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Chapter 9

Simulation

Chapter Overview

9.1   Introduction

This chapter defines simulation and describes how simulation can be performed in Excel. We illustrate some simple tools available in Excel and then discuss how to use some new functions to create a simulation in Excel. The reader should be aware that there are several simulation software packages available that have more advanced features than what is available in Excel; since Excel is primarily a spreadsheet software, simulation features are not highlighted. However, we want to demonstrate, that when developing a spreadsheet-based DSS, simulation can be performed with the basic tools available in Excel. We revisit the simulation topic in Chapter 20 with VBA. We have several DSS applications which use simulation, such as Poker Simulation, Birthday Simulation, Queuing Simulation, and Reliability Analysis.

9.2   Defining Simulation

Simulation is a modeling tool that is used to imitate a real-world process and better understand system behavior. Authentic system behavior can be estimated through the use of distributions. From these distributions, a user can generate random numbers to evaluate multiple strategies and predict future performance. Simulation is a useful tool in that it can make observations through trial and error without the cost of materials, labor, and time that would be necessary to observe the same results of the original process.

For example, if the supervisor of a paper mill wants to determine how production will be affected by a change in the rate of raw material input, it would be very costly to observe the actual affects in the mill. He would have to instruct employees to change the rate of placing raw materials in the first machine, possibly change the processing times of some following machines, react to any malfunctions caused by this change, and possibly get an undesirable result, thus having to change everything back to how it originally was. By using simulation, he could see what would happen numerically without actually modifying the actual process. If the simulation model showed an increase in production by changing the rate of incoming raw materials, the supervisor could implement the change; otherwise, he would not need to waste time if production did not increase.

Simulation can be used differently from optimization in that instead of seeking to optimize an objective function value, it simulates the behavior of a system to assess its performance under several scenarios that may be used in "what-if" analysis. There are many applications in which simulation becomes useful; we will discuss these applications in the following sections. Excel offers two simple tools for performing simulation: *Data Tables* and the *Scenario Manager*. We will then explain how some Excel functions can be used to perform more advanced simulation.

9.2.1 Data Tables

We use *Data Tables* to determine how some outputs vary in response to changes in input. *Data Tables* use a spreadsheet to refer to cells that may contain formulas or functions for some output and input of a problem. For example, you may have input as sales price and output as profit. The input cell would just contain a numerical value, but the output cell would contain a formula that calculates profit based on quantity sold and

profit per unit. You could then use a *Data Table* to vary the price values and observe the change in profit.

There are two types of *Data Tables*: **one-way data tables** and **two-way data tables**. A *one-way data table* enables you to determine how changing one input will change any number of outputs. In the above example of price and profit, price is the only input value that needs changing, so you would create a *one-way data table*. It is also possible to observe the number of customers as price values while observing profit. Since this would only require changing one input (price) you would use the *one-way data table*. *Two-way data tables* allow you to determine how changing two inputs affect a single output. For example, if you varied price and location values as input, you could observe profit. Since two inputs are changing, you would use a *two-way data table*.

To use *Data Tables*, you must first prepare your spreadsheet. To do this, create a list of inputs and outputs. These cells may contain values or formulas. You should label them for easy reference. Next, create a list of the various input values that you want to experiment with. If you are creating a *one-way data table*, you should put these values in a single column. If you are creating a *two-way data table*, you should create one column and one row of varying input values for the two inputs of interest. Then, enter the output formulas that you want the *Data Table* to calculate for observation. For *one-way data tables*, these output cells (again, more than one output can be observed using *one-way data tables*) should be in the columns adjacent to the input column. For *two-way data tables*, this output cell (again, only one output value is observed with *two-way data tables*) should be placed in the upper corner of the data table (in the cell above the column of input value and next to the row of input values).

Once you've prepared your spreadsheet, highlight the entire table area that you just created (without column or row titles). Now, select *Data > Table* from the Excel menu. A small dialog box will appear that asks you for the row and/or column input cell (see Figure 9.1).
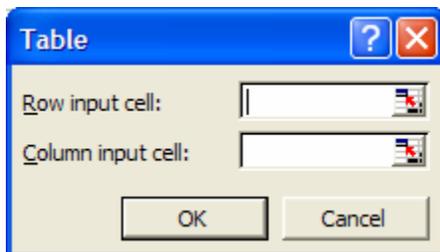

Figure 9.1   The *Row Input* and *Column Input* cells refer to the initial list of inputs and outputs.

If you are creating a *one-way data table*, the column input cell will be the only reference you give. This reference is to the cell in your original list of inputs and outputs that contains the current value of the input that you wish to vary. The formula you copied for the output cell(s) should be in terms of this input cell location. If you are creating a *two-way data table*, you will reference both a row and column input since you are varying two inputs. Again, these references should be to the input cells in the initial list of inputs and outputs. Once you have specified the input cell references, click *OK* and observe the values calculated in the data table.

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Let's consider an example to illustrate both *one-way* and *two-way data tables*. In Figure 9.2, we provide a list of inputs and outputs for ticket sales. The *Total Profit* is calculated by finding the unit profit (price minus cost per ticket) and multiplying this value by the number of salespersons and the average number of tickets sold per person. Using the input cells shown, the formula for the total profit would be:

=(B6-B7)*B4*B5

B8	▼		*fx* =(B6-B7)*B4*B5	
		A		B
1				
2	**Ticket Sales**			
3				
4		Number of salespersons		3
5		Avg num tickets sold per person		15
6		Price per ticket		$20.00
7		Cost per ticket		$15.00
8		Total profit		$225.00
9				

Figure 9.2 The initial list of inputs and outputs contains values and formulas.

The first data table we want to create will show the different profit values as we vary the price per ticket. Since we are only varying one input, this will be a *one-way data table*. Let's begin by creating a column of various prices per ticket. We will vary these prices from $16.00 to $24.00 per ticket. Then, we copy the formula used in cell B8 to a cell in the first row of the data table (above the first price value in the input columns). Note that the output formula can either be copied directly or we can just refer to the original formula cell in the list of inputs and outputs; that is, we could again type "=(B6-B7)*B4*B5" orsimply refer to this formula in the initial table by typing "=B8". Figure 9.3 shows the preparation for this data table.

		Total profit	
		$225.00	
	$16.00		
	$18.00		
	$20.00		
	$22.00		
	$24.00		
	$30.00		

(left axis label: Price per ticket)

Figure 9.3 Preparation for the one-way data table comparing price per ticket and total profit.

We now select *Data > Table* from the menu. Since we are varying price as the input, and we have these varying values in the first column of the data table, we should specify the location of the price value in our initial list of inputs and outputs as the *Column Input*; this is, cell B6 (see Figure 9.4).

4

Figure 9.4   Using the price per ticket initial cell reference as the *Column Input*.

Once we press *OK*, Excel completes the data table. In Figure 9.5, we find that various profit values have been calculated for each possible price per ticket listed.

Figure 9.5   The final data table shows various profit values as the price per ticket changes.

Now suppose that we are curious to see how the combination of price per ticket and number of salespersons affects the total profit. Since we are varying two inputs, we should now create a *two-way data table*. We again begin by creating a column of varying prices per ticket. Then, we create a row for varying numbers of salespersons (above the first price value in the input column). Now, in the cell above the input column of prices and next to the input row of number of salespersons, we again copy the formula for total profit. Figure 9.6 illustrates the preparation for this data table.

Total profit	Number of salespersons				
$225.00	1	2	3	4	5
$16.00					
$18.00					
$20.00					
$22.00					
$24.00					
$30.00					

(Left side label: Price per ticket)

Figure 9.6 The preparation for the two-way data table that varies both price per ticket and number of salespersons.

We now select *Data > Table* from the menu. This time, we will reference both the *Row Input* and *Column Input*. The *Row Input* will be the number of salespersons, which is cell B4, in the initial list of inputs and outputs. The *Column Input* will again be cell B6 for price per ticket (see Figure 9.7).

