 3, ... If $F_1(t)$ is caused by $(F_1(t-1), F_2(t-1))$, $(F_1(t-2), F_2(t-2)), ..., (F_1(t-n), F_2(t-n))$, then the fuzzy logical relationship is presented by $(F_1(t-n), F_2(t-n)), ..., (F_1(t-2), F_2(t-2)),\ (F_1(t-1), F_2(t-1)) \rightarrow F_1(t)$ and is called two factors *n*-order fuzzy time series forecasting model, where $F_1(t), F_2(t)$ are called the main factor and the secondary factor fuzzy time series respectively. If a fuzzy logical relationship is presented as

$$(F_1(t-n), F_2(t-n), ..., F_m(t-n)), ..., \ (F_1(t-2), F_2(t-2), ..., F_m(t-2)), \ (F_1(t-1), F_2(t-1), ..., \ F_m(t-1)) \rightarrow F_1(t) \qquad (8)$$

then the fuzzy logical relationship is called m factors n-order fuzzy time series forecasting model, where $F_1(t)$ are called the main factor fuzzy time series and $F_2(t), ..., F_m(t)$ are called the secondary factor fuzzy time series. The application of multivariate high order fuzzy time series can be found in Lee *et al* (2006) and Jilani *et al* (2007).

Because $f_j(t) = f_i(t-1) \circ R_{ij}(t, t-1)$ is equivalent to the fuzzy rule "IF $f_i(t-1)$, THEN $f_j(t)$", and the fuzzy relation $R_{ij}(t, t-1) =$ $f_i(t-1) \times f_j(t)$, then we can view the fuzzy rule as the fuzzy relation and vice versa.

4. Designing fuzzy time series model

Like in modeling traditional time series data, we use training data to set up the relationship among data values at different times. In fuzzy time series, the relationship is different from that in traditional time series. In fuzzy time series, we exploit the past experience knowledge into the model. The experience knowledge has form "IF … THEN …". This form is called fuzzy rules. So fuzzy rules is the heart of fuzzy time series model. Furthermore, main step to modeling fuzzy time series data is to identify the training data using fuzzy rules. Constructing fuzzy time series model, referred from Song, Q. & Chissom, B.S.(1993), can be done by the following steps: (1) define the universes of discourse; (2) collect the training data or linguistic values; (3) define fuzzy sets on the universes of discourse; (4) set up fuzzy relationships using training data or linguistic values; (5) sum up all the relationships defined in Step 4. Based on Theorem 2, the summation will be the fuzzy time series model; (6) calculate the forecasting outputs; (7) defuzzify the output of the model. If the goal of output of the model is fuzzy set, then Step 7 is not necessary. The Step 7 is used if we want the real output.

Let $A_{1,k}(t-i), ..., A_{N_i,k}(t-i)$ be N_i fuzzy sets with continuous membership function that are normal and complete in fuzzy time series $F_k(t-i)$, i =1, 2, 3,…, n, k = 1, 2, …, m, then the rule:

$$R^j: \ IF \ (x_1(t-n) \ is \ A^j_{i_1,1}(t-n) \text{ and ...and } x_m(t-n) \text{ is } A^j_{i_m,m}(t-n)) \text{ and } \ldots$$

$$\text{and } (x_1(t-1) \ is \ A^j_{i_1,1}(t-1) \text{ and ...and } x_m(t-1) \text{ is } A^j_{i_m,m}(t-1)), \ THEN \ x_1(t) \text{ is } A^j_{i_1,1}(t) \qquad (9)$$

is equivalent to the fuzzy logical relationship (8) and vice versa. So (9) can be viewed as fuzzy relation in $U \times V$ where $U = U_1 \times ... \times U_{mn} \subset R^{mn}$, $V \subset R$ with $\mu_A(x_1(t-n), ..., x_1(t-1), ..., x_m(t-n), ..., x_m(t-1)) =$ $\mu_{A_{i_1,1}}(x_1(t-n))...\mu_{A_{i_1,1}}(x_1(t-1))...\mu_{A_{i_m,m}}(x_m(t-n)...\mu_{A_{i_m},m}(t-1)$, where $A =$ $A_{i_1,1}(t-n) \times ... \times A_{i_1,1}(t-1) \times ... \times A_{i_m,m}(t-n) \times ... \times A_{i_m,m}(t-1)$.

