CHAPTER 1

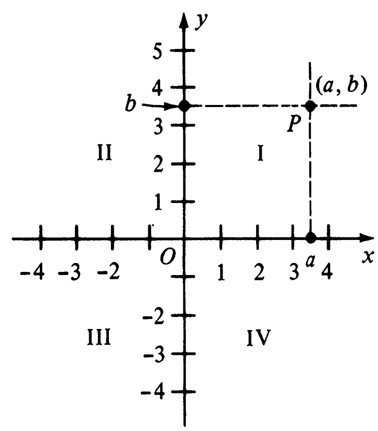
COORDINATE SYSTEM

1. RECTANGULAR COORDINATE SYSTEM

The Cartesian plane was named after Rene Descartes. It is also called the ***x-y* plane**. A rectangular, or Cartesian coordinate system may be introduced in a plane by considering two perpendicular coordinate lines in the plane which intersect in 0 on each line. Upon these lines, called x- and y-axes, directed segments measured from their intersection 0 (the origin) are taken to represent pairs of real numbers.

Usually one of the lines is horizontal with positive direction to the right, and the other line is vertical with positive direction upward, as indicated by the arrowheads. The two lines are called coordinate axes and the point 0 is called the origin. The plane is then called a coordinate plane or, with the preceding notation for coordinate axes, and xy-plane.

We choose equal uniform scales on both axes. Such pairs (a, b) of ordered numbers locate points P, and conversely. That is, to every point P belongs a unique pair of numbers and to every pair of numbers there corresponds a point P. This is a one-to-one correspondence. These number pairs are called coordinates of P: the x-coordinate is the abscissa, the y-coordinate is the ordinate. It is to be understood that the first number in a pair represents a segment length in the x-direction, the second in the y-direction.



We sometimes say that P has coordinates (a, b). Conversely, every ordered pair (a, b) determines a point P in the xy-plane with coordinates a and b. Specifically, P is the point of intersection of lines perpendicular to the x-axis and y-axis at the points having coordinates a and b, respectively. The symbol P(a, b) will denote the point P with coordinates (a, b). To plot a point P(a, b) means to locate, in a coordinate plane, the point P with coordinates (a, b).

The coordinate axes divide the plane into four parts called the first, second, third, and fourth quadrants and labeled I, II, III, and IV, respectively. Signs of number pairs in these quadrants are, respectively (+, +) (-, +) (-, -) (+, -).

Exercises

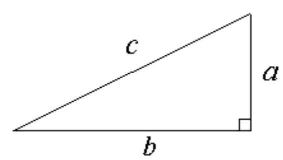
1. Plot the points whose coordinates are (4,3), (4, 3), (-4,3), (4, -3), (5,0), (0, -2), (0,0), − − , (V3,V3), and(V5,-6).
2. In which quadrant does a point lie if
   a. both coordinates are positive ?
   b. both are negative?
3. Where may a point lie if

a. its ordinate is zero,
b. its abscissa is zero?

4. What points have their abscissas equal to 2? For what points are the ordinates equal to 2?
5. Where may a point be if
   a. its abscissa is equal to its ordinate,
   b. its abscissa is equal to the negative of its ordinate?
6. Determine the coordinate of the point symmetric to (5, 4) with respect to
   a. the origin
   b. the x-axis
   c. the y-axis
7. Find the projections of each of the following segments on the x-axis and on the y-axis respectively
   a. from A(-3, -5) to B(4, -6)
   b. from A(2, -3) to B(-2, 5)

2. DISTANCE OF TWO POINTS

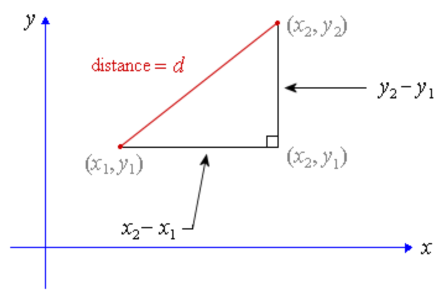
Recall Pythagoras' Theorem: For a right-angled triangle with hypotenuse length c,



$$c = \sqrt{a^2 + b^2}$$

We use this to find the distance between any two points (x_1, y_1) and (x_2, y_2) on the Cartesian plane.

The point (x_2, y_1) is at the right angle. We can see that:



- The distance between the points (x_1, y_1) and (x_2, y_1) is simply $|x_2 - x_1|$ and
- The distance between the points (x_2, y_2) and (x_2, y_1) is simply $|y_2 - y_1|$.

Using Pythagoras' Theorem we have the distance between (x_1, y_1) and (x_2, y_2) given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example

Find the distance between the points (3, -4) and (5, 7).