	A	B	C	D	E	F	G	H
1								
2	**Ticket Sales**							
3			Table			?✕		
4	Number of salespersons	3	Row input cell:	B4				
5	Avg num tickets sold per person	15						
6	Price per ticket	$20.00	Column input cell:	B6				
7	Cost per ticket	$15.00						
8	Total profit	$225.00		OK	Cancel			
9								
19								
20	*Two-way data table*		Total profit		Number of salespersons			
21			$225.00	1	2	3	4	5
22			$16.00					
23			$18.00					
24			$20.00					
25			$22.00					
26			$24.00					
27			$30.00					

(Column label near rows 22–27: Price per ticket)

Figure 9.7 The number of salespersons is the *Row Input* and the price per ticket is the *Column Input*.

We press *OK* to see the completed data table. For any combination of price per ticket and number of salespersons (among the varying values specified), we can find the calculated total profit (see Figure 9.8).

Total profit	Number of salespersons				
$225.00	1	2	3	4	5
$16.00	$15.00	$30.00	$45.00	$60.00	$75.00
$18.00	$45.00	$90.00	$135.00	$180.00	$225.00
$20.00	$75.00	$150.00	$225.00	$300.00	$375.00
$22.00	$105.00	$210.00	$315.00	$420.00	$525.00
$24.00	$135.00	$270.00	$405.00	$540.00	$675.00
$30.00	$225.00	$450.00	$675.00	$900.00	$1,125.00

(Left side label: Price per ticket)

Figure 9.8   The final data table shows various profit values for combinations of number of salespersons and prices per ticket.

9.2.2 Scenario Manager

The *Scenario Manager* allows you to vary up to 32 input cells for various values, or scenarios, and observe the results of several output cells. The *Scenario Manager* will create a *Scenario Report,* which shows the resulting output values for each scenario of input values.

The spreadsheet preparation for using the *Scenario Manager* is simpler than that for *Data Tables*. All we need is the initial list of inputs or outputs. Appropriate values and formulas should be filled in in these cells. Now, we go to *Tools > Scenarios* to view the *Scenario Manager* dialog box shown below in Figure 9.9.
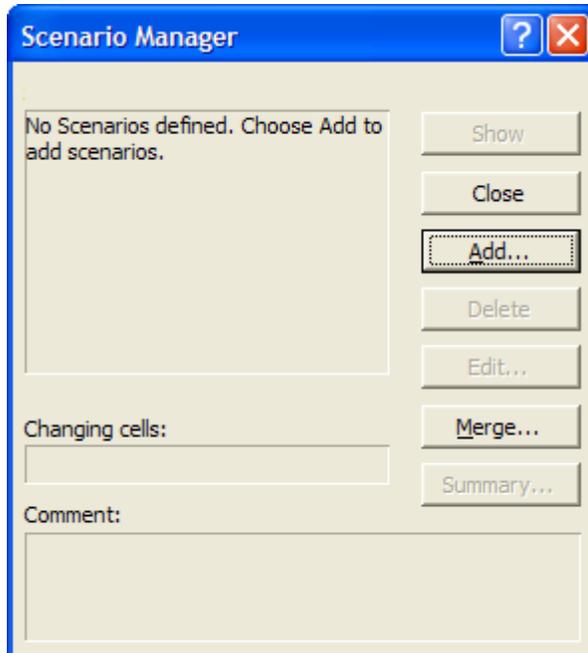

Figure 9.9   The Scenario Manager dialog box.

To create a scenario, simply press the *Add* button. Another dialog box, in which the inputs are selected for the scenario, will appear (see Figure 9.10). These cell references

should be to the list of inputs created in the spreadsheet preparation. We also name the scenario at this time.
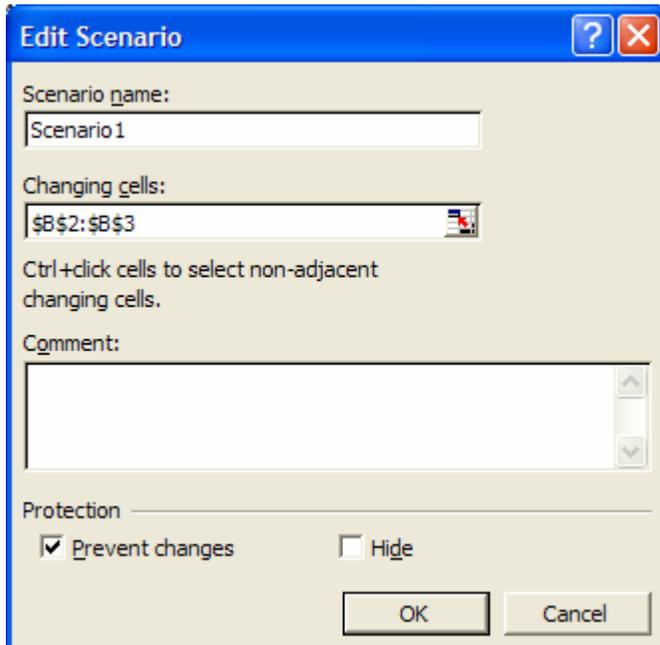

Figure 9.10   Creating the scenario name and selecting the input cells to change.

Next we specify the values that these inputs should take for the scenario that we are creating (see Figure 9.11).
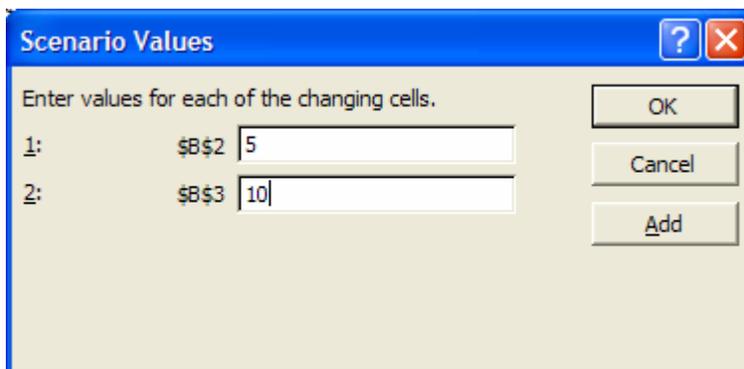
Figure 9.11   Specifying the values of the changing input cells.

The scenario will now be listed in the original *Scenario Manager* dialog box. At this point, we can *Add* more scenarios, *Edit* a scenario we have already created, or *Delete* any of the listed scenarios. When we are done specifying the various scenarios of input values, we can click *Summary* to create the *Scenario Report*. The *Scenario Summary* dialog box asks us to select the outputs that we want to observe for the various scenarios of inputs (see Figure 9.12). We can then choose to create a *Scenario Summary*, or *Scenario Report*, or a *Scenario Pivot Table* report. These will be created as new sheets in the current workbook.
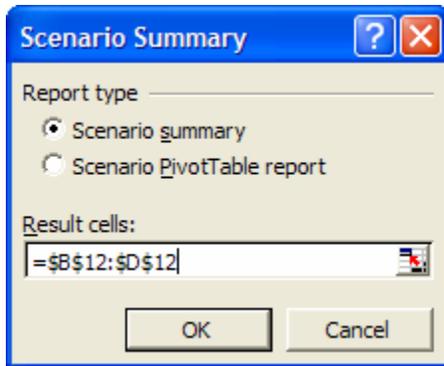
Figure 9.12 Selecting the output cells to put in the Scenario Report.

Let's consider an example to illustrate the benefits of using the *Scenario Manager*. In the table shown in Figure 9.13, there is a list of inputs for a company's sales. These inputs are: tax rate; Year 1 sales; sales growth; year 1 price; year 1 cost; interest rate; cost growth; and price growth. Then, there is a table of outputs for five years. The outputs are: unit sales; unit price; unit cost; revenue; costs; before tax profits; tax, after tax profits; and total NPV. The unit sales, unit price, and unit cost are actually inputs calculated using the growth rates from the input table.

	A	B	C	D	E	F	G
1							
2	**Company Sales**						
3							
4	*Inputs*	Tax Rate	0.4				
5		Year 1 Sales	10000				
6		Sales Growth	0.1				
7		Year 1 Price	$9.00				
8		Year 1 Cost	$6.00				
9		Interest Rate	0.15				
10		Cost Growth Rate	0.05				
11		Price Growth Rate	0.03				
12							
13	*Outputs*	Year	1	2	3	4	5
14		Unit Sales	10000	11000	12100	13310	14641
15		Unit Price	$9.00	$9.27	$9.55	$9.83	$10.13
16		Unit Cost	$6.00	$6.30	$6.62	$6.95	$7.29
17		Revenues	$90,000.00	$101,970.00	$115,532.01	$130,897.77	$148,307.17
18		Costs	$60,000.00	$69,300.00	$80,041.50	$92,447.93	$106,777.36
19		Before Tax Profits	$30,000.00	$32,670.00	$35,490.51	$38,449.83	$41,529.81
20		Tax	$12,000.00	$13,068.00	$14,196.20	$15,379.93	$16,611.92
21		After Tax Profits	$18,000.00	$19,602.00	$21,294.31	$23,069.90	$24,917.89
22							
23		NPV	$70,054.34				

Figure 9.13 The initial input and output cells.

