Endah Retnowati
e.retno@uny.ac.id

Problem Solving in Mathematics

Day – 1 on 8 October 2014. QITEP in Mathematics, PPPPTK Yogyakarta
Differentiated Instruction For Senior High School Mathematics Teachers

Outline

- Problem
- Problem solving
- Problem based learning
- Heuristic plan/strategy or heuristics
- How students solve problem
- The problem with PBL
- How to create effective PBL

- Where is my name tag?
- What is her name?
- I hope today I will be learning new material. What could it be?
- Can I have my lunch break extended 15 minutes?
- I could not access the internet.
- What is a square-root of 100?

A question is categorised a question as a problem if it cannot be answered immediately using knowledge available at one's disposal (or long term memory) and so has to be consciously worked out

Suppose five days after the day before yesterday is Friday. What day of the week is tomorrow?

Problem solving has two aspects

(1) the *process*: a set of activities, and

(2) the *product*: the actual solution

an *ill-defined problem* has multiple acceptable products and many possible ways for reaching them,

a *well-defined problem* has only one possible product and one agreed process for reaching it

Do this:

- Write a mathematics problem solving on a piece of paper.

What Polya (1965) offers?

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back

Schoenfeld (1979) offers:

1. Draw a diagram if at all possible
2. If there is an integer parameter, look for an inductive argument
3. Consider arguing by *contrapositive* or *contradiction*
4. Consider a similar problem with fewer variables

- Contrapositive:
  - Instead of proving the statement "If X is true, then Y is true," you can prove the equivalent statement "If Y is false, then X must be false"
- Contradictions:
  - Assume, for the sake of argument, that the statement you would like to prove is false. Using this assumption, go on to prove either that one of the given conditions in the problem is false, that something you know to be true is false, or that you wish to prove is true. If you can do any of these, you have proved what you want.

Brunning, Scraw, Norby & Ronning (2004)

(1) identifying the problem,
(2) representing the problem,
(3) selecting an appropriate strategy,
(4) implementing the strategy, and
(5) evaluating solutions.

Understand the problem

- What is given
- What is unknown
- What operations are allowed
- Represent the problem

Devise a plan

- Determine a general course of attack
- Restate the problem (the unknown) so that it is more like a familiar problem
- Find a related problem

Carry out the plan

- Carry out the computations
- Carry out the need operations

Look back

- Look over the process you went through
- Try to see how this experience can be helpful in solving other problems

Krulik (1995)

1. Read and think
2. Explore and plan
3. Select a strategy
4. Find an answer
5. Reflect and extend

1. Read and think

- Identify the fact
- Identify the question
- Visualise the situation
- Describe the setting
- Restate the action

2. Explore and plan

- Organise the information
- Is there sufficient information
- Is there too much information
- Draw a diagram or construct a model
- Make a chart, a table, a graph, or a drawing

3. Select a strategy

- Pattern recognition
- Working backwards
- Guess and test
- Simulation or experimentation
- reduction./expansion
- Organised listing/exhaustive listing
- Logical deduction
- Divide and conquer

4. Find an answer

- Estimate
- Use computational skills
- Use algebraic skills
- Use geometric skills
- Or calculator if it may

5. Reflect and extend

- Check your answer
  - Is the computation correct?
  - Is the question answered?
  - Is the answer reasonable?
- Find alternate solutions
- What is….?
- Extend to either:
  - A generalisation or
  - A mathematical concept
- Discuss the solutions
- Create interesting variations on the original problem

Problem Based Learning

- Anybody knows what it is?

Teaching component skills

general problem solving strategy

Teaching within specific domain

specific problem solving strategy

Problem solving example:

For Gilda's party, the Hoagie House prepared a huge sub sandwich of a 7-foot long Italian roll. Gilda wants to feed 16 friends. How many cuts must she make?

HEURISTIC (RULE OF THUMBS) "General problem solving strategy"

- Pattern recognition
- Working backwards
- Guess and test
- Simulation or experimentation
- reduction./expansion
- Organised listing/exhaustive listing
- Logical deduction
- Divide and conquer