Let $F_1(t-1), F_2(t-1), ..., \ F_m(t-1) \rightarrow F_1(t)$ be m factor one-order fuzzy time series forecasting model. Then $F_1(t-1), F_2(t-1), ..., \ F_m(t-1) \rightarrow F_1(t)$ can be viewed as fuzzy time series forecasting model with m inputs and one output. In this paper, we will design m factor one-order time invariant fuzzy time series model using table lookup scheme and singular value decomposition method. But this method can be generalized to m-factor n-order fuzzy time series model. Table lookup scheme is used to construct fuzzy logical relationships and then the singular value decomposition method is used to reduce the fuzzy logical relationships.

Suppose we are given the following N training data: $(x_{1p}(t-1), x_{2p}(t-1), ..., x_{mp}(t-1); x_{1p}(t))$, $p = 1, 2, 3, ..., N$. Constructing fuzzy logical relationships from training data using the table lookup scheme is presented as follows:

Step 1. Define the universes of discourse for main factor and secondary factor. Let $U = [\alpha_1, \beta_1] \subset R$ be universe of discourse for main factor, $x_{1p}(t-1), x_{1p}(t) \in [\alpha_1, \beta_1]$ and V = $[\alpha_i, \beta_i] \subset R, i = 2, 3, ..., m$, be universe of discourse for secondary factors, $x_{ip}(t-1) \in [\alpha_i, \beta_i]$.

Step 2. Define fuzzy sets on the universes of discourse. Let $A_{1,k}(t-i), ..., A_{N_i,k}(t-i)$ be N_i fuzzy sets in fuzzy time series $F_k(t-i)$ that are continuous, normal and complete in $[\alpha_k, \beta_k] \subset R, k = 2, 3, ..., m$, i =0,1.

Step 3. Set up fuzzy relationships using training data. For each input-output pair $(x_{1p}(t-1), x_{2p}(t-1), ..., x_{mp}(t-1); x_{1p}(t))$, determine the membership values of $x_{kp}(t-1)$ in $A_{i_k,k}(t-1)$ and membership values of $x_{1p}(t)$ in $A_{i_1,1}(t)$. Then for each $x_{kp}(t-i)$, determine $A_{i_k^*,k}(t-i)$ such that $\mu_{A_{j^*,k}(t-i)}(x_{k,p}(t-i)) \geq \mu_{A_{j,k}(t-i)}(x_{k,p}(t-i))$, j = 1, 2, …, N_k. Finally, for each input-output pair, obtain a fuzzy logical relationship as $((A_{j_1^*,1}(t-1), A_{j_2^*,2}(t-1), ..., A_{j_m^*,m}(t-1)) \to A_{i_1^*,1}(t)$. If we have some fuzzy logical relationships with the same antecedent part but different consequent part, then the fuzzy logical relationships are called the conflicting fuzzy relation. So we must choose one fuzzy logical relationship of conflicting group that has the maximum degree.

For a fuzzy logical relationship generated the input-output pair $(x_{1p}(t-1), x_{2p}(t-1), ..., x_{mp}(t-1); x_{1p}(t))$, we define its degree as $(\mu_{A_{j_1^*,1}(t-1)}(x_{1p}(t-1))\mu_{A_{j_2^*,2}(t-1)}(x_{2p}(t-1))...\mu_{A_{j_m^*,m}(t-1)}(x_{mp}(t-1))\mu_{A_{i_1^*,1}(t)}(x_{1p}(t))$. From this step we have the following M collections of fuzzy logical relationships designed from training data:

$$R^l: \ (A^l_{j_1^*,1}(t-1), A^l_{j_2^*,2}(t-1), ..., A^l_{j_m^*,m}(t-1)) \to A^l_{i_1^*,1}(t), \ l = 1, 2, 3, …, M. \quad (10)$$

Step 4. Determine the membership function for each fuzzy logical relationship resulted in the Step 3. We view each fuzzy logical relationship as fuzzy relation in $U \times V$ with $U = U_1 \times ... \times U_m \subset R^m$, $V \subset R$, then the membership function for the fuzzy logical relationship (10) is defined by

$$\mu_{R^l}(x_{1p}(t-1), x_{2p}(t-1), ..., x_{mp}(t-1); x_{1p}(t))$$

$$= \mu_{A_{j_1^*,1}(t-1)}(x_{1p}(t-1))\mu_{A_{j_2^*,2}(t-1)}(x_{2p}(t-1))...\mu_{A_{j_m^*,m}(t-1)}(x_{mp}(t-1))\mu_{A^l_{i_1^*,1}(t)}(x_{1p}(t))$$

Step 5. For given input fuzzy set $A'(t-1)$ in input space U, compute the output fuzzy set $A'_l(t)$ in output space V for each fuzzy logical relationship (10) as $\mu_{A'_l}(x_1(t)) = \sup_{x\in U}(\mu_{A'}(x(t-1))\mu_{R^l}(x(t-1); x_1(t)))$ where $x(t-1) = (x_1(t-1), ..., x_m(t-1))$.