For this example, let's focus on the company's after tax profits for each of the five years as well as their total NPV. We want to consider three different scenarios for year 1 sales, sales growth, and year 1 price and choose various values of these inputs for Best, Worst, and Most Likely scenarios. We begin by adding a new scenario (see Figure 9.14).
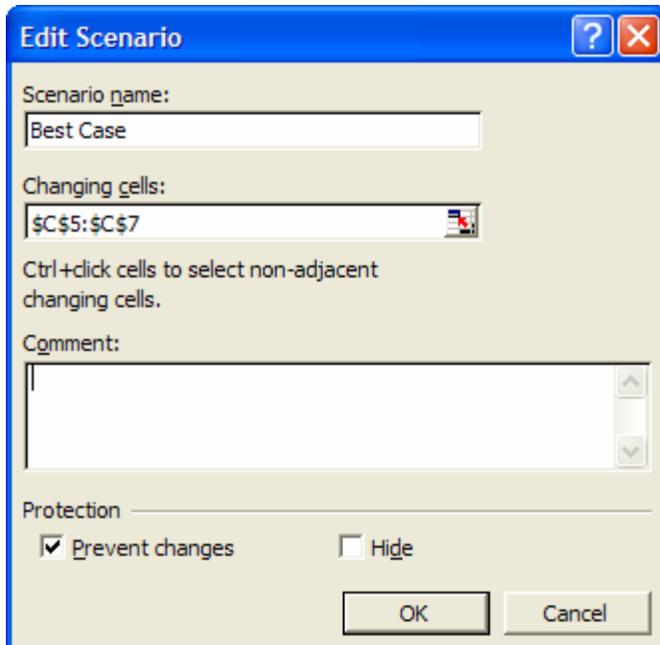
Figure 9.14   Naming the Best Case scenario and selecting the changing input cells.

We select the input cells, or changing cells, for the year 1 sales, sales growth, and year 1 price: *C5:C7*. We name this scenario the Best Case scenario and give the values 20,000, 0.2, and 10 for these inputs, respectively (see Figure 9.15). Then, we press *Add* on this dialog box to directly create the next scenario.
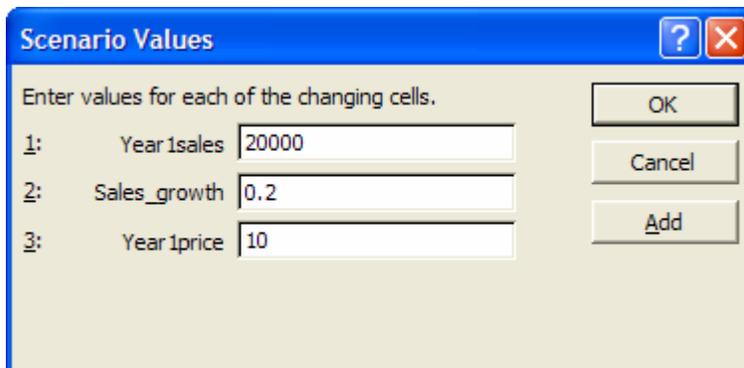
Figure 9.15   Specifying the values for the Best Case inputs.

Let's name this the Worst Case scenario and select the same input cells. We give them the values 5,000, 0.02, and 5, respectively.

Finally, we create the Most Likely scenario using the same input cells with the values 10,000, 0.1, and 7.5. Figure 9.16 illustrates the list of scenarios that we have created in the Scenario Manager dialog box.
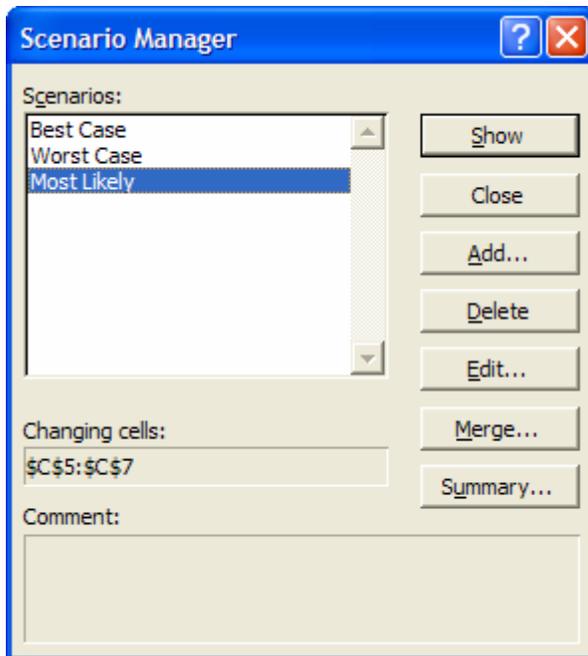
Figure 9.16   Creating the three scenarios.

We now choose *Summary* and select the "after tax profit row" (for all five years), which is in cells *C21* and *G21,* and the NPV cell, which is cell *C23*, as the outputs (see Figure 9.17). We then press *OK* to create the *Scenario Report*.
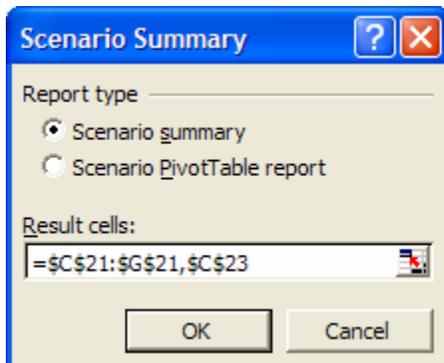
Figure 9.17   Selecting the output cells to create the Scenario Report.

The *Scenario Report* (see Figure 9.18) exhibits the output cells, or result cells, for each scenario as well as the current values from the initial tables. Now, we can easily observe the NPV of the Best Case, Worst Case, and Most Likely scenarios by looking at the last row of the *Scenario Report*. This is especially helpful if output and input cells are spread out on the spreadsheet, since results are grouped together in the report.

	A	B	C	D	E	F	G	H
1								
2		**Scenario Summary**						
3				Current Values:	Best Case	Worst Case	Most Likely	
5		Changing Cells:						
6			Year1sales	20000	20000	5000	10000	
7			Sales_growth	0.2	0.2	0.02	0.1	
8			Year1price	$10.00	$10.00	$5.00	$7.50	
9		Result Cells:						
10			AfterTaxYear1	$48,000.00	$48,000.00	-$3,000.00	$9,000.00	
11			AfterTaxYear2	$57,600.00	$57,600.00	-$3,519.00	$9,405.00	
12			AfterTaxYear3	$69,016.32	$69,016.32	-$4,090.33	$9,741.10	
13			AfterTaxYear4	$82,560.80	$82,560.80	-$4,718.50	$9,980.12	
14			AfterTaxYear5	$98,588.50	$98,588.50	-$5,408.35	$10,087.17	
15			NPV	$226,892.67	$226,892.67	-$13,345.75	$32,063.83	

Figure 9.18 The Scenario Report shows the outputs for the various input values in each scenario.

Note that we have named our input and output cells in order to see the names shown in the first column of the *Scenario Report.* If you do not name your cells, the cell reference will be shown instead (i.e. *C23* instead of "NPV"). Also note that some icons are provided next to the row numbers and column letters to hide some parts of the report and to show others.

9.2.3 Generating Random Numbers within Distributions

Despite its benefits, creating many scenarios of possible input values with *Data Tables* or the *Scenario Manager* can be tedious. We will therefore explain how to use some important Excel functions to generate various input data values for several scenarios, or *runs* of a simulation.

