TOPIC 7
Computer Application For Finding Maximum And Minimum Value Of A Function

In spite of **plot**, there is built-in function in Matlab that are very useful to drawing graph of a function, i.e. **ezplot** and **fplot**.

ezplot

It is easy to use function plotter, especially for explicit, implicit and parametric function.

1. Explicit function, f=f(x)

This is an example of explicit function:

y=cos(x) $$f(x)=\frac{\sin(x)}{1+x^2}$$

A. ezplot(f) plots the expression f = f(x) over the default domain -2*pi < x < 2*pi.
>>ezplot('cos(x)')
>>ezplot('sin(x)/(1+x^2)')

B. ezplot(f, [a,b]) plots f = f(x) over a < x < b
>>ezplot('cos(x)', [0, pi])

C. ezplot(f, [xmin, xmax,ymin,ymax]) plots f=f(x) = 0 over xmin < x < xmax and ymin < y < ymax.
>> ezplot('sin(x)/(1+x^2)', [-4, 4,-0.5,0.5])

2. Implicit function, f = f(x,y)

Example:

$$x^2-y^2=1 \qquad \frac{1}{y}-\log(y)+\log(-1+y)+x=1$$

A. ezplot(f) plots f(x,y) = 0 over the default domain -2*pi < x < 2*pi and -2*pi < y < 2*pi
>> ezplot('x^2-y^2-1')

B. ezplot(f, [xmin,xmax,ymin,ymax]) plots f(x,y) = 0 over xmin < x < xmax and ymin < y < ymax.
>> ezplot('x^3 + y^3 - 5*x*y + 1/5',[-3,3])

C. ezplot(f, [a,b]) plots f(x,y) = 0 over a < x < b and a < y < b.
>> ezplot('x^3 + y^3 - 5*x*y + 1/5',[-3,3,-5,5])

If f is a function of the variables u and v (rather than x and y), then the domain endpoints a, b, c, and d are sorted alphabetically. Thus,

ezplot('u^2 - v^2 - 1',[-3,2,-2,3]) plots u^2 - v^2 - 1 = 0 over -3 < u < 2, -2 < v < 3.

3. Parametric function

Example:

$$x = \sin(t)$$

$$y = \cos(t)$$

$$x^2 + y^2 = \sin^2 t + \cos^2 t = 1$$

A. ezplot(x,y):

ezplot(x,y) plots the parametrically defined planar curve x = x(t) and y = y(t) over the default domain 0 < t < 2*pi.

>> ezplot('sin(t)','cos(t)')

B. ezplot(x,y, [tmin,tmax])

ezplot(x,y, [tmin,tmax]) plots x = x(t) and y = y(t) over tmin < t < tmax.

>>ezplot('sin(3*t)*cos(t)','sin(3*t)*sin(t)',[0,pi])

ezplot(f, [a,b], FIG), ezplot(f, [xmin,xmax,ymin,ymax], FIG), or ezplot(x,y, [tmin,tmax], FIG) plots the given function over the specified domain in the figure window FIG.

Fplot

1. fplot(f,[a,b]) : plot f

plots the function f over a < x < b

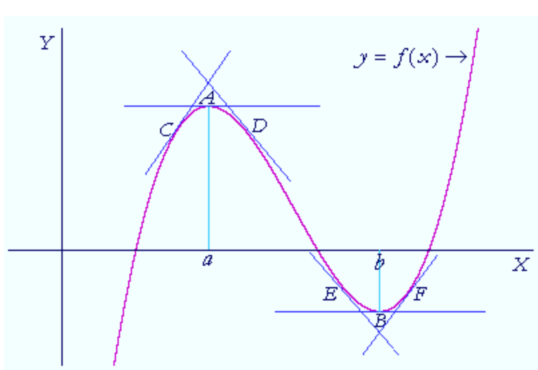
>> fplot('sin(1 ./ x)', [0.01 0.1])

2. fplot(f,[xmin,xmax,ymin,ymax]) :

plots the function f over xmin <x<xmax and ymin <y<ymax

>> fplot('[tan(x),sin(x),cos(x)]',2*pi*[-1 1 -1 1])

Finding the minimum value of a function



We say that a function f(x) has a relative maximum value at x = a, if f(a) is greater than any value in its immediate neighborhood. We call it a "relative" maximum because other values of the function may in fact be greater.

