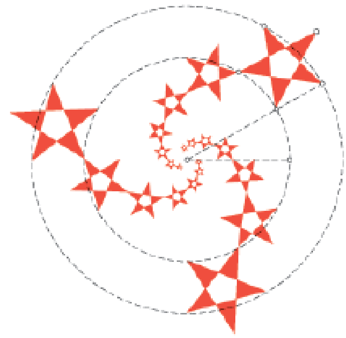
YOGYAKARTA STATE UNIVERSITY
MATHEMATICS AND NATURAL SCIENCES FACULTY
MATHEMATICS EDUCATION STUDY PROGRAM

TOPIC 9:

SIMILARITY



Ratios and proportions have many applications. Architects use them when they make scale models of buildings. Interior designers use scale drawings of rooms to decide furniture size and placement. Similar triangles can be used to find indirect measurements. Measurements of distances such as heights of tall buildings and the widths of large bodies of water can be found using similar triangles and proportions.

Polygons with the same shape are known as *similar polygons*. To be similar, all pairs of corresponding sides must be in proportion, and each pair of corresponding angles must be equal. It is not enough for just one

of those conditions to be true. Congruent figures have the same shape and size. Similar figures are not usually the same size, but they are identical in shape. The secret to having an identical shape is having all angles identical in degree measure. Studying triangles with identical angles but different side-lengths led to the discovery of the trigonometric functions.

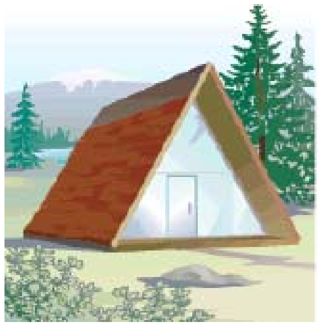
Let's begin by looking at ratios. To discuss similarity, we need to know about proportions. If you compare two quantities, then you have used a ratio. A *ratio* is the comparison of two numbers using division.

The ratio of x to y can be written $\frac{x}{y}$ or x:y. Ratios are usually expressed in simplest form.

Since $\frac{2}{4}$ and $\frac{3}{6}$ are both equal to $\frac{1}{2}$, they are equal to each other. A statement that two ratios are

equal is called a *proportion*. A proportion can be written in one of the following ways: $\frac{2}{4}=\frac{3}{6}$

The first and last numbers in a proportion are called the *extremes*. The middle numbers are called the *means*.



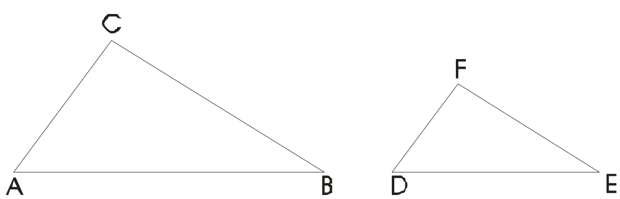
Theorem (Theorem of Proportional segment)

A line is paralllel to one side of a triangle if and only if it divides the other two sides proportionally

Definition (similar triangles)

Suppose there is a correspondence between two triangles. Two triangles is said to be similar if their correspond angles are congruent and their correspond sides are proportional.

We symbolize $\Delta ABC \sim \Delta DEF \Leftrightarrow \angle BAC \cong \angle FDE, \angle ABC \cong \angle DEF, \angle BCA \cong \angle EFD,$ and $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$.



Two congruent triangles have proportion 1. For any triangle and a positive number k, there is a triangles which is similar to it with proportion k. These theorems can be used to determined whether two triangles are similar.

Theorem (S-S-S theorem)

If the lengths of the corresponding sides of two triangles are proportional, then the triangles are similar.