Simulation is useful because it can handle variability of parameters in a model. That is, you may not know many settings in a process with full certainty; you may be aware of a range of numbers into which values fall, but not know the exact figures. For example, assume that there is a shipment company that is dependent upon a variety of suppliers before its workers can organize and ship their products. The company's staff knows which suppliers will be delivering goods within a few days, but they do not know the exact day or time when each delivery will arrive. This variability affects their business, since they cannot satisfy their customers' demands until they receive products from their suppliers. To put this situation into a model, we would not be able to assign a constant rate to the arrival of goods from suppliers. Therefore, we would want to study the arrival rates over a time and match the distribution to the arrival rate that closely matches the suppliers' patterns. Once we know the distribution of a certain parameter, we can generate random numbers within this distribution to observe the effect on satisfying customer demand. Therefore, it is now appropriate to introduce the concepts of *distributions* and *random numbers*.

In Chapter 4, we mentioned that the RAND and RANDBETWEEN functions are used to generate random numbers. The **RAND** function does not have any parameters; it returns a randomly chosen fractional number between 0 and 1. The RAND formula is:

   =RAND()

RAND is useful for generating test data or any data with random patterns. You can manipulate this RAND value if you want to generate values outside the interval between 0 and 1. If you want to generate numbers between 1 and *n*, multiply this RAND value by *(n-1)* and add 1.

   =RAND()*(n-1) + 1

To generate a random number between some lower and upper bound, replace 1 with the lower bound and n with the upper bound in the above formula. We demonstrate this particular use of RAND in in Figure 9.19 with *n* = 10. Here we want to generate heights, widths, and depths to calculate some probable packaging volumes. We type the following function to create random numbers between 1 and 10:

   =RAND()*9 + 1

C6	▼		=	=RAND()*9+1		
	A	B	C	D	E	
1						
2						
3			Packaging Volumes			
4			Heights	Widths	Depths	
5			1.72	4.97	2.24	
6			9.53	7.76	8.06	
7			3.88	1.20	4.29	
8			3.76	9.25	5.08	
9			4.17	6.82	9.81	
10			7.38	3.54	6.50	
11			9.66	4.67	9.46	
12			5.09	5.47	9.37	
13			4.59	1.94	3.57	
14			5.14	6.77	7.44	
15						

Figure 9.19 The RAND function can be manipulated to change the range of the random numbers produced. Here the range is changed from the default of 0 to 1, to 1 to 10.

We can convert these random numbers into random integers by changing the format of the cells in which we entered this function. Simply use the *Number* tab of the *Formatting* dialog box and set the number of decimal places to 0.

We can also generate a random integer by using the **INT** function with the RAND function. The INT function rounds a number down to the nearest integer.

   =INT(number)
   =INT(range_name)
   =INT(cell_referenced)

To generate random integers, we can apply the INT function to the RAND function by typing:

   =INT(RAND())

      =INT(RAND()*(n-1) + 1)

For example, the following formula would generate the numbers 1, 2, 3, 4 (random integers between 1 and 5):

      =INT(RAND()*4 + 1)

Note that the integer 5 would never be generated using this formula since the INT function always rounds down to the nearest integer. That is, the random number 1.6 would be transformed to the integer 1. You could also use the **TRUNC** function to create an integer by removing the decimal part of a number without rounding. The TRUNC function is used similarly to the INT function.

An easier way to manipulate the RAND function is to use another function called **RANDBETWEEN**. This function takes two parameters, which are the lower and upper limits of the range. The format of this function is the following:

      =RANDBETWEEN(lower_limit, upper_limit)

Note: to use the RANDBETWEEN function, you will need to ensure that the *Analysis Toolpack* has been selected as an *Add-In*. Go to *Tools > Add-Ins* to check.

Using the same example, if we wanted to create random numbers between 1 and 10, we would type the following:

      =RANDBETWEEN(1,10)

Note: When using either RAND or RANDBETWEEN, the numbers generated will change with every operation performed in the worksheet. That is, after creating the table in Figure 9.19, if we were to then type new text or data into another cell on the same worksheet, the initial random numbers shown would automatically recalculate new random numbers in our range. To prevent this automatic recalculation from occurring, go to *Tools > Options* from the menu and, on the *Calculation* tab, check *Manual*. We can now use *F9* to recalculate the random numbers if necessary. (Note, however, that some other functions may not recalculate automatically when copied after changing the *Calculation* option to *Manual*.)

The RAND and RANDBETWEEN functions can also be used with the distribution functions discussed in Chapter 7. The general distribution function has the following format:

      =DIST(x_value, distribution_parameters, cumulative_value)

For these parameters, the *x-value* is the number for the distribution. The *cumulative value* is *TRUE* if we use the cumulative distribution function (cdf), or *FALSE* if we use the probability mass function (pmf). However, to generate a random number within a given distribution, we must use the inverse functions of these distribution functions. These inverse functions return the inverse of the cumulative probability functions. That is, if probability = DIST(x,...), then DISTINV(probability,...) = x.

Some of the inverse functions of the more common distributions are **BETAINV**, **BINOMINV**, **LN** or **LOGINV** (the *Exponential* inverse), and **NORMINV**. We can also find these functions in the *Statistical* category of the list of functions when we choose *Insert > Function* from the menu. The format of these inverse functions is:

> =DISTINV(probability, distribution_parameters)

The *probability* parameter is a number between 0 and 1 associated with the given distribution. We will use the RAND function as the value for this parameter to generate a number between 0 and 1. For example, to generate random numbers from the*Normal* distribution, we would follow the next format:

> =NORMINV(RAND(), mean, std dev)

In Figure 9.20, we have entered this function in a column of cells for a *Normal* distribution with a mean of 50 and a standard deviation of 15. The function is entered as the following:

> =NORMINV(RAND(), 50, 15)

Note that most of the numbers generated are in the range between 35 (that is, 50 – 15) and 65 (that is, 50 + 15), with the majority being closer to 50.

Figure 9.20 To generate a random number within a distribution, we use the RAND function as the first parameter of the inverse distribution functions.

<table>
<tr>
<td rowspan="4" align="center">

Summary</td>
<td>**Data Tables:** Used to determine how outputs vary in response to changes in input.</td>
</tr>
<tr>
<td>**Scenario Analysis:** Performs all possible alternative actions and notes the varying results from these different situations.</td>
</tr>
<tr>
<td>**Random Number Generation in Distributions:** Use the **RAND** function as the first parameter in the inverse distribution functions.</td>
</tr>
<tr>
<td>**Inverse Functions:** **NORMINV, LN** or **LOGINV** (the *Exponential* inverse)**, BETAINV, BINOMINV, CHIINV, FINV,** and **GAMMAINV.**</td>
</tr>
</table>

9.3   Applications

We will now explain three common applications of simulation in Excel. To illustrate these examples, we will use the ideas of scenario analysis and random number generations in distributions. Please note that there are many other areas and models for which simulation is a beneficial tool besides the following.

9.3.1 News Vendor Problem

A bookstore must determine how many 2006 comic calendars to order in September of 2005. It costs $2.30 to order each calendar, and the store sells each one for $4.70. After January 1, 2006, any unsold calendars can be returned to the supplier for a salvage value of $0.75 each. Our best guess is that the number of calendars demanded is governed by the following probabilities:

Demand	Probability
150	0.3
200	0.3
250	0.4

So, how many calendars should the company order? Let's set up our spreadsheet by listing the inputs and outputs. For inputs, you know the ordering cost, sales price, and salvage value. You are also provided with a table of possible demands with their corresponding probabilities (see Figure 9.21).

	C14	▼	f_x =IF(RandProb<=CumProb1, Demand1, IF(AND(RandProb>CumProb1,RandProb<=CumProb2),

		A	Demand2,IF(AND(RandProb>CumProb2,RandProb<=CumProb3),Demand3,)))

	A					
1						
2	**News Vendor Problem**					
3						
4	*Inputs*		Ordering Cost	$2.30		
5			Sales Price	$4.70		
6			Salvage Value	$0.75		
7						
8			**Demand**	**Probability**	**Cumulative**	**Random Probability**
9			150	0.30	0.30	0.74
10			200	0.30	0.60	
11			250	0.40	1.00	
12						
13			**Number Ordered**	203.82		
14			**Expected Number Sold**	250.00		
15			**Number Returned**	0.00		
16						

Figure 9.21   Preparing the spreadsheet using formulas to determine unknown values.