We say that a function f(x) has a relative minimum value at x = b, if f(b) is less than any value in its immediate neighborhood. Again, other values of the function may in fact be less. With that understanding, then, we will drop the term relative.

The value of the function, the value of y, at either a maximum or a minimum is called an **extreme** value. A value of x at which the function has either a maximum or a minimum is called a **critical** value. In the figure, the critical values are x = a and x = b.

MATLAB has **fminbnd** function to find the value of x where an extreme value occurs. Below is the step to find an critical and extreme value of a function using MATLAB.

Example:

Determine the minimum and maximum value of $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 4$

Step 1: define a function

Matlab has **inline** function to define a function

```
>>f= inline('1/3*x^3-2*x^2+3*x+4')
```

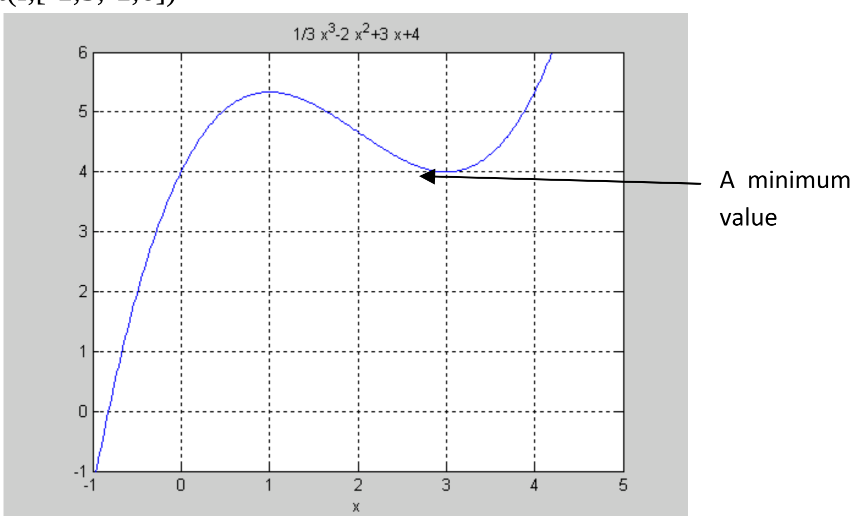
Step 2: determine the interval of a minimum value

At first, plot the function to determine the interval, using ezplot.

```
>>ezplot(f),grid
```

Specify the interval in plotting make the graph more clearly.

```
>>ezplot(f,[-1,5,-1,6])
```



From the graph we know that the minimum value is in interval (2 ,4).

Step 3: determine critical value using fminbnd

Syntax: **fminbnd(f,x1,x2)**

notes : f is a function that defined with **inline** in step 1

x1,x2 are (x1,x2) in the interval where minimum value occurs.

So the command to find critical value is:

```
>> xmin =fminbnd(f,2,4)
        xmin =
                3.0000
```

Step 4: determine extreme value

The extreme value can be determined by evaluate the critical value in the function.

```
>> ymin =f(xmin)
        ymin =
                4.0000
```

Based on step 3 and 4, we get the minimum of $f(x)=\frac{1}{3}x^3-2x^2+3x+4$ occurs at the point **(3,4).**

Finding the maximum value of a function

Example:

Determine a maximum value of $f(x)=\frac{1}{3}x^3-2x^2+3x+4$

Step 1: define the x-axes reflection of defined function

Because of fminbnd is merely to find the minimum value, so to find the maximum, we must reflect the function to x-axes