Step 6. Compute fuzzy set $A'(t)$ as the combination of M fuzzy sets $A'_1(t), A'_2(t), A'_3(t), . . A'_M(t)$ by

$$\mu_{A'(t)}(x_1(t)) = \max_{l=1}^{M}(\mu_{A'_1(t)}(x_1(t), ..., \mu_{A'_M(t)}(x_1(t)))$$

$$= \max_{l=1}^{M}(\sup_{x\in U}(\mu_{A'}(x(t-1))\mu_{R^l}(x(t-1); x_1(t))) = \max_{l=1}^{M}(\sup_{x\in U}(\mu_{A'}(x(t-1))\prod_{f=1}^{m}\mu_{A_{f,f}(t-1)}(x_f(t-1))\mu_{A^l_{i_1,1}}(x_1(t)))).$$

Step 7. Calculate the forecasting outputs. Based on the Step 6, if we are given input fuzzy set $A'(t-1)$, then the membership function of the forecasting output $A'(t)$ is

$$\mu_{A'(t)}(x_1(t)) = \max_{l=1}^{M}(\sup_{x\in U}(\mu_{A'}(x(t-1))\prod_{f=1}^{m}\mu_{A_{f,f}(t-1)}(x_f(t-1))\mu_{A^l_{i_1,1}}(x_1(t)))). \quad (11)$$

Step 8. Defuzzify the output of the model. If the goal of output of the model is fuzzy set, then stop in the Step 7. We use this step if we want the real output. For example, if given the input fuzzy set $A'(t-1)$ with Gaussian membership function $\mu_{A'(t-1)}(x(t-1)) = \exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^*(t-1))^2}{a_i^2})$, then the forecasting real output using the Step 7 and center average defuzzifier is

$$x_1(t) = f(x_1(t-1), ..., x_m(t-1)) = \frac{\sum_{j=1}^{M} y_j \exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^{*j}(t-1))^2}{a_i^2+\sigma_{i,j}^2})}{\sum_{j=1}^{M}\exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^{*j}(t-1))^2}{a_i^2+\sigma_{i,j}^2})} \quad (12)$$

where y_j is center of the fuzzy set $A^j_{i_1,1}(t)$.

If the number of training data is large, then the number of fuzzy logical relationships may be large too. So increasing the number of fuzzy logical relationships will add the complexity of computation. To overcome the complexity of the computation, we will reduce the fuzzy logical relationships using singular value decomposition method. The steps to reduce the fuzzy logical relationships using singular value decomposition are presented as follows:

Step 1. Compute the firing strength of the fuzzy logical relationship (10) for each training datum $(x;y)$ = $(x_1(t-1), x_2(t-1), ..., x_m(t-1); x_1(t))$ defined by

$$L_l\,(x;y) = \frac{\prod_{f=1}^{m} \mu_{A_{j_f},f(t-1)}(x_f\,(t-1))\mu_{A_{j_1,1}^l}(x_1(t))}{\sum_{k=1}^{M}\prod_{f=1}^{m} \mu_{A_{j_f},f(t-1)}(x_f\,(t-1))\mu_{A_{j_1,1}^k}(x_1(t))} \tag{13}$$

Step 2. Construct N x M matrix $L = \begin{pmatrix} L_1(1) & L_2(1) & \cdots & L_M(1) \\ L_1(2) & L_2(2) & \cdots & L_M(2) \\ \vdots & \vdots & \vdots & \vdots \\ L_1(N) & L_2(N) & \cdots & L_M(N) \end{pmatrix}$. (14)

Step 3. Compute singular value decomposition of L. Based on the Section 2, *SVD* of L is written as $L = USV^T$, where U and V are N x N and M x M orthogonal matrices respectively, S is N x M matrix whose entries $s_{ij} = 0, i \neq j$, $s_{ii} = \sigma_i \quad i = 1, 2, ..., r$ with $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_r \geq 0$, $r \leq \min(N, M)$.

Step 4. Determine the number of fuzzy logical relationships that are designed as r with $r \leq \text{rank}(L)$.