Domain specific strategy is needed

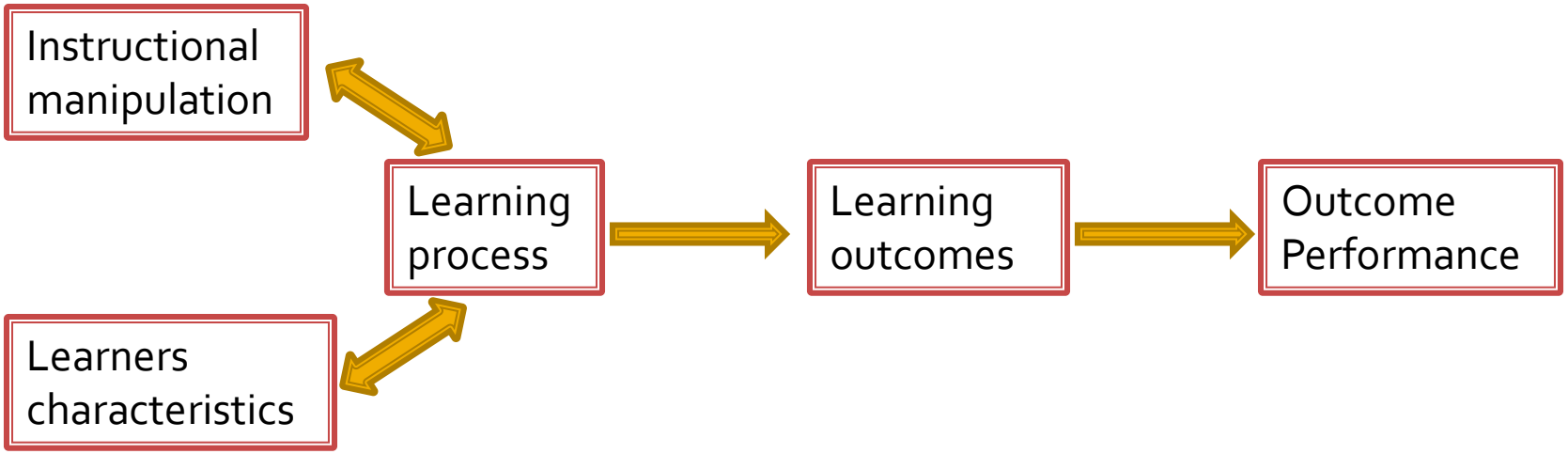
- Problem 4: Two points on the surface of the unit sphere (in 3-space) are connected by an arc A which passes through the interior of the sphere. Prove that if the length of A is less than 2, then there is a hemisphere H which does not intersect A.

- Problem 5: Let a, b, and c be positive real numbers. Show that not all three of the terms $a(1-b)$, $b(1-c)$, and $c(1-a)$ can exceed $\frac{1}{4}$.

The main purpose of learning through problem solving

- Schema construction and automation
- Transfer knowledge to new/advanced problem solving

Cognitive approach on learning process



Instructional manipulations

The sequence of external events including the organisation and content of instructional materials and behaviors of the teacher.

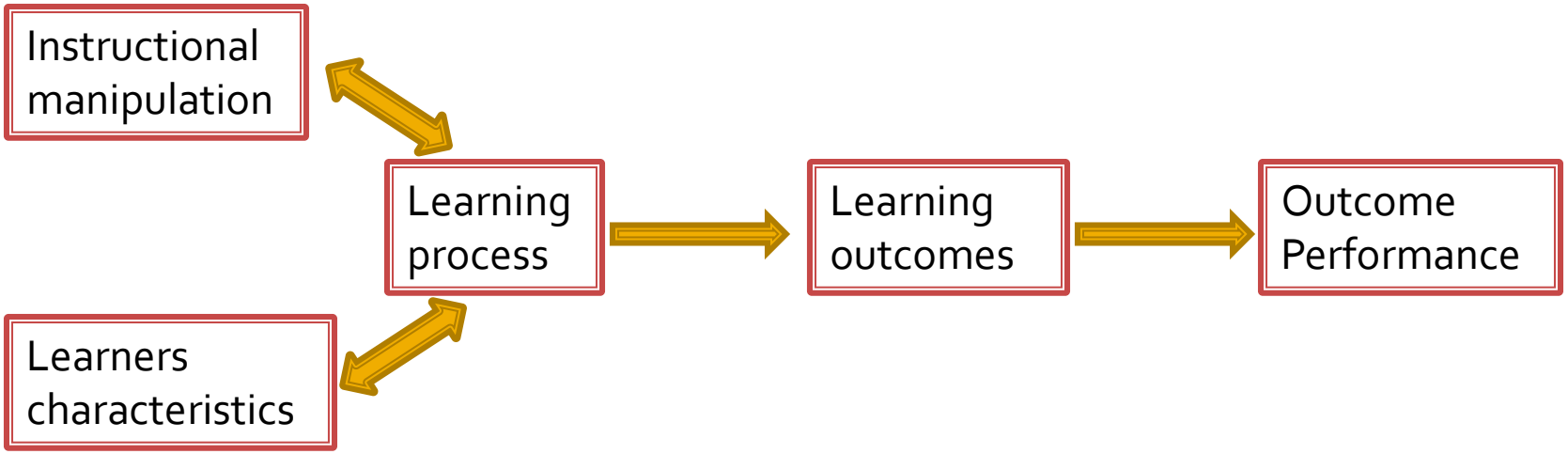
This includes what is taught, how it is taught and depends on the characteristics of the teacher and on the curriculum

Learner characteristics

The learner's existing knowledge, including facts, procedures and strategies that may be required in the learning situation

The nature of the learner's memory system, including its capacity and mode of representation in memory.