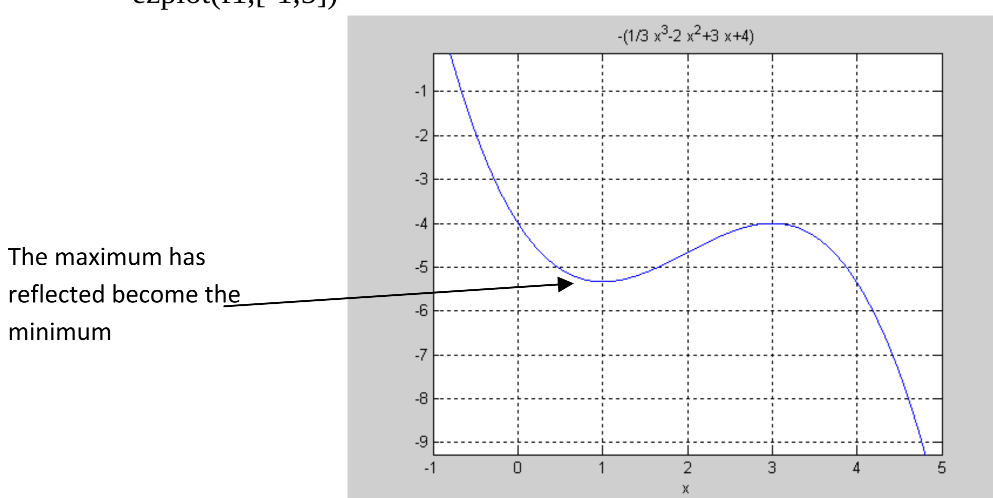
The reflected function is defined by give the **– (minus)** sign in front of predefined function.

```
>>f1= inline('-(1/3*x^3-2*x^2+3*x+4)')
```

Step 2: determine the interval of the maximum value

At first, plot the f1 graph.

```
>>ezplot(f1,[-1,5])
```



The maximum of the graph occurs at the interval (0,2).

Step 3: determine the critical value using fminbnd

```
>> xmax =fminbnd(f1,0,2)
       xmax =
          1.0000
```

Step 4: determine the extreme value

The extreme value can be determined by evaluate the critical value in the **predefined function (f), not f1.**

```
>> ymax=f(xmax)
       ymax =
          5.3333
```

Based on step 3 and 4, we get the maximum of $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 4$ occurs at the point **(1, 5.33).**

Exercise:

Find the minimum and maximum of a function $f(x) = 4x - x^3$. Use the interval (-3 3) to drawing the graph becomes more clearly.

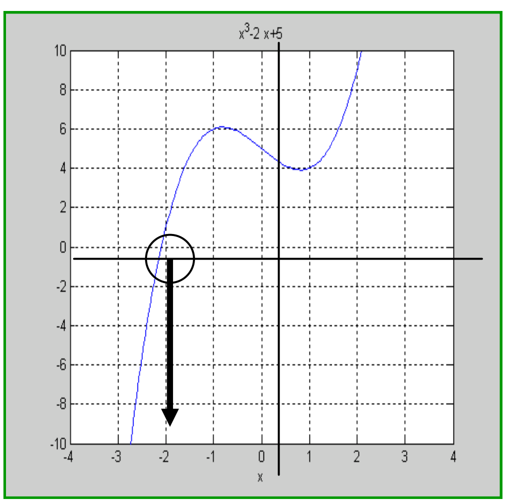
Finding the Zero point of a function

Matlab has the **fzero** function to find a zero of a function. **X=fzero(f,xo)** tries to find a zero of the function **f** near **xo**,

It first finds an interval containing **xo** where the function values of the interval endpoints differ in sign, then searches that interval for a zero.

>>f= inline('x^3-2*x+5')

>>ezplot(f)



From the graph is known that a zero of the function is near x=-2, so:

\>> tnol=fzero(f,-2)

\>> ynol = f(tnol) % check tnol