Step 5. Partition V as $V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$, where V_{11} is r x r matrix, V_{21} is (M-r) x r matrix, and construct $\bar{V}^T = \begin{pmatrix} V_{11}^T & V_{21}^T \end{pmatrix}$.

Step 6. Apply *QR*-factorization to $\bar{V}^T$ and find M x M permutation matrix E such that $\bar{V}^T E = QR$ and Q is r x r orthogonal matrix, $R = [R_{11}\ R_{12}]$, R_{11} is r x r upper triangular matrix.

Step 7. Assign the position of entries one's in the first r columns of matrix E that indicates the position of the r most important fuzzy logical relationships.

Step 8. Construct fuzzy time series forecasting model (11) and (12) using the r most important fuzzy logical relationships.

5. Applications of the proposed method

In this section, we apply the proposed method to forecast the inflation rate in Indonesia. The proposed method is implemented using Matlab 6.5.1. In this paper, we apply six-factors one-order fuzzy time series model to predict inflation rate. The main factor is inflation rate and the secondary factors are the interest rate of Bank Indonesia certificate, interest rate of deposit, money supply, total of deposit and exchange rate. The data of the factors are taken from January 1999 to February 2003. The data from January 1999 to January 2002 are used to training and the data from February 2002 to February 2003 are used to testing. First, we will construct fuzzy logical relationship using table lookup scheme and then using the singular value decomposition method, the resulted fuzzy logical relationships will be reduce based on r most important fuzzy logical relationships. The procedure to forecasting inflation rate based on the table lookup scheme is given by the following steps:

Step 1. Define the universes of discourse for main factor and secondary factor. In this paper, we will predict the inflation rate of k^{th} month using data of inflation rate, interest rate of Bank Indonesia certificate, interest rate of deposit, money supply, total of deposit and exchange rate of (k-1)th month. The universes of discourse of interest rate of Bank Indonesia certificate, interest rate of deposit, exchange rate, total of deposit, money supply, inflation rate are defined as [10, 40], [10, 40], [6000, 12000], [360000, 460000], 40000, 90000], [-2, 4] respectively.

Step 2. Define fuzzy sets that are continuous, complete and normal on the universe of discourse such that the fuzzy sets can cover the input spaces. We define sixteen fuzzy sets $B_1, B_2, ..., B_{16}$, sixteen fuzzy sets $C_1, C_2, ..., C_{16}$, twenty five fuzzy sets $D_1, D_2, ..., D_{25}$, twenty one fuzzy sets $E_1, E_2, ..., E_{21}$, twenty one fuzzy sets $F_1, F_2, ..., F_{21}$, thirteen fuzzy sets $A_1, A_2, ..., A_{13}$ on the universes of discourse of the interest rate of Bank Indonesia certificate, interest rate of deposit, exchange rate, total of deposit, money supply, inflation rate respectively. We use Gaussian membership function for all fuzzy sets.

Step 3. Set up fuzzy logical relationships using training data. We use 36 pair of training data and based on this step, we have 36 fuzzy relations in the form:

$$(B_{j_2}^l(t-1), C_{j_3}^l(t-1), D_{j_4}^l(t-1), E_{j_5}^l(t-1), F_{j_6}^l(t-1), A_{j_1}^l(t-1)) \rightarrow A_{j_*}^l(t)$$

Table 1 presents all fuzzy logical relationships.

Apply the Step 4 to Step 7, for given input fuzzy set $A'(t-1)$, then the membership function of the forecasting output $A'(t)$ is

$$\mu_{A'(t)}(x_1(t)) = \max_{l=1}^{M}(\sup_{x\in U}(\mu_{A'}(x(t-1))\prod_{f=1}^{m}\mu_{A_{f,f(t-1)}}(x_f(t-1))\mu_{A_{1,1}^{l}}(x_1(t))))$$

$$= \max_{l=1}^{M}(\prod_{f=1}^{m}\exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^{*j}(t-1))^2}{a_i^2+\sigma_{i,j}^2})\mu_{A_{1,1}^{l}}(x_1(t))) \qquad (15)$$

Step 8. Defuzzify the output of the model. The predicting real output of the model using (15) and center average defuzzifier is $x_1(t) = f(x_1(t-1),...,x_m(t-1)) = \dfrac{\sum_{j=1}^{M} y_j \exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^{*j}(t-1))^2}{a_i^2+\sigma_{i,j}^2})}{\sum_{j=1}^{M}\exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^{*j}(t-1))^2}{a_i^2+\sigma_{i,j}^2})}$.