The expected number of magazines sold will be based on these demands and probabilities. First, we compute the cumulative probabilities of each demand (since demands are increasing). We then create a cell that will generate a random probability; we do this using the RAND function. To determine the expected number of magazines sold, we use the IF function to compare this random probability to the ranges of cumulative probabilities. We have named the cell ranges to make our formulas easier to read (see Figure 9.21).

We will then use the NORMINV function, combined with the RAND function, to determine how many calendars to order (see Figure 9.22). We set the mean to 200 (since this is the average demand) and the standard deviation to 15 (even though the standard deviation of the demand value is 50, we want to keep the order number closer to the mean).

Next, we list the outputs; these are costs, sales revenue, salvage revenue, and profit. The costs are equal to the number ordered times the ordering cost. The sales revenue is the expected number sold times the sales price. The salvage revenue is the number returned times the salvage value. The number returned is considered to be the difference between the number ordered and the expected number sold; however, if the number sold is greater than the number ordered, this value is 0. The profit is then the sales revenue plus the salvage revenue minus the costs (see Figure 9.22).

	C13	▼	*fx* =NORMINV(RAND(), 200, 15)		
	A	B	C	D	E
1					
2	**News Vendor Problem**				
3					
4	*Inputs*	Ordering Cost	$2.30		
5		Sales Price	$4.70		
6		Salvage Value	$0.75		
7					
8		**Demand**	**Probability**	**Cumulative**	**Random Probability**
9		150	0.30	0.30	0.74
10		200	0.30	0.60	
11		250	0.40	1.00	
12					
13		**Number Ordered**	203.82		
14		**Expected Number Sold**	250.00		
15		**Number Returned**	0.00		
16					
17					
18	*Outputs*	**Costs**	$468.79		
19		**Sales Revenue**	$1,175.00		
20		**Salvage Revenue**	$0.00		
21		**Profit**	$706.21		
22					

Figure 9.22 Using the NORMINV function with the RAND function to generate a random number from the Normal distribution.

To determine how many calendars to order, we press *F9* to generate new random numbers and observe the profit. In order to determine the corresponding number of calendars to order on average, we refer to the average profit. Pressing *F9* multiple times and observing various profit and order values helps us to answer the question, but it will be tedious to keep track of the average values. A better method is to simulate several runs of this data for various random values of expected number of magazines sold and number of magazines ordered. To do this, we created the table shown in Figure 9.23. Notice that we have named several cells to make the formula clearer.

| D25 | ▼ | *fx* =IF(C25<=CumProb1, Demand1, IF(AND(C25>CumProb1,C25<=CumProb2),Demand2,IF(AND(C25>CumProb2,C25<=CumProb3),Demand3,))) |

	A	B	C	D	E	F	G	H	I
23									
24	*Runs*	Number Ordered	Rand Prob for Demand	Expected Number Sold	Number Returned	Costs	Sales Revenue	Salvage Revenue	Profit
25	1	221.21	0.19	150.00	71.21	$508.78	$705.00	$53.41	$249.63
26	2	188.24	0.54	200.00	0.00	$432.95	$940.00	$0.00	$507.05
27	3	208.02	0.57	200.00	8.02	$478.46	$940.00	$6.02	$467.56
28	4	211.85	0.26	150.00	61.85	$487.26	$705.00	$46.39	$264.13
29	5	200.75	0.55	200.00	0.75	$461.72	$940.00	$0.56	$478.84
30	6	175.77	0.25	150.00	25.77	$404.26	$705.00	$19.32	$320.06
31	7	203.61	0.41	200.00	3.61	$468.30	$940.00	$2.71	$474.41
32	8	215.41	0.34	200.00	15.41	$495.44	$940.00	$11.56	$456.11
33	9	179.96	0.46	200.00	0.00	$413.90	$940.00	$0.00	$526.10
34	10	195.31	0.95	250.00	0.00	$449.22	$1,175.00	$0.00	$725.78
35	11	226.09	0.54	200.00	26.09	$520.00	$940.00	$19.57	$439.57
36	12	199.43	0.91	250.00	0.00	$458.68	$1,175.00	$0.00	$716.32
37	13	189.31	0.98	250.00	0.00	$435.42	$1,175.00	$0.00	$739.58
38	14	194.98	0.59	200.00	0.00	$448.46	$940.00	$0.00	$491.54
39	15	237.64	0.84	250.00	0.00	$546.58	$1,175.00	$0.00	$628.42
40	16	215.72	0.54	200.00	15.72	$496.15	$940.00	$11.79	$455.64
41	17	210.11	0.62	250.00	0.00	$483.26	$1,175.00	$0.00	$691.74
42	18	169.54	0.11	150.00	19.54	$389.94	$705.00	$14.65	$329.71
43	19	214.80	0.80	250.00	0.00	$494.04	$1,175.00	$0.00	$680.96
44	20	188.79	0.48	200.00	0.00	$434.21	$940.00	$0.00	$505.79
45									
46	Averages	202.33	0.55	205.00	12.40	$465.35	$963.50	$9.30	$507.45

Figure 9.23 Making runs to compare profit values for several random input values.

This table displays 20 runs, or 20 rows of various input generated from changing random values. From the values stored in each run, we can easily find the average profit and average number of calendars ordered.

9.3.2 Game of Craps

In the game of Craps, a player rolls two dice. If the first roll yields a sum of 2, 3, or 12, the player loses. If the first roll yields a sum of 7 or 11, the player wins. Otherwise, the player continues rolling the dice until she matches the value thrown on the first roll or rolls a sum of 7. Rolling a match for the first roll wins the game, rolling a value of 7 before a match loses the game. How many times will a player win on average in 1, 2, 3, 4, or 5 rolls?

To answer this question, we begin by listing the known inputs in a spreadsheet (see Figure 9.24). We already understand the conditions for winning and losing on one roll and on multiple rolls. Knowing that the player rolls two dice, we can calculate the probability of winning or losing on the first roll. However, if the player makes multiple rolls in the game, we do not know exactly how many rolls will be needed.

	A	B	C	D	E	F
1						
2	**Game of Craps**					
3						
4	*Input*			**One Roll**		
5		Losing	Probability		Winning	Probability
6		2	1/36		7	1/6
7		3	1/18		11	1/18
8		12	1/36			
9						
10				**Multiple Rolls**		
11		First	Probability		Second	Probability
12		4	1/12		2	1/36
13		5	1/9		3	1/18
14		6	5/36		4	1/12
15		8	5/36		5	1/19
16		9	1/9		6	5/36
17		10	1/12		7	1/6
18					8	5/36
19					9	1/9
20					10	1/12
21					11	1/18
22					12	1/36
23						

Figure 9.24 The conditions for winning and losing on one or multiple rolls.

Let's determine possible outcomes for 1, 2, 3, 4, and 5 rolls in a game. To do so, we will generate random results for rolling two dice for each roll for 20 games, or runs. To generate the result of a roll, we simply use the RANDBETWEEN function with the interval (2, 12). Then, we use the IF function to check the conditions for winning and losing and output the player's status after the roll. For the first roll, we can use the following IF function (these cell references are for the first row in the simulation table):

=IF(OR(C26=2,C26=3,C26=12),"Lose",IF(OR(C26=7,C26=11),"Win",
    "Continue Play"))

The IF function, along with the OR function, checks if any of the losing conditions or any of the winning conditions are met for the first roll. If the player neither wins nor loses, then she continues to play (see Figure 9.25). We should also use Conditional Formatting to identify cells with "Win" values, since this will helpanswer the question stated in the problem description.

	D26	▼		*fx*	=IF(OR(C26=2,C26=3,C26=12),"Lose",IF(OR(C26=7,C26=11),"Win", "Continue Play"))				
	A	B	C	D	N	O	P	Q	R
24									
25	*Output*	Game	Roll Value	**First Roll** **Win or Lose**					
26		1	4	Continue Play					
27		2	3	Lose					
28		3	2	Lose					
29		4	5	Continue Play					
30		5	7	Win					
31		6	10	Continue Play					
32		7	5	Continue Play					
33		8	8	Continue Play					
34		9	10	Continue Play					
35		10	10	Continue Play					
36		11	4	Continue Play					
37		12	12	Lose					
38		13	2	Lose					
39		14	7	Win					
40		15	3	Lose					
41		16	4	Continue Play					
42		17	11	Win					
43		18	8	Continue Play					
44		19	12	Lose					
45		20	5	Continue Play					
46									

Figure 9.25 Recording the outcome of rolling two dice for 20 different games.

