## Integrate GeoGebra in the Mathematics Teaching and Learning

Ali Mahmudi Email: ali\_uny73@yahoo.com Phone Number: +6281328728725

Department of Mathematics Education Faculty of Mathematics and Natural Science Yogyakarta State University, Indonesia

#### **Abstract**

The use of technology, especially computer software, in the mathematics teaching and learning is a necessity. This article discusses the GeoGebra software and its integration in the mathematics teaching and learning activities. Unlike most other software, GeoGebra can be downloaded free from the internet. GeoGebra can be used to demonstrate, visualize and as a tool for constructing mathematical concepts. By using GeoGebra, mathematical concepts can be visualized and manipulated quickly, accurately and efficiently. In this article also provided some examples of the use GeoGebra for teaching and learning. The integration of GeoGebra in the mathematics teaching and learning has great potential to make mathematics teaching and learning more interesting, dynamic, efficient and meaningful.

Keywords: Mathematics, Teaching & learning, GeoGebra, Integrate

### 1. Introduction

The use of technology, especially computers with a variety of relevant programs, in the mathematics teaching and learning is a necessity. The National Council of Teachers of Mathematics (NCTM) Principles and Standars for School Mathematics dedicated technology as one of their six principles for school mathematics: "Technology is essential in teaching and learning mathematics; it influence the mathematics that is taught and enhances students' learning (NCTM, 2000).

By integrating technology in the mathematics teaching and learning, teachers can provide a creative environment and opportunities to support students in building knowledge and skills. Students with a different potential or capacity level can get benefit from the use of technology in different ways. Gifted students, for example, can be supported by nurturing their interest and mathematical skills. On the other hand, lower students can be provided with activities that meet their special needs and assist them to overcome their individual difficulties. Therefore, students "may focus more intently on computer tasks" and "may benefit from the constraints imposed by a computer environment" (NCTM, 2000). Thus, the use of technology in enabling learning to meet student learning needs.

Mafi (2011) states that in today's world, use of technology are not just confined to industrialized countries. These tools are available to all communities. For this reason, aplication of technology in teaching mathematics was taken into special consideration and many advantages of it in facilitating learning and teaching mathematics.

According to Larijani, Khalifeh and Ahmad (2012), there are some advantages of using technology in mathematics teaching and learning: enhanching imagination, raising understanding and realization of students, learning in a knowledge sharing and experimental-exploratory environment, changing belief of students toward mathematics, saving time due to lack of manual calculation and facilitating teaching concepts to students.

One of the technology, especially computer software, that can be used as a media of mathematics teaching and learning is GeoGebra. This article discusses the GeoGebra and its integration in the mathematics teaching and learning activities.

#### 2. What is GeoGebra?

GeoGebra was created by Markus Hohenwarter in 2001. Unlike most other software, GeoGebra can be downloaded free from the internet: www.geogebra.org. GeoGebra works on a very broad spectrum on a computer system that has a Java program. GeoGebra complement a variety of computer programs for learning algebra already exist, such as Derive, Maple, MuPad and computer program for learning geometry, such as Geometry Sketchpad or CABRI Geometry's.

Unlike other programs that are used specifically to teach and learn the geometry or algebra separately, GeoGebra is designed to teach and learn algebra and geometry simultaneously. GeoGebra is a dynamic geometry program in which students work with points, vectors, segments, lines and conic sections. On the other hand, equations and coordinates can be entered directly. Functions can be defined algebraically and then changed dynamically afterwards.

GeoGebra is very useful for teachers and students. Not as to the use of commercial software that is usually only used in schools, GeoGebra can be installed on personal computers and used whenever and wherever by students and teachers. GeoGebra offers an effective opportunity to create creative and interactive learning environment that allows students to explore various mathematical concepts. GeoGebra offers a novel dynamically connected learning environment (Hohenwarter et al, 2008).

Various advantages of using GeoGebra in the mathematics teaching and learning described by Dikovic (2009) as follows.

- a. In comparisson to a graph calculator, GeoGebra is more user-friendly. GeoGebra offers easy-to-use interface, multilingual menus, commands and help.
- b. Geogebra encourages students' project in mathematics, multiple presentations and experimental and guided discovery learning
- c. Students can personalize their own creations through the adaptation of interface (e.g. font size, language, quality of graphics, color, coordinates, line thickness, line style and other features)
- d. GeoGebra was created to help students gain a better understanding of mathematics. Students can manipulate variables easily by simply dragging "free" objects around the plane of drawing, or by using sliders. Students can generate changes using a technique of manipulating free objects, and then they can learn how the dependent objects will be affected. In this way, students have the opportunity to solve problems by investigating mathematical relations dynamically.
- e. The algebra input allows the user to generate new objects or to modify those already existing, by the command line. The worksheet files can easily be published as web pages

Other advanteges of using GeoGebra in mathematics teaching and learning are as follows.