The comparison of prediction and true values of inflation rate using table lookup scheme is shown in Figure 2(d).

To know the most fuzzy logical relationships, we apply the singular value decomposition method to a matrix of firing strength of rules. The procedure to discard the less important fuzzy logical relationships is presented as follows:

Step 1. Compute the firing strength of the fuzzy logical relationship in Table 1 based on (13) for each training datum.

Step 2. Construct *36* x *36* matrix $L = \begin{pmatrix} L_1(1) & L_2(1) & \cdots & L_{36}(1) \\ L_1(2) & L_2(2) & \cdots & L_{36}(2) \\ \vdots & \vdots & \vdots & \vdots \\ L_1(36) & L_2(36) & \cdots & L_{36}(36) \end{pmatrix}$, where $L_j(i)$, *i,j* = 1, 2, …, 36, is computed using Step 1.

Step 3. Compute singular value decomposition of *L* as $L = USV^T$. There are thirty four nonzero singular values of *L*. The distribution of the singular values of *L* can be seen in Figure 1.

Step 4. Determine the number of fuzzy logical relationships that are designed as *r* with $r \le \text{rank}(L)$. Based on the Figure 1, the singular values decrease strictly after the first twenty nine singular values. So we choose arbitrarily the first eight, twenty, and twenty nine singular values. Apply the *QR* factorization to get a permutation matrix *E* and then assign the position of entries one's in the first *r* columns of matrix *E* that indicates the position of the *r* most important fuzzy logical relationships.

Table 1. Six-factors one-order fuzzy logical relationship groups for inflation rate using table lookup scheme

rule	$((x_2(t-1), x_3(t-1), x_4(t-1), x_5(t-1), x_6(t-1), x_1(t-1))$	$\to x_1(t)$	rule	$((x_2(t-1), x_3(t-1), x_4(t-1), x_5(t-1), x_6(t-1), x_1(t-1))$	$\to x_1(t)$
1	(B14, C14, D13, E12, F2, A11)	→ A8	19	(B3, C1, D13, E3, F7, A8)	→ A6
2	(B15, C14, D12, E13, F2, A8)	→ A5	20	(B3, C2, D10, E2, F7, A6)	→ A4
3	(B15, C14, D12, E13, F3, A5)	→ A4	21	(B3, C2, D12, E3, F8, A4)	→ A7
4	(B14, C13, D10, E15, F2, A4)	→ A4	22	(B3, C2, D15, E6, F8, A7)	→ A8
5	(B10, C11, D9, E16, F2, A4)	→ A4	23	(B3, C2, D15, E7, F8, A8)	→ A9
6	(B7, C8, D4, E13, F2, A4)	→ A3	24	(B3, C2, D15, E7, F14, A9)	→ A6
7	(B4, C5, D5, E13, F2, A3)	→ A3	25	(B3, C2, D15, E9, F9, A6)	→ A7
8	(B3, C3, D7, E11, F3, A3)	→ A4	26	(B3, C3, D16, E11, F9, A7)	→ A7
9	(B3, C2, D11, E10, F4, A4)	→ A5	27	(B4, C3, D19, E13, F9, A7)	→ A6
10	(B3, C2, D5, E4, F4, A5)	→ A5	28	(B4, C3, D24, E15, F10, A6)	→ A7
11	(B3, C2, D7, E9, F4, A5)	→ A8	29	(B4, C3, D21, E14, F10, A7)	→ A8
12	(B2, C2, D5, E6, F8, A8)	→ A8	30	(B4, C3, D23, E14, F11, A8)	→ A9
13	(B2, C2, D7, E7, F5, A8)	→ A5	31	(B5, C3, D15, E10, F12, A9)	→ A5
14	(B2, C2, D7, E7, F5, A5)	→ A4	32	(B5, C3, D12, E10, F13, A5)	→ A6
15	(B2, C1, D7, E7, F5, A4)	→ A6	33	(B5, C5, D16, E12, F13, A6)	→ A6
16	(B1, C1, D9, E7, F5, A6)	→ A7	34	(B5, C5, D19, E16, F12, A6)	→ A8
17	(B2, C1, D11, E8, F6, A7)	→ A6	35	(B5, C5, D19, E17, F14, A8)	→ A8
18	(B2, C1, D12, E4, F7, A6)	→ A8	36	(B5, C5, D19, E18, F16, A8)	→ A9

As the result of taking the first eight, twenty and twenty nine singular values, we reduce the number of fuzzy logical relationships from thirty six to eight, twenty, twenty nine respectively. The positions of the eight most important fuzzy logical relationships are identified as 4, 8, 15, 18, 22, 30, 31, 36. The positions of the twenty most important fuzzy logical relationships are identified as 1, 4, 5, 6, 7, 8, 9, 10, 12, 15, 18, 20, 22, 27, 30, 31, 32, 33, 34, 36. The positions of the twenty nine most important fuzzy logical relationships are identified as 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 15, 16, 17, 18, 19, 20, 22, 25, 26, 27, 28, 30, 31, 32, 33, 34, 36. The resulted fuzzy logical relationships are used to design fuzzy time series forecasting model by (11) and (12).