For all sequential rolls, we again use the IF function, but this time, we are checking the current roll with the first roll. Therefore, we will need to create a new column for the second roll values (again using the RANDBETWEEN function) and compare these results with the results of the first roll value (see Figure 9.26). The comparative function with new winning and losing conditions can then be written as follows (these cell references are for the first row in the simulation table):

=IF(D26="Continue Play",IF(E26=$C26,"Win",IF(E26=7,"Lose",
    "Continue Play")),"")

Now, we can create two more columns (one for the roll value and one for the comparative function) for the third, fourth, and fifth rolls (see Figure 9.26). Notice that we have used column absolute referencing to refer to the result of the first roll; this way we can copy and paste the two columns for each roll. We have also used Conditional Formatting on the sequential columns with comparative functions to identify the "Win" values more easily.

F26   =IF(D26="Continue Play",IF(E26=$C26,"Win",IF(E26=7,"Lose","Continue Play")),"")

	Game	Roll Value	First Roll Win or Lose	Second Roll Value	Second Roll Win or Lose	Third Roll Value	Third Roll Win or Lose	Forth Roll Value	Forth Roll Win or Lose	Fifth Roll Value	Fifth Roll Win or Lose
Output	1	4	Continue Play	6	Continue Play	7	Lose	5		8	
	2	3	Lose	2		10		3		4	
	3	2	Lose	6		10		8		9	
	4	5	Continue Play	9	Continue Play	5	Win	9		12	
	5	7	Win	5		11		3		10	
	6	10	Continue Play	7	Lose	3		3		6	
	7	5	Continue Play	12	Continue Play	4	Continue Play	4	Continue Play	5	Win
	8	8	Continue Play	6	Continue Play	10	Continue Play	8	Win	12	
	9	10	Continue Play	10	Win	3		12		3	
	10	10	Continue Play	3	Continue Play	12	Continue Play	4	Continue Play	6	Continue Play
	11	4	Continue Play	4	Win	6		5		10	
	12	12	Lose	2		9		9		10	
	13	2	Lose	10		11		7		5	
	14	7	Win	9		3		12		6	
	15	3	Lose	3		4		7		2	
	16	4	Continue Play	10	Continue Play	7	Lose	5		4	
	17	11	Win	4		11		8		6	
	18	8	Continue Play	4	Continue Play	5	Continue Play	9	Continue Play	6	Continue Play
	19	12	Lose	5		3		4		11	
	20	5	Continue Play	5	Win	5		7		2	

	1 Roll	2 Rolls	3 Rolls	4 Rolls	5 Rolls
Number of Wins	3	3	1	1	1
Percentage of Games Won	0.15	0.15	0.05	0.05	0.05

Figure 9.26 Recording the number of wins when there are 1, 2, 3, 4, or 5 rolls in a game.

Now, we create a summary table below the simulation table. We want to count the number of wins for each roll among 20 games, or runs. To perform this conditional counting, we use the **COUNTIF** function. We will discuss this function in more detail in Chapter 10 with Database functions. The structure for this function is as follows:

=COUNTIF(table_range, condition)

For counting the number of wins in the first roll, we use this function:

=COUNTIF(D26:D45, "Win")

We copy this formula to calculate the number of wins in each roll for all 20 games. What we have done is simply taken each of these resulting numbers and divided them by 20 to find the percentage of games won for each roll. (We can also press *F9* to generate new rolls and simulate another 20 games, or runs, of data.)

9.3.3 Bidding

A contractor is planning to make a bid on a construction project. She believes that it will cost $12,000 to complete the project. Three competitors are going to bid against her. Based on past history, she believes that each competitor's bid is equally likely to be any value between her cost and triple her cost of completing the project. She also believes

that each competitor's bid is independent of the other bids. Which bid will maximize her expected profit?

We begin by listing the known inputs. We know that the cost estimate is $12,000. Suppose we also know that the competitors will bid somewhere between this cost and three times the cost. To estimate these values, we use the RAND function and create the interval (cost, 3*cost). As described in the previous section, we can use these lower and upper bound values to generate random numbers as follows:

=(3*Cost - Cost)*RAND() + Cost

Note that we could have also used the RANDBETWEEN function with this interval; however, we want non-integer values. Once we have generated possible bids for the competitors, we must determine possible bids for the contractor. She assumes that she will bid anywhere between $12,000 and $20,000 for the job. In this case, we have used the RANDBETWEEN function since it is just an estimate.

Now we can determine the output, which is the profit, by comparing the contractor's bid to the competitors' bids. If the contractor's bid is the minimum value, then her profit is her bid minus her cost; however, if her bid is not the minimum, then she will lose the bid and not gain any profit (see Figure 9.27).

	C12	▼	*fx* =IF(Bid <= MIN(C8:C10), (Bid-Cost), 0)	
	A	B	C	D
1				
2	**Bidding**			
3				
4	*Inputs*	Cost	$12,000.00	
5		Bid	$14,453.00	
6				
7		Competitor	Bid	
8		1	$28,781.93	
9		2	$34,459.32	
10		3	$30,092.22	
11				
12	*Output*	Profit	$2,453.00	
13				

Figure 9.27   Calculating the inputs and output using several formulas.

If we limit these formulas to determine which bid will generate the maximum profit, we will have to generate several random numbers (by pressing *F9*) and keep track of the profit and bid values for comparison. Several simulation runs would accomplish this in a more efficient manner. In Figure 9.28, we have repeated the above formulas for 20 runs of the simulation. We then have several instances of the contractor's bid and the competitors' bids, and for each instance we can calculate the profit.

	F17	▼	f_x =IF(B17 <= MIN(C17:E17), (B17-Cost), 0)					
	A	B	C	D	E	F	G	H
15								
16	*Runs*	**Bid**	**Competitor 1 Bid**	**Competitor 2 Bid**	**Competitor 3 Bid**	**Profit**		**Bid**
17	1	$15,690.00	$14,993.28	$14,050.81	$22,029.33	$0.00		$15,690.00
18	2	$13,662.00	$17,395.10	$34,302.13	$22,282.02	$1,662.00		$13,662.00
19	3	$17,280.00	$22,006.05	$30,509.48	$22,700.79	$5,280.00		$17,280.00
20	4	$11,584.00	$18,369.15	$24,449.36	$26,229.12	-$416.00		$11,584.00
21	5	$18,975.00	$26,060.56	$33,813.77	$26,125.09	$6,975.00		$18,975.00
22	6	$12,847.00	$19,597.15	$12,669.49	$16,131.09	$0.00		$12,847.00
23	7	$17,207.00	$32,864.08	$35,163.63	$35,085.23	$5,207.00		$17,207.00
24	8	$18,934.00	$19,302.54	$23,290.85	$24,499.14	$6,934.00		$18,934.00
25	9	$19,677.00	$17,097.63	$16,938.32	$12,330.32	$0.00		$19,677.00
26	10	$14,426.00	$21,881.31	$29,408.37	$32,708.86	$2,426.00		$14,426.00
27	11	$15,035.00	$12,743.63	$24,231.15	$19,210.92	$0.00		$15,035.00
28	12	$19,826.00	$18,405.85	$24,876.44	$20,627.62	$0.00		$19,826.00
29	13	$17,342.00	$20,054.33	$14,642.96	$29,610.41	$0.00		$17,342.00
30	14	$15,828.00	$27,079.44	$32,249.80	$26,351.58	$3,828.00		$15,828.00
31	15	$19,995.00	$14,972.32	$24,257.29	$35,931.19	$0.00		$19,995.00
32	16	$11,464.00	$24,788.02	$16,230.10	$16,856.95	-$536.00		$11,464.00
33	17	$17,425.00	$30,644.89	$32,527.09	$13,592.90	$0.00		$17,425.00
34	18	$11,362.00	$14,421.39	$13,547.21	$20,214.11	-$638.00		$11,362.00
35	19	$15,451.00	$21,232.78	$20,191.66	$19,355.12	$3,451.00		$15,451.00
36	20	$19,908.00	$31,998.96	$34,837.88	$27,856.80	$7,908.00		$19,908.00
37								
38			**Max Profit**	$7,908.00				
39			**Bid for Max Profit**	$19,908.00				

Figure 9.28 Generating simulation runs to find the max profit.

