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Worked Examples in Mathematics

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Abstract: *Providing worked examples to students when learning mathematics is not unusual. Cognitive Load theorists suggest that worked examples should be designed in such a way they do not trigger extraneous cognitive process. Cognitive Load Theory (CLT) is an instructional design theory that has been used to generate instructional procedures using our knowledge of human cognitive architecture. The theory reveals that worked example instructions works precisely effective for new learners than problem solving instructions. Acquiring knowledge from worked examples provides learner relevant knowledge to enhance learning and problem solving. Nevertheless, designing worked examples should refer to the split attention and redundancy effects. The current research shows that worked examples could be learn individually or in groups. However, if more complex material is presented, learning them individually is better than in groups.*

Keywords: worked example, problem solving, mathematics, cognitive load theory

1. Introduction: What a worked example is

While a problem solving comprises a problem statement only, a worked example covers includes a problem statement, a solution and also a commentary (Mayer, 1999). A commentary is added next to the solution steps in order to describe the underlying reasons why the step is selected. A solution included in a worked example demonstrates a step by step solving the problem. It is suggested that a useful worked example should show an effective procedure as applied by experts (see Atkinson, Derry, Renkl, & Worthan, 2000 for review).

Worked examples are not only often used by teacher in a classroom but also commonly appear in textbooks. Giving worked examples is usually aimed to elucidate a concept or procedure. Therefore it is important to create a worked example that facilitates learners to understand the concept or procedure. Understanding occurs when learners could construct knowledge by connecting it with prior knowledge and rebuilding new knowledge structure (Pass, Renkl, & Sweller, 2004). This process is primarily handled by working and long term memory in our cognitive system. As a consequence, worked example designers should refer to how our cognitive system works for learning by worked examples.

2. Why Learning by Example is Effective: A Cognitive Load Theory

Cognitive Load Theory (CLT) has been developed by John Sweller and his colleagues since 1980s. It is an instructional design theory that has been used to generate instructional procedures using our knowledge of human cognitive architecture (Pass et al., 2004; Sweller, 2004, 2006). Human cognitive architecture may describe to us the system in our brain that guides cognitive processes such as responding, thinking and understanding. The major features are working memory which holds information or knowledge we are attending to during a cognitive activity and long term memory where knowledge are stored to and retrieved from. Because working memory is severely limited in capacity and duration when processing novel information (Cowan, 2001; Miller, 1956), CLT suggests that instructions should be presented to minimise irrelevant cognitive load (Pass et al., 2004; Sweller, 2004, 2006).

Extraneous cognitive load might be triggered when working memory load is allocated to activity that is not relevant directly to understanding. A means end analysis is an example. Means ends analysis is a generic strategy to solve a problem by breaking down a problem into sub-problems and testing them one by one to find the solution (Ayres &

Sweller, 1990). This strategy is usually applied by novices because of the lack of prior knowledge to understand the problem solving. Solving problem this way is inefficient and inhibits learning because working memory load is not directly allocated for constructing knowledge covering the problem. Instead it is overburden by irrelevant activities, such as creating sub-sub goals, searching and examining one step and if fail, move to the next one.

Worked examples can be very effective to prevent extraneous cognitive activity like means ends analysis. Sweller and Cooper (1985) found when lack of prior knowledge, learners are benefited from learning by examples and by problem solving later, rather than learning by problem solving only. Worked examples as noted previously provides an opportunity to acquire knowledge on how to solve problem and also the underlying reason. The explicit guidance of worked examples makes learning new material easier. Thus working memory load can be directly allocated to knowledge construction. A worked example effect describes that learning by pairs of worked example and problem solving is more advantageous than learning by problem solving only (Pass et al., 2004; Sweller, 2004, 2006).

Nevertheless, worked examples can be less effective due to its presentations. For an instance, when a diagram and the required explanation is separated. This presentation causes learners to search and match labels or symbols in the diagram with the text to gain understanding. A mental integration required in this process will impose a heavy extraneous cognitive load. Tarmizi and Sweller (1988) suggested to integrate both relevant source of information. Ward and Sweller (1990) shows when symbols are used to represent a word problem, integrating them in the problem statement is also easier to understand. In cognitive load theory, this effect is called a split attention effect (Pass et al., 2004; Sweller, 2004, 2006).

It should be noted that integration of two sources of information is ultimately needed if they are required each other to be understood. However, if two sources of information are put together when one source can already be understandable, the redundancy effect may occur (Pass et al., 2004; Sweller, 2004, 2006). Chandler and Sweller (1991) found that redundancy effect will impose extraneous cognitive load and hinder learning. It is suggested the elimination one source is better than integrate both sources.

3. Can worked example be learned in groups?

Using the integration principle for designing worked examples in Geometry, Retnowati, Ayres, and Sweller (2010) found that they are learnable individually or in collaborative learning. Similar results also found for Arithmetic worked examples designed using the principle (Retnowati, 2011, September). However, the current research reveals when the problem is more complex to the learners, the worked example is better learned individually than in groups. The presentation of others during learning more complex worked examples seems to extraneous. In the presented poster, examples of the worked example designs examined in this research are exhibited.

4. Conclusions

It can be concluded that the effectiveness of worked example for learning new topic depends on the presentation. If extraneous cognitive activity imposed by learning worked example can be minimized, then learning is optimal. This can be done by following the integration principle (to avoid split attention effect) or the elimination principle (to avoid the redundancy effect). Its complexity should be taken account if learners are allocated to learn worked examples in individual or group settings.

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Invitation Letter

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November 24-27 2012, Beijing, China

Dear Dr. Endah Retnowati

It's my pleasure to invite you to attend the Second International STEM in Education Conference with the theme of Instructional Innovations and Interdisciplinary Research in STEM Education (STEM2012), in Beijing. Your paper has been accepted to the conference, and I wanted to express my wish that you will be able to attend the conference in Beijing Normal University on November 24th through November 27th, 2012. Please note that we do require that in order for the paper to be published in the conference proceedings, at least one author must register for and attend the conference to present the paper.

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I hope all is well and I look forward to seeing you in Beijing in November.

Sincerely,

ShengQuan Yu

Sincerely,
Dr. Shengquan Yu
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