Cognitive approach on learning process



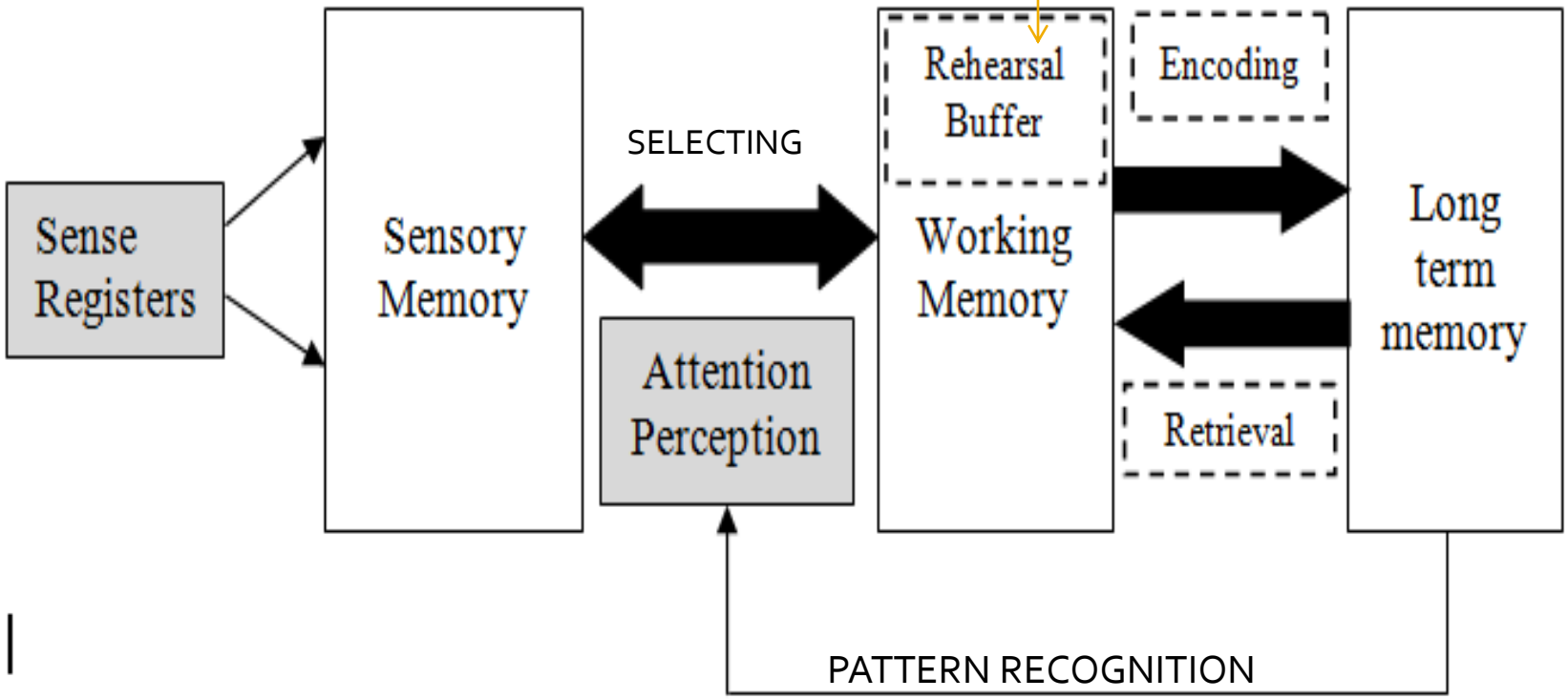
Outcomes

- Learning outcomes
  - The cognitive changes in the learner's knowledge or memory system, including newly acquired facts, procedures and strategies
- Outcome performance
  - The learner's performance & behaviour on tests that measure the amount of retention or the ability to transfer knowledge to new learning tasks

Two main points on learning process:

- Learner centered approach
  - Learner characteristics
  - Learning process and outcomes
  - Instructional manipulations affects changes in the learner's knowledge
- Learner is an active information processor
  - Knowledge is constructed by learner
    - HOW AND WHERE IS KNOWLEDGE IS CONSTRUCTED?