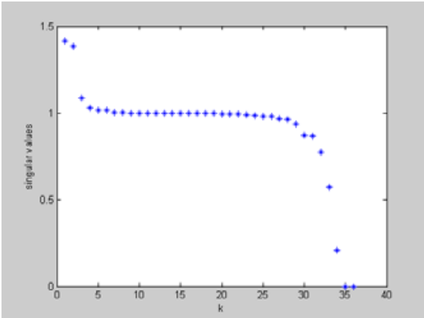

Figure 1. Distribution of singular values of matrix *L*

The MSE of training and testing data from the different number of reduced fuzzy logical relationships are shown in Table 2. From Table 2, the predicting inflation rate using the proposed method results more accuracy than that using the other methods. From Figure 1, the singular values are “small” after the first twenty nine singular values, so the forecasting inflation rate using the first twenty nine singular values gives a better accuracy than using the first eight and twenty singular values respectively.

Table 2. Comparison of MSE of training and testing data
using the different methods

Method	Number of fuzzy relations	MSE of training data	MSE of testing data
Proposed method	8	0.485210	0.66290
	20	0.312380	0.30173
	29	0.191000	0.21162
Table lookup scheme	36	0.063906	0.30645
Neural network		0.757744	0.42400

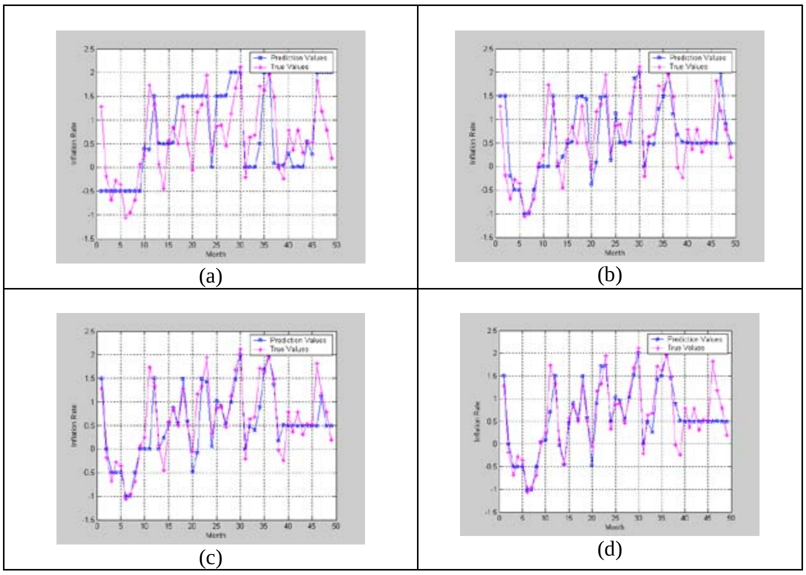

Figure 2. Prediction and true values of inflation rate using proposed method: (a) eight fuzzy logical relationships, (b). twenty fuzzy logical relationships, (c) twenty nine fuzzy logical relationships, (d) thirty six fuzzy logical relationships

6. Conclusions

In this paper, we have presented a new method to design fuzzy time series model. The method combines the table lookup scheme and singular value decomposition method, where defined fuzzy sets are continuous, normal and complete in the universe of discourse. Based on the training data, the table lookup scheme is used to construct fuzzy logical relationships and then we apply the singular value decomposition and *QR*-factorization method to the firing strength matrix of the fuzzy logical relationships to remove the less important fuzzy logical relationships. The position of the entries one's of the permutation matrix yields the position of the most important fuzzy logical relationships. Then, the proposed method is applied to forecast the inflation rate. Furthermore, predicting inflation rate using the proposed method yields more accuracy than that using the neural network method. The precision of forecasting depends also to taking factors as input variables and the number of defined fuzzy sets. In the next work, we will design how to select the important variables to improve prediction accuracy.

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