When the 20 runs are complete, we can determine the max profit value by using the MAX function and the corresponding bid value. To find this bid value, we use the VLOOKUP function; however, since we are looking for the bid value based on the location of the max profit value, we must have the profit column in the first column of the lookup table. Therefore, we simply copy the bid column to the right of the profits and only select the data in columns F, G, and H as the lookup table. We can repeat this 20-run simulation by generating a new set of random numbers (by pressing *F9*) to compare the max profit and bid values.

9.4 <u>Summary</u>

➢ Simulation is a modeling tool used for analyzing a process running under different parameters.
➢ Simulation is used not to find the best result given certain inputs, but rather to find the best inputs given a desired result.
➢ Scenario Analysis performs all possible alternative actions and notes the varying results from these different situations.
➢ To perform random number generation in distributions, use the RAND() function as the first parameter in the inverse distribution functions.
➢ Inverse Distribution functions are the following: NORMINV, LN (the Exponential inverse), BETAINV, BINOMINV.
➢ Applications of simulation include the News Vendor Problem, Cash Flows, the Game of Craps, and Bidding.

9.5 Exercises

9.5.1 Review Questions

1. What is a simulation?
2. How do simulation models differ from optimization models?
3. How do you activate the *Scenarios* function?
4. How is the *Scenarios* function a useful tool?
5. Name four of the distribution functions under the *Statistical* category of functions in Excel.
6. Name four of the inverse distribution functions under the *Statistical* category of functions in Excel.
7. How can you generate a random number within a distribution?
8. Which inverse distribution function is appropriate to use for generating a random number within an Exponential distribution?
9. If given a gamma distribution with an alpha value of 5 and a beta value of 8, what is the appropriate functional expression to generate a random number?
10. Give an example of an application of Excel simulation.

9.5.2 Hands-On Exercises

1. A student needs a grade average of at least 93.0 points to receive an A in one of his classes. The student's grade average is computed by averaging the student's score on five different tests given in the class. Each test is scored out of 100 points, and the highest grade the student can receive is 100 points since no extra credit is offered. The student received the following grades on the first three tests: 86, 94, and 88. He is trying to determine if it is possible for him to earn an A in the class and if so, what grades he must earn on the final two tests in order to earn an A. Perform an Excel scenario analysis to determine if the student can possibly receive an A in the class, and, if he can, determine what combinations of grades on the final two tests will enable him to do so.

2. Generate five random numbers for each of the following distributions:
   a. Exponential distribution with a mean of 12 and a standard deviation of 2.
   b. Beta distribution with an alpha of 4 and a beta of 3.
   c. Chi-squared distribution with 6 degrees of freedom.
   d. Standard normal cumulative distribution.

3. In chapter 4, problem 3, you completed the following table, which determines the height of a cylinder given its volume and radius. Use scenario analysis to complete the table again for each of the following cylinder volumes: 10, 15, 20, 25, and 30 ft^3. Recall that the height of a cylinder can be computed with the following formula: Height = Volume/(Radius^2).

Volume (ft^3):	10
Radius (ft)	**Height (ft)**
5	
2	
4	
3	

4. A bakery receives its ingredients from one of three vendors. Each vendor under consideration charges different prices for each different ingredient. The tax rates vary by the state in which the vendors are located, and the shipping costs vary by the distances the supply must travel. The table below displays the prices, tax rates, and shipping costs per pound for each vendor. The management has to choose a vendor for each ingredient. It has been estimated that 200 lbs of ingredients will be needed per month. Use scenario analysis to help the management to choose a vendor for each ingredient.

	Vendor A	Vendor B	Vendor C
Price of White Flour (per lb)	$ 1.15	1.2	1.1
Price of Wheat Flour (per lb)	$ 1.35	1.25	1.23
Price of Sugar (per lb)	$ 0.89	0.95	1.05
Tax Rate	5.0%	8.0%	6.5%
Shipping Costs (per lb)	$ 0.16	0.14	0.18

5. A materials engineer is conducting a test on the fracture toughness of an expensive ceramic material. The engineer wants to report on the results of fifty trials but can only afford to actually conduct ten trials since testing the material renders it unusable. Her results so far have followed a chi-squared distribution with 3.5 degrees of freedom. Using random number generation, predict forty more possible trial results for the engineer to use.

6. A company is opening up a new distribution center in Gainesville, Florida. Five different banks have offered the company loan plans; each plan's cash flow is provided below. Initially assume that the interest rate is 5%.
a. Create a spreadsheet that finds the net present value and the internal rate of return of each loan.
b. Run several scenarios that change the interest rate to 3%, 3.5%, 4%, and 4.5%.
c. View the scenario summary. Which interest rate would the company prefer? Which bank can provide the most economical loan plan for the company?

Year	Bank A	Bank B	Bank C	Bank D	Bank E
0	$1,000,000	$1,000,000	$1,000,000	$1,000,000	$1,000,000
1	-$100,000	0	-$150,000	0	-$500,000
2	-$200,000	-$300,000	-$150,000	0	-$100,000
3	-$300,000	-$300,000	-$150,000	0	-$100,000
4	-$400,000	-$300,000	-$150,000	0	-$100,000
5	-$500,000	-$300,000	-$150,000	-$250,000	-$100,000
6	0	-$300,000	-$150,000	-$250,000	-$100,000
7	0	0	-$150,000	-$250,000	-$100,000
8	0	0	-$150,000	-$250,000	0
9	0	0	-$150,000	-$250,000	0
10	0	0	-$150,000	-$250,000	0

7. Suppose that you have just been hired by Sturdy Boxes, Inc. to manage the production floor. On your first day, you realize that the garbage compactor is full of cardboard boxes. After questioning the workers, you find out that those boxes were not cut and shaped correctly in the Flex-O machine. The problem is that the Flex-O machine's blades are not replaced quickly enough after it has been discovered that misshapen boxes are being produced. After studying daily production reports, you determine that the time for the first failure to occur follows a normal distribution with an average of four hours and a standard deviation of 15 minutes. If the time to process each box has a normal distribution with an average of 30 seconds and a standard deviation of three seconds, how many boxes should be processed before the blades on the Flex-O machine should be replaced? Use ten scenarios with thirty runs each and find the minimum value to approximate the number of boxes to produce before replacing the blade. Your objective is to use each blade as long as possible but change it before failures start to occur.

8. A PVC pipe manufacturer has recently approximated the effect of certain factors on the amount of time it takes to produce a certain length of PVC pipe. These factors include the pressure and temperature of the liquid PVC and the radius and thickness of the pipe. The time that it takes to produce PVC pipe is shortest when the liquid PVC pressure is at 20 psi, the liquid PVC temperature is 240 degrees Fahrenheit, the radius is at its minimum setting of one-half inch, and the thickness of the pipe is at its minimum setting of one-eighth inch. The time to produce a 12-foot pipe at these settings follows a normal distribution with a mean of 85 seconds and a standard deviation of six seconds. Below are the approximated effects of each factor on the mean time necessary to produce a 12-foot PVC pipe. Note that P is the liquid PVC pressure setting, T is the liquid PVC temperature setting, R is the radius of the pipe, and TH is the thickness of the pipe.