LEARNING
PROCESS: organise information, build connection, construct knowledge

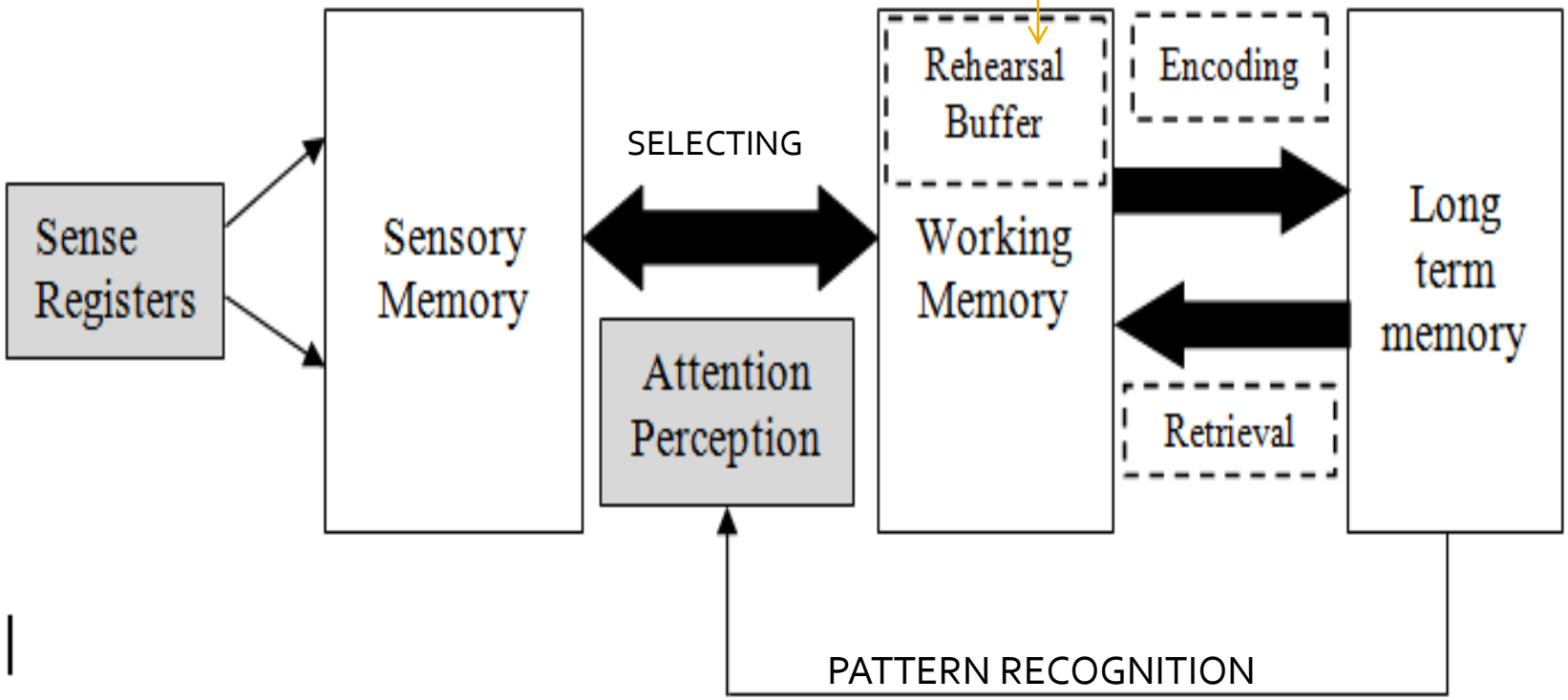


STEPS ON SENSORY MEMORY

- ATTENTION AND PERCEPTION
- PATTERN RECOGNITION

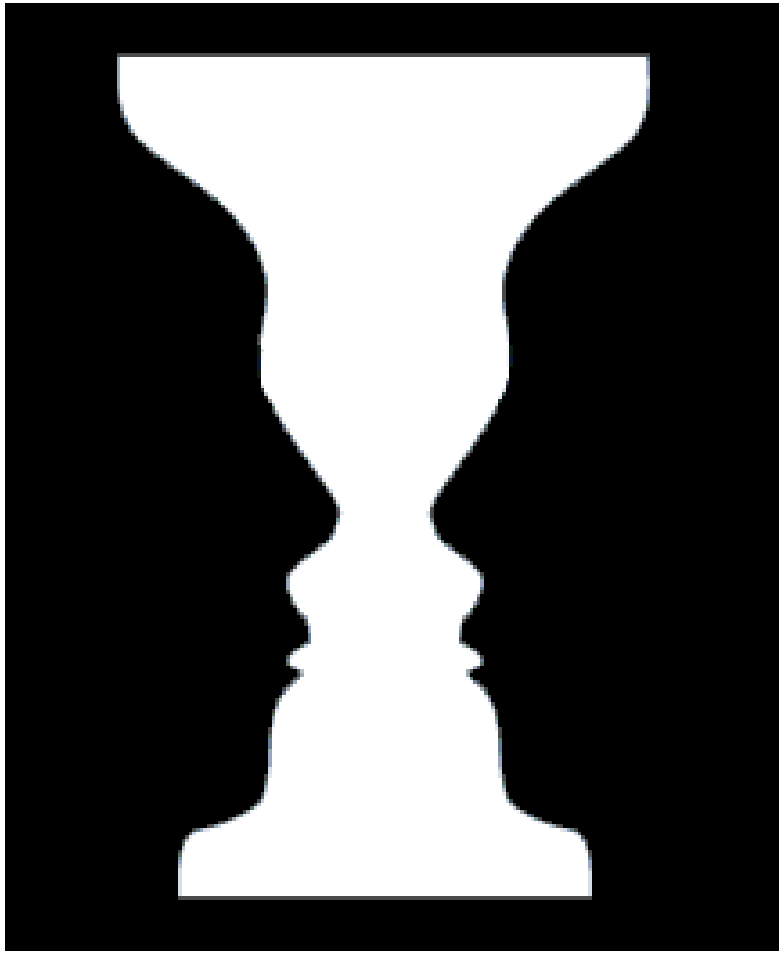
PRIOR KNOWLEDGE
CONTEXT

LEARNING PROCESS:
organise information, build connection among information and integration with prior knowledge, and eventually construct knowledge, encode knowledge to LTM