- a. GeoGebra can produce a sketch of geometry quickly and accurately than by using a pencil or ruler
- b. Manipulation facility (dragging) can provide a clearer visual experience to the students in understanding the concept of geometry.
- c. GeoGebra can be used to evaluate a geometrical construction.
- d. GeoGebra facilitate teacher and students to investigate or demonstrate the properties of a geometrical object.

Studies on the use of Dynamic Geometry Software (DGS) indicates that DGS, such as GeoGebra, can be effectively integrated in the learning activities in mathematics and has the potential to create learning activities based on student activities (Hohenwarter, et al, 2008). Moreover, research conducted by Larijani et al (2012) show that the average of grades of the group taught by use of GeoGebra is higher than the average of control group. So it's concluded that

using GeoGebra in the mathematics teaching and learning has positive effect. GeoGebra enables all students to solve the exercises at the same time and see the correct answer.

Another research that was done by Saha et al (2010) on "the influence of GeoGebra on mathematical understanding with emphasis on ordinate geometry". The findings of this research indicate that there was significant difference in grades of students in posttest which is oriented to GeoGebra group. The finding showed that computer aided learning are a useful supplement in traditional teaching.

Although it has many advantages, according to Dikovic (2009), there are some things to consider in using GeoGebra, as follows.

- a. Students without previous programming experience will hardly enter algebraic commands in the input box. Although the basic commands are not difficult to learn, students may feel embarrased or quite at a loss of what to do
- b. Some methodological approaches (e.g. independent exploring and experimenting) can not be appropriate for many students
- c. In a technical sense, GeoGebra does not have an in-built support for animation. So, including the modules for animating in GeoGebra should become an important technical element for future versions.
- d. Future extensions of the software GeoGebra will surely include more symbolic features of computer algebra systems which will further increase possible complex applications in the mathematical analysis and 3D extensions.
- e. Limited research on the impact of GeoGebra on teaching and learning of mathematics

Research also indicates that solely providing teachnology, including GeoGebra, to teacher in the majority of cases is insufficient for a successful integration of technology into teaching practises. Moreover, although the potential benefits of technology use for teaching and learning are well known and extensively examined, the process of integration technology into mathematics classrooms proved to be slower than initially expected (Cuban, Kilkpatrick & Peck, 2011).

For high quality professional development, "it is important to know in which way a software package can be introduced to novices most effectively (Mously, Lambdin, & Koc, 2003) in order to minimize unecessary difficulties during the introduction process for teachers and to facilitate the first contact with the new software tool as much as possible. Teachers who feel comfortable with operating a new software tool are more likely to integrate this tool into their teaching practise than teachers who experienced initial difficulties.

In line with the above description, the NCTM Principles and Standards for School Mathematics states that "effective use of technology in the mathematics classroom depends on the teacher." (NCTM, 2000). Moreover, successful integration of technology in learning is also very dependent on the availability of computers, availability of time, support from colleagues and from the school management.

# 3. Integration of GeoGebra in the Mathematics Teaching and Learning

GeoGebra can be used to assist the learning of mathematics ranging from elementary school through college. Many teachers and researchers from around the world have developed a worksheet and methods of using GeoGebra at all levels of schooling. In junior high school or college, GeoGebra is very useful to support the learning of mathematical concepts. Some examples of construction GeoGebra for learning mathematics can be seen in GeoGebra Wiki (www.geogebra.org/wiki).

There are some alternatives to use or integrate GeoGebra in the mathematics teaching and learning. According to Hohenwarter & Fuchs (2004), GeoGebra is very useful as a medium of learning mathematics in a variety of activities as follows.

- a. Tool for demonstration and visualization. In the traditional learning, teachers use GeoGebra to demonstrate and visualize certain mathematics concepts.
- b. Tool for construction. GeoGebra can be used to visualize the construction of certain mathematical concepts, such as construction of circle that through all points of triangle.
- c. Tool for discovery process. GeoGebra can used as a tool for students to construct a mathematical concept, such as the locus of points or characteristics of parabola

Here are some examples of the use or integration of GeoGebra in mathematics teaching and learning.

## **Example 1: Derivative, Roots and Extreme Points**

Figure 1 below visualize the polynomial  $f(x) = ax^3 + bx^2 + cx + d$ , the roots, extreme points or first and second derivatives. Parameter polynomial function f(x) can be modified by using the "slider" which allows students to do the following things.

- a. Investigate the effect of changing the parameters of the initial function (for example, the parameter *d* will change or move the graph of the function to the "top" and the "down" without changing its shape
- b. Explore the relationship between the initial function and its derivatives (for example, parameter d does not affect the derivation, f has an extreme points and the first derivative f' has a root.