Liquid PVC pressure factor = 1 + (20 – P)/(20 * 10)
Liquid PVC temperature factor = 1 + (240 – T)/(240 * 10)
Radius factor = 1 + (R – 0.5)/(0.5 * 10)
Thickness factor = 1 + (TH – 0.125)/(0.125 * 10)

Use scenario analysis to find the approximate time to produce a 12-foot PVC pipe for each of the following settings:

a. P = 21 psi, T = 230° F, R = 1", TH = 0.25
b. P = 18 psi, T = 250° F, R = 2", TH = 0.35
c. P = 24 psi, T = 235° F, R = 5", TH = 0.5
d. P = 17 psi, T = 245° F, R = 3", TH = 0.25

9. A wholesaler receives hundreds of products from several manufacturers and distributes the products among the company's retail stores. Most of the products have a fairly constant supply and demand, but the distribution manager has realized that the demand rate and supply time of several of the products fluctuate significantly. At times, the demand is so high that the wholesaler cannot provide the supply in time to meet the demand. At other times, the supply arrives well before the demand and sits in inventory until it is distributed. The distribution

manager would like to order all of the products at an appropriate time so that they will arrive in time to meet demand without accumulating too much. After studying past trends, the distribution manager approximated the distributions of the demand rate and supply time for one of the products as shown in the following table. Assume that the product is perishable and only good for the month in which it is ordered. Also assume that the product is ordered in batches of 100. The supply time follows a normal distribution with the following parameters:

Mean = ceiling[20 * (1 + (D − 200) / 200)], where D is the amount of demand
Standard deviation = 3

Month	Demand
January	~Norm (20000, 3000)
February	~Norm (17500, 4500)
March	~Norm (13000, 1900)
April	~Norm (9500, 1100)
May	~Norm (7800, 900)
June	~Norm (7000, 600)
July	~Norm (7400, 700)
August	~Norm (8500, 950)
September	~Norm (9000, 1250)
October	~Norm (10000, 2500)
November	~Norm (15000, 4000)
December	~Norm (25000, 1500)

a. Use ten runs to find the approximate number of batches to order to meet the demand and the approximate number of days to allow for delivery so that the delivery will arrive in time for the next month.
b. Perform scenario analysis to find the results for each of the following distributions if the time of supply arrival has a Normal distribution with the following parameters:

Mean = ceiling[A * (1 + (D − B) / B)], where A is the value of the mean for an order size of B batches

Standard deviation = S

i. A = 15, B = 160, S = 5
ii. A = 25, B = 220, S = 2
iii. A = 12, B = 210, S = 4

10. A local company currently sells 40,000 units of a product for $45 each. The unit variable cost of production for the product is $5. The company is thinking about cutting the product price by 30%. They are sure this will increase sales by an amount between 10% and 50%. Perform a sensitivity analysis to show how profit will change as a function of the percentage increase in sales (ignore fixed costs):

a. Using Data Tables
b. Using the Scenario Manager

11. A GMC dealer believes that demand for 2005 trucks will be normally distributed with a mean of 200 and a standard deviation of 30. His cost of receiving a truck is $25,000, and he sells each truck for $40,000. Half of all leftover trucks can be sold for $30,000. He is considering ordering 200, 220, 240, 260, 280, or 300 trucks. How many should he order?

12. The game of keno is played as follows: 80 balls (numbered 1 to 80) are mixed up and 20 balls are randomly drawn. Before the 20 balls are drawn, the player chooses 10 different numbers. If at least 5 of the numbers are drawn, the player wins. What is the probability that the player will win?

13. Suppose you are bidding for an oil well that you believe will yield $40 million (including the cost of developing and mining the oil) in profits. Three competitors are bidding against you and each competitor's bid is assumed to follow a normal random variable with a mean of $30 million and a standard deviation of $4 million. What should you bid (within $1 million)?

14. Two basketball teams are ready for the best-of-seven NBA finals. The two teams are evenly matched, but the home team wins 60% of the games between the two teams. The sequence of home and away games is to be chosen by Team A. Team A has the home edge and will be the home team for four of the seven games. They have the following sequence choices for the home games:

    a. Sequence 1: Team A, A, B, B, A, B, A
    b. Sequence 2: Team A, A, B, B, B, A, A

    Use simulation to prove that each sequence gives Team A the same chance of winning.

15. The game of Chuck-a-Luck is played as follows: You pick a number between 1 and 6 and toss three dice. If your number does not appear on any of the dice, you lose $1. If your number appears x times you win $x. On the average, how much money will you win or lose on each play of the game?

16. In August 2004, a car dealer is trying to determine how many 2005 cars should be ordered. Each car ordered in August 2004 costs $10,000. The demand for the dealer's 2005 models has the probability distribution shown in the table below:

Cars Demanded	Probability
20	.30
25	.20
30	.25
40	.25

Each car sells for $15,000. If the demand for 2005 cars exceeds the number of cars ordered in August 2004, the dealer must reorder at a cost of $12,000 per car. Excess cars may be disposed of at $9,000 per car. Use simulation to

determine how many cars should be ordered in August 2004. For your optimal order quantity, find the expected profit.

17. In the above problem, suppose now that the demand for cars is normally distributed with a mean of 40 and a standard deviation of 7. Use simulation to determine an optimal order quantity. For this optimal order quantity, determine the expected profit.

18. A ticket from Miami to Orlando sells for $150. The plain can hold 100 people. It costs $8,000 to fly an empty plain. The airline incurs variable costs of $30 per person on the plane. If the flight is overbooked, anyone who cannot get a seat receives $300 in compensation. On the average, 95% of all people who have a reservation show up for the flight. To maximize the expected profit, how many reservations for the flight should be taken?

19. An emergency room at a nearby hospital has an average of 2 patients arriving every hour (exponentially distributed). Upon entering, each patient fills out a form; this always takes 5 minutes. Then, each patient is processed by one of two registration clerks. This takes an average of 7 minutes (exponentially distributed). Then each patient walks 2 minutes to a waiting room and waits to meet the doctor. The time it takes the doctor to see a patient averages 20 minutes with a standard deviation of 10 minutes (normally distributed).

    a. On average, how long does a patient spend in the emergency room?
    b. On average, how much of this time is spent waiting for a doctor?
    c. What percentage of the time is the doctor busy?

20. A person plays a game in which 2 fair dices are thrown. If the total outcome is 7 or 11, the person wins the game. Use simulation to find the probability of a win.

21. Jane is buying a house that costs $210,000. A financial institution is willing to lend her the money. The bank provides different rates depending on the amount of down payment and the duration of the loan. The table below presents the interest rate for different loan durations and amount of down payment. Use two-way Data Tables to calculate the monthly payments that Jane will be making to pay-off the loan under each scenario.

Down Payment	Loan Duration (in years)				
	10	15	20	25	30
0%	5.50%	5.75%	5.80%	5.85%	5.90%
5%	5.45%	5.70%	5.75%	5.80%	5.85%
10%	5.40%	5.65%	5.70%	5.75%	5.80%
15%	5.35%	5.60%	5.65%	5.70%	5.75%
20%	5.30%	5.55%	5.60%	5.65	5.70%

22. A drive-through only, fast food restaurant is considering hiring new employees. It has been estimate that each additional employee will reduce the service time by 10%. The following data have been collected (the times are in minutes):

Time between arrivals	Frequency
4.0	20
4.5	35
5.0	32
5.5	10
6.0	3
Total	100

Service time	Frequency
2.0	5
3.5	15
4.5	18
5.0	30
5.5	32
Total	100

The manager of the restaurant would like to perform a simulation analysis to see if hiring new employees would be beneficial.

a. Simulate the arrival and service time of 100 customers using the information provided above.
b. Estimate the expected waiting time per customer, expected total time in the system, the longest waiting time, and the percentage of time the server (drive through window) is idle.
c. Assume that one additional employee was hired. Recalculate the expected waiting time per customer, expected total time in the system, the longest waiting time, and the percentage of time the server is idle.

23. The coffee shop in a university town faces the problem of many bagels to order per day from a nearby bakery. For planning purposes, the bakery requires that a fixed amount of bagels be ordered every day in a given month. At the end of the month, the order size may be changed.

Based on past observations, the coffee shop manager has identified the following distribution of daily demand for bagels for the current month:

Sales	Frequency
20	30
25	55
40	15

It costs $0.1 to buy a bagel from the bakery, regardless of the quantity ordered. The bagels are sold for $1.05. Left over bagels are disposed at the end of the day. The manager is interested to identify an order size for the current month that would maximize the profits.

Randomly generate the demand for the 30 days of the current month using the distribution given above. Consider order sizes from 20 to 40. For each order size,

calculates the corresponding profits. Based on the results of the simulation what should the order size for the current month be?

24. The tables below present the distribution of the inter-arrival time and service time of the jobs in a service center.

Time between arrivals	Probability
30 minutes	0.20
45	0.35
60	0.25
75	0.15
90	0.05
Total	1

Service time	Probability
20 minutes	0.15
35	0.20
60	0.30
70	0.25
95	0.10
Total	1

Estimate the expected waiting time per job, expected total time in the service center, and the longest waiting time for 100 jobs under the following two different service rules:

First In First Out (*FIFO*) – jobs are served in the order they arrive.
Last In First Out (*LIFO*) – the last job to arrive