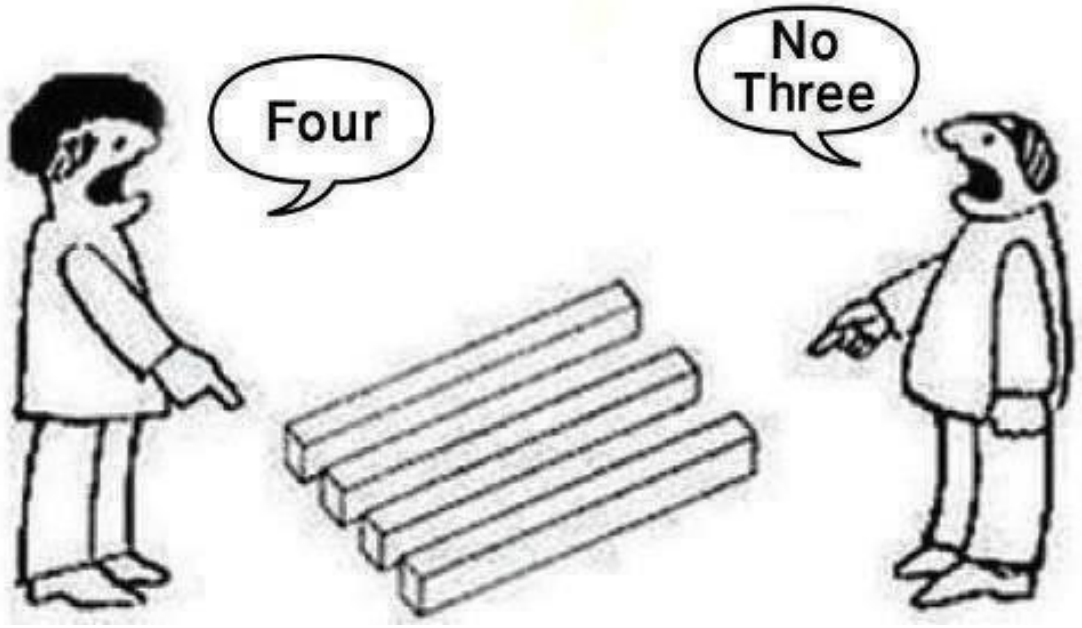


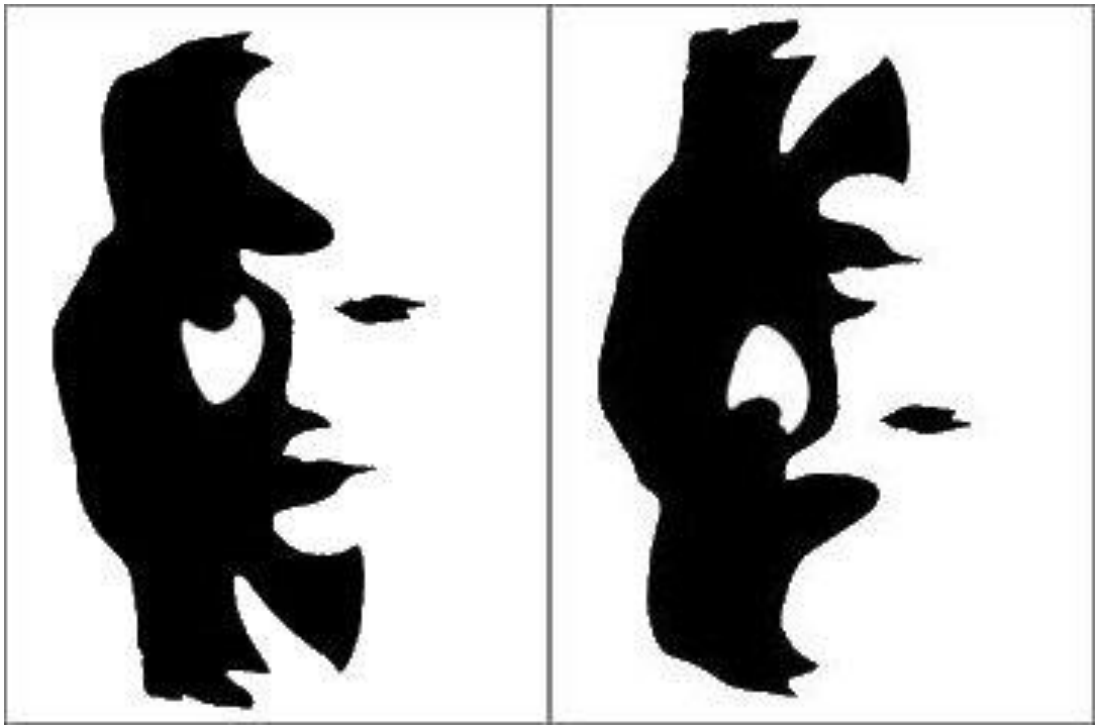

Tuesday
29.5.13
BEAR


Thursday 5th December
5.12.13
2013
1/2 of 8 is 4
20 (10 school days)
Billy
Kadir
Saman
Brian
Shane
Host Points
Jemma
Gracia
Shane
Griffin



It is really confusing!!!

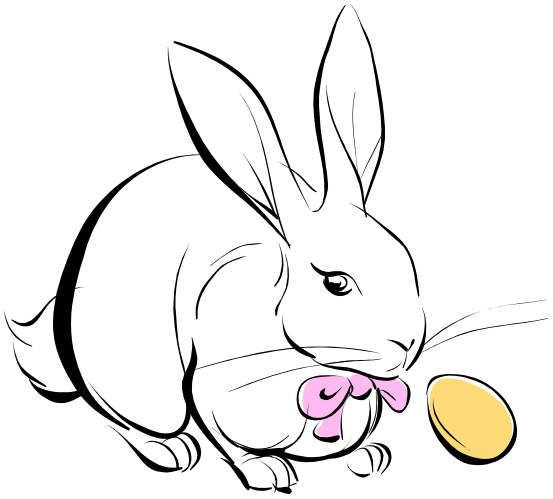





HALF FULL
HALF EMPTY

PERCEPTION

- The assignment of meaning to incoming stimuli
  - Is the detection of incoming stimuli by your senses
  - Is the process by which stimuli are perceived, recognised and understood



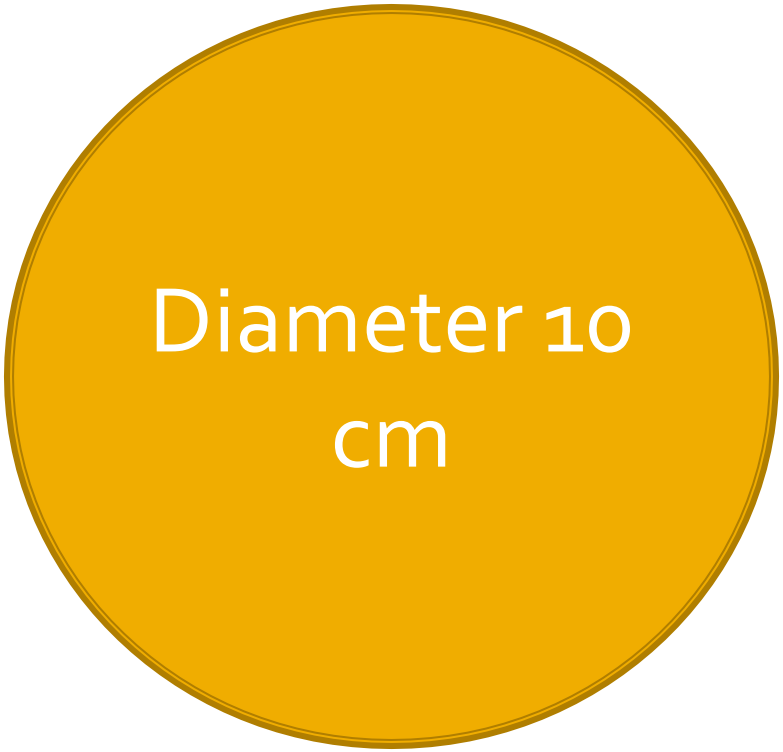
Steps of perception

Detection of a stimuli through senses

Storage of some representation of the stimuli in memory system

Pattern recognition

Assignment of meaning to stimuli


Diameter 10
cm

Example: steps of perception

Detection of a stimuli through senses: stimulus may be seen/heard

Storage of some representation of the stimuli in memory system: stored these into icon/echo

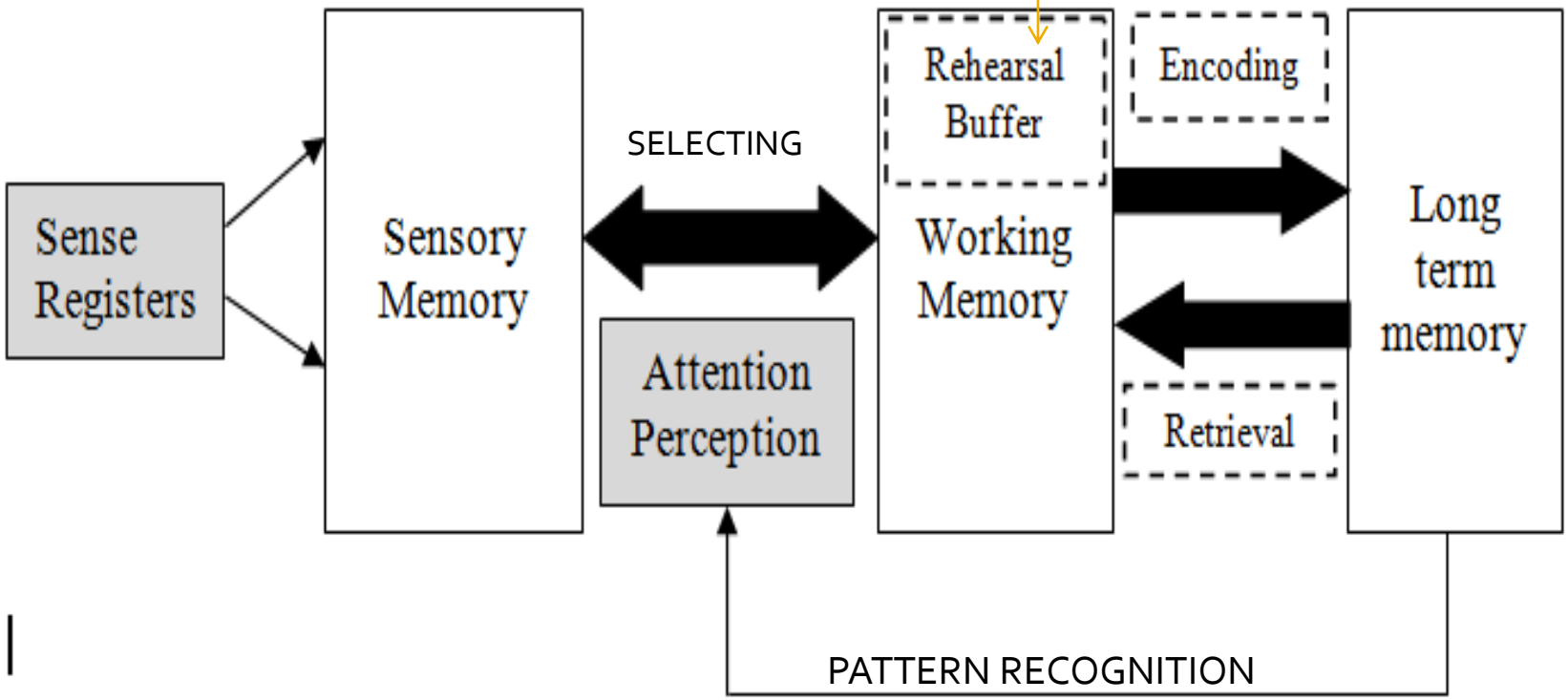
Pattern recognition: circle/writing/sound information from LTM used to recognise pattern