<u>Answer</u>

Here, $x_1 = 3$ and $y_1 = -4$; $x_2 = 5$ and $y_2 = 7$. So the distance is given by:

Exercises

1. Draw the triangle with the given vertices and find the lengths of the sides
   a. A(-1, 1), B(-1, 4), C(3, 4)
   b. A(2, -1), B(4, 2), C(5,0)
   c. A(0, 0), B(5, -2), C(-3, 3)
2. Draw the triangle having the vertices A(6, 2), B(2, -3), and C(-2, 2) and show that the triangle is isosceles
3. Show that the points A(-2, 0), B(2, 0), and C(0, 2V3) are vertices of an equilateral triangle.
4. Determine whether the points A(-5, 6), B(2, 5), and C(1, -2) are the same distance from D(-2, 2)
5. If (x, 4) is equidistant from (5, -2) and (3, 40, find x
6. Find the point on the y-axis that is equidistant from (-4, -2) and (3, 1)

3. DIVISION OF A LINE SEGMENT

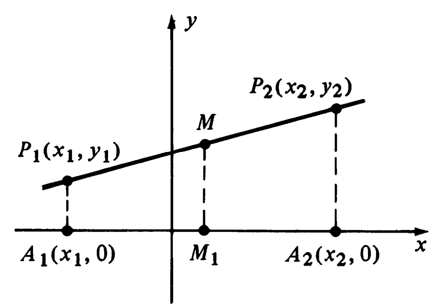
Let $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ be the extremities of a line segment, and let M(x, y) lies on line AB such that . We will determine the coordinate of M.

We have and . From similarity,

or

Hence

By the same way, we obtain



If M is the midpoint of the segment, we obtain

and

This property may be generalized by letting P(x, y) be any division point of the line through P_1 and P_2. If the ratio of P_1P to P_1P_2 is a number r, then

and .

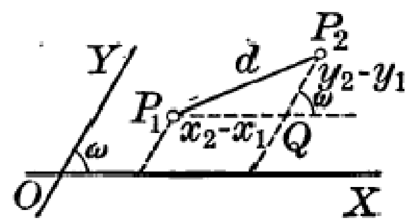
Exercises

1. Find the coordinates of the point that divides the segment from A(3, 4) to B(-2, 7) in the ratio 3:5.
2. What are the coordinates of the point P such that AP=3/5AB, where A has the coordinates (-2, -1) and B has the coordinates (5, 6) ?
3. Two points of a given segment are A(-3, 2) and B(5, -4). Find the coordinates of the points P and Q on the line containing A and B such that
   a. AP=7/3AB
   b. AP=7/3QB
4. One end point of a segment is (3, -4) and the mid-point of the segment is (-2, 1). What are the coordinates of the other end point of the segment ?
5. In what ratio does the point (0, 2) divide the segment whose end points are (-3, 0) and (3, 4)
6. Find y if the point (2, y) lies on the line joining the points (-3, 4) and (6, -3)

4. OBLIQUE COORDINATE SYSTEM

On rectangular coordinate system, the angle formed by x-axis and y-axis is right angle. When the axes are not rectangular, we called oblique coordinate system. It is the custom to designate the angle by ω. Then by the law of cosines, the distant formula will be

$$d^2 = P_1Q^2 + QP_2^2 - 2P_1Q.QP_2\cos\,(180^0 - \omega)$$



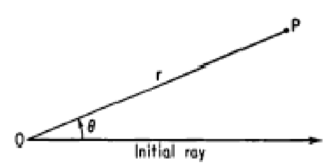
That is, since cos(180-ω) = - cosω we have

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + 2(x_2 - x_1)(y_2 - y_1)cos\omega}$$

Exercises

1. Find the distance between points A(-2,9) and B(1,1) in oblique coordinate system with ω=60^o.
2. Exchange the coordinates of points A(-2,9) and (1,1) in oblique coordinate with ω=60^o into rectangular coordinate.

5. POLAR COORDINATE SYSTEM



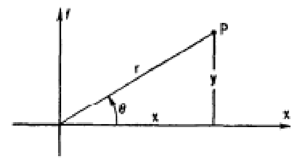
A second serviceable system in our assumed plane has for reference an initial ray, or `half' line, and its initial end point 0, called the pole. An ordered pair of numbers (r, θ) then locates a point P whose directed distance from 0 is r, the radius vector of P; and this vector makes the angle 0 with the initial ray. The angle of the pair is positive when measured from the initial ray in the counterclockwise direction, negative when clockwise. A negative distance r is to be interpreted as the extension of the radius vector "backward" through the pole 0.

For example, (-2, 30°) is plotted by drawing a radius vector at +30° from the initial ray, then extending this line backward from 0 a length two units. It should be noted that although a single point is determined by a given pair of polar coordinates, the converse is not true. A selected point has an unlimited number of coordinate pairs. For example, the pairs (2, 30°), (-2, 210°), (-2, -150°), and (2, -330°) all designate the same point. If the angle is given in radian measure, there will be no symbol attached.

Exchange of Systems.

The rectangular and polar coordinate systems may be exchanged one for the other by making the pole and the origin coincident, and the x-axis co incident with the initial ray as shown. Thus a point P may have coordinates (x, y) in the rectangular system and (r, 0) in the polar system. Relationships between these two sets of coordinates are apparently

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad \text{and} \quad \begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = \arctan\,(y/x). \end{cases}$$



These relationships permit the transfer of coordinates in one system to the other. For example, (V3, 1) in rectangular coordinates can be written as (2, 30°) in polar coordinates; whereas (-1, 5π/6) polar, is (V3/2, -1/2) rectangular.

Exercises

1. Change the following polar coordinates to rectangular coordinates and plot the points (a, 0), (0, a), (1, π/2), (-1, 7π/2), (-1, -,π/2), (-2, 120°), (-1, 7π)
2. Determine for each of the following points in rectangular coordinates four sets of polar coordinates with 0≤θ≤2π, (0, 1), (1, 0), (-1, 0), (0, -1), (3, 1), (1, -1)