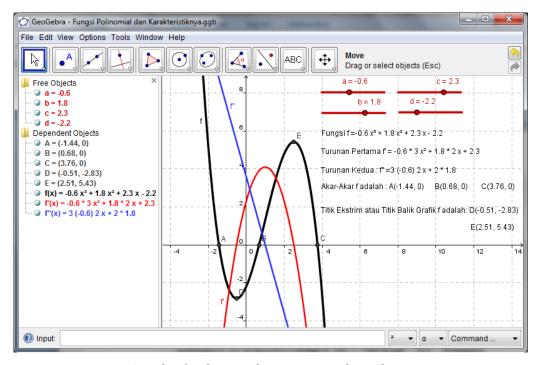


Figure 1. Graph of Polinomial Function and its Characteristics

Systematic analysis of the characteristic polynomial and its derivatives can help students have a better understanding of the algebraic manipulation of functions. GeoGebra also allows students to visualize the characteristics of a particular type of function and improving the skills of students to draw graphs of functions and derivatives.

## **Example 2: The introduction concept of integral**

GeoGebra can be used to introduce the concept of integral. Students are

required to make graphs of certain functions, such as 
$$f(x) = \frac{x^3}{4} - \frac{x^2}{2} - x + 2$$

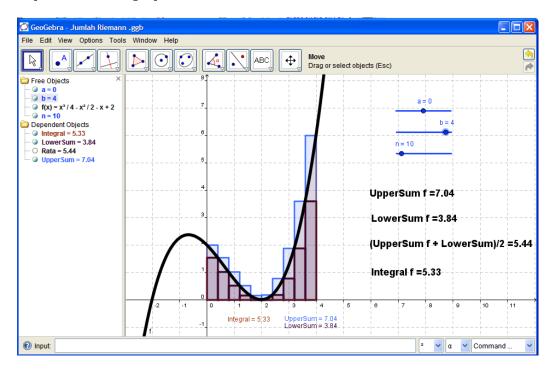


Figure 2. Introduction of Concept of Integral

Students may be asked to determine the upper-sum and lower-sum of the function or the number partition of rectangle n, where n is a parameter. By using GeoGebra, can be determined the integral of the function. With menu "slider" to change the value of n, it would appear that the integral value will be close to the average Upper-Sum and Lower-Sum on the graph. In this way, students will gain a good understanding of the concept of integral.

# **Example 3: Locus**

GeoGebra can be used to facilitate in explaining the concept of locus. Suppose there is a problem as follows.

Given line segment AB with A (2,4) and B (6,8). Through point B created a line g and through point A created line h that perpendicular to line g. Determine the locus of intersection of line g and h.

Using GeoGebra, students can explore and found that the locus is a circle as shown in the figure below.

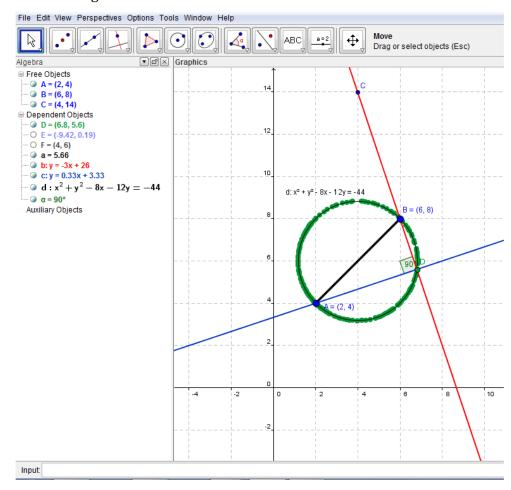


Figure 3. Locus

Formally, it can be proved as follows. Suppose the point that it meets the condition is  $T(x_0,y_0)$ . Equation line g that through point B is  $y-8=m_1(x-6)$  and line h that through point A has equation:  $y-4=m_2(x-2)$ . Because  $g\perp h$ , so satisfy  $m_2=-\frac{1}{m}$ .

$$y_0 - 8 = m_1(x_0 - 6)$$
 .....(1)

and

$$y_0 - 4 = m_2(x_0 - 2)$$

Because  $m_2 = -\frac{1}{m_1}$  , so  $y_0 - 4 = -\frac{1}{m_1}(x_0 - 2)$  that equivalent to

$$m_1 = \frac{2 - x_0}{y_0 - 4} \tag{2}$$

By subtituting (2) to (1) obtained:

$$x_0^2 + y_0^2 - 8x_0 - 12y_0 + 44 = 0$$
 ......(3)

Because  $T(x_0, y_0)$  is any point in the locus, so generally can be concluded:

$$x^{2} + y^{2} - 8x - 12y + 44 = 0$$
 ..... (4)

This is an equation of the circle. Thus, the locus is a circle.

#### 4. Conclusion

By using GeoGebra, students can "see" an abstract mathematical concepts. Students also can make a connections and discover mathematical concepts. Students' ability to access electronic solutions can potentially increase students' interest towards mathematics teaching and learning and enhance students' cognitive abilities. Thus, the use of GeoGebra in the mathematics teaching and learning can provide a range of benefits and the potential to make mathematics teaching and learning more dynamic and meaningful.

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