Assignment of meaning to stimuli: select information to assign meaning that is undertaken in working memory

PERCEPTION IS AFFECTED BY

- Nature of stimulus (context of stimulus)
- Background of knowledge
- Pattern recognition occurs when elements match!!

LEARNING PROCESS:
organise information, build connection among information and integration with prior knowledge, and eventually construct knowledge, encode knowledge to LTM



ATTENTION

Allocation of cognitive resources to a task

Critical for learning – to process information learners have to pay attention

BUT

Human's have extremely limited processing capacity!

Limitations

- Generally people cannot attend to more a few things at once
- Under many conditions multi-tasking is not very effective because attention is divided too much, leading to poor executions of tasks (divided attention)
- Automation of skills can compesate for limited attention capacity

Attention Allocation

- The type of TASK influences attention allocation
  - Nature of task
  - Nature of need
  - Motivations
- Attention is allocated differently according to the tasks provided

RESOURCE LIMITED

A task where performance will improve if more attention is shifted to that task

*CONCENTRATION

DATA-LIMITED

Performance is limited by the quality of the presented task

Some tasks are so complex that some individuals can never apply enough resources to them because of lack of knowledge

DISTRACTIONS

- Students are easily distracted
  - Teacher is giving important explanations, students mind starts to 'wander' –tuning in to other conversations (sounds), looking out the window (visuals), thinking of other matters (internal cognition)….etc
  - Concentration is dependant on attention

Pattern recognition

- Prior knowledge used to make decisions about the meaning of the stimuli
- When stimuli in the environment are recognised as something stored in memory
- Two systems for recognising paterns:
  - Parts to whole (Example?)
  - Whole to parts (Example?)

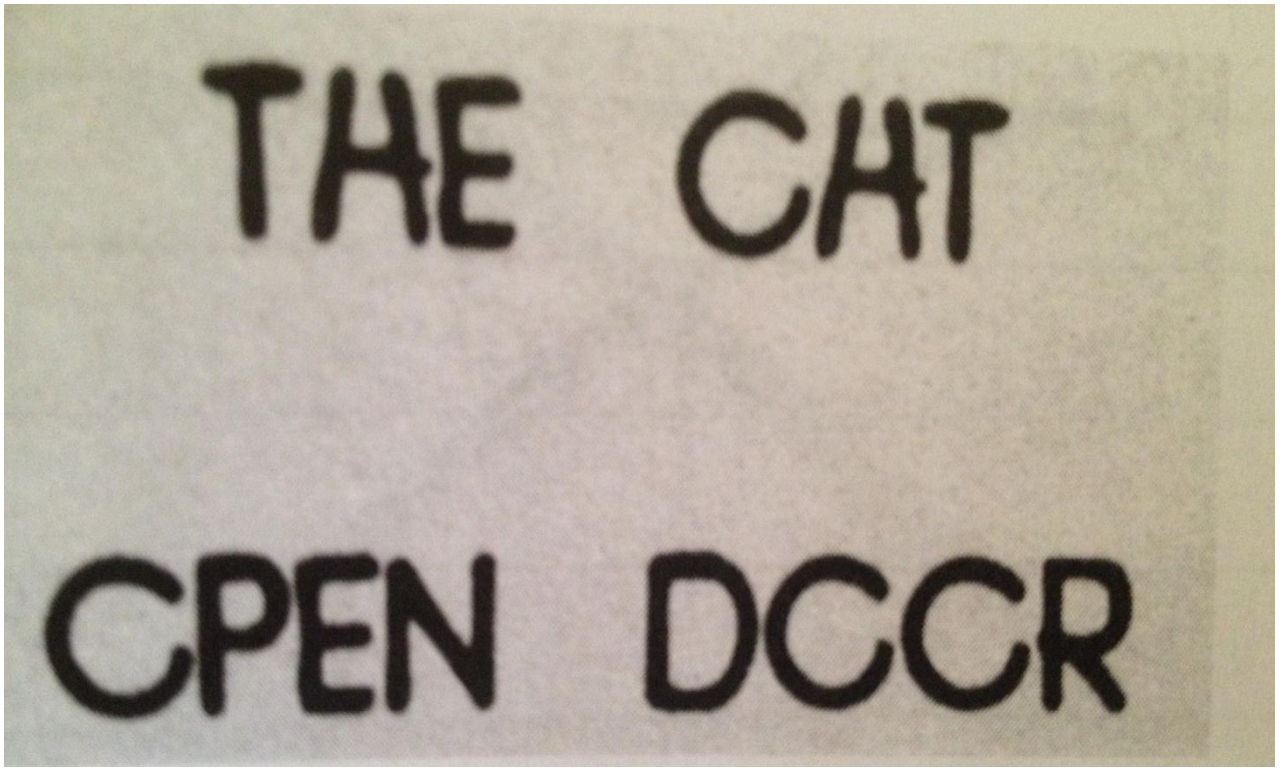
Theory of pattern recognition (??!!??)

Gestalt theory (PLEASE SEARCH)

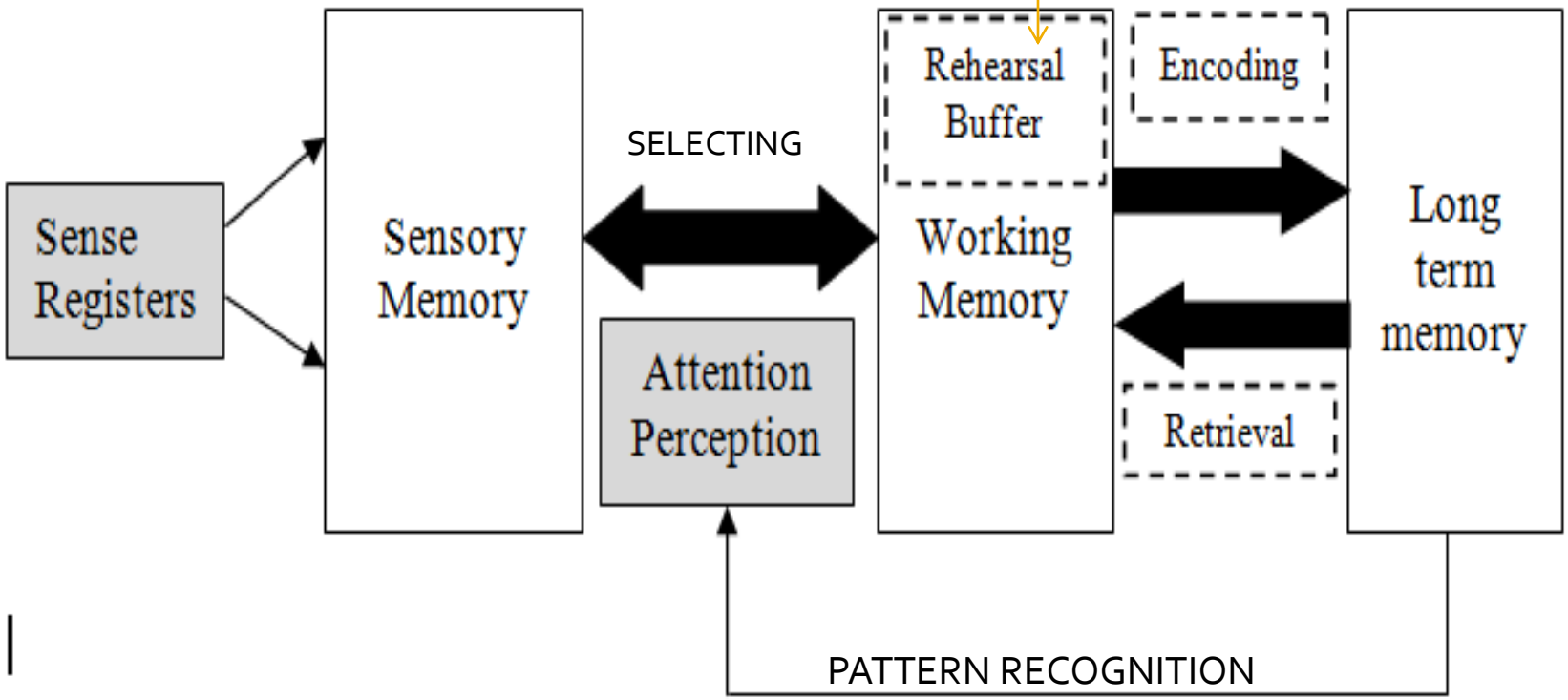
PRIOR KNOWLEDGE

- Directly affects perception process
- Allows perception occurs
- Guides perception of new information

EXAMPLE