Theorem (A-A)

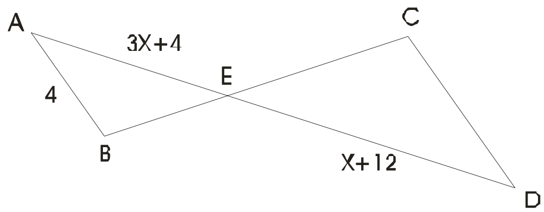
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Theorem (S-A-S)

If the lengths of two pairs of corresponding sides of two triangles are proportional and the corresponding included angles are congruent, then the triangles are similar.

Example

Given $\overline{AB} // \overline{CD}$, *AB*=4, *CD*=8, *AE*=3x+4, and *DE*=x+12. Prove that $\Delta ABE \sim \Delta DCE$, and then find *AE* and *DE*.



<u>Solution:</u>

Compare ΔABE and ΔDCE.

$\angle A \cong \angle D$	(Line $\overleftrightarrow{AB}$ and line $\overleftrightarrow{CD}$ which is $\overline{AB} // \overline{CD}$, is cut by a transversal $\overleftrightarrow{BC}$ then $\angle$A and $\angle$D are alternate interior angles)
$\angle B \cong \angle C$	(Line $\overleftrightarrow{AB}$ and line $\overleftrightarrow{CD}$ which is $\overline{AB} // \overline{CD}$, is cut by a transversal $\overleftrightarrow{BC}$ then $\angle$B and $\angle$C are alternate interior angles).

Based on A-A theorem, we conclude that $\Delta ABE \sim \Delta DCE$.

Next, we'll find *AE* and *DE*.

Since $\Delta ABE \sim \Delta DCE$, then $\frac{AB}{DC} = \frac{AE}{DE} = \frac{BE}{CE}$. From the equality

$$\frac{AB}{DC} = \frac{AE}{DE} \Leftrightarrow \frac{4}{8} = \frac{3x+4}{x+12}$$

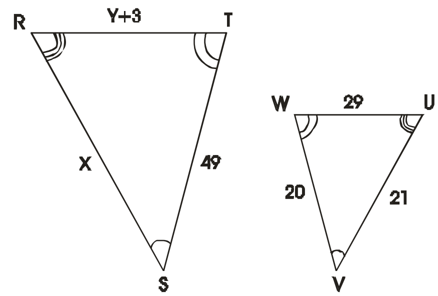
$$\Leftrightarrow x+12 = 2(3x+4)$$

$$\Leftrightarrow 5x = 4 \Leftrightarrow x = \frac{4}{5}.$$

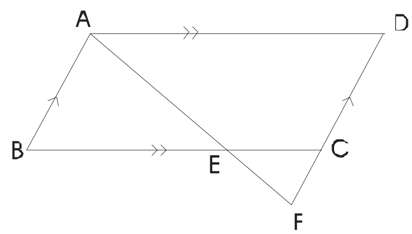
So, $AE = 3x+4 = 3.\frac{4}{5}+4 = 6\frac{2}{5}$ and $ED = x+12 = \frac{4}{5}+12 = 12\frac{4}{5}$.

PROBLEMS

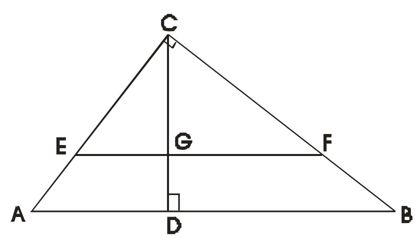
1. Are any two equilateral triangles similar ? Explain your answer.
2. Prove that the altitude to the hypotenuse of a right triangle divides it into two triangles which are similar to the given triangle and to each other.
3. Given $\Delta RST \sim \Delta UVW$. Find x and y.



4. Find three pairs of similar triangles if $\overline{AB} // \overline{CD}$ and explain why.



5. Given $\overline{EF} // \overline{AB}$, *AD*=9, *DB*=16, and *EC*=2(*AE*)=8. Find *AE*, *AC*, *CF*, *CB*, *CD*, and *EF*.



6. Find the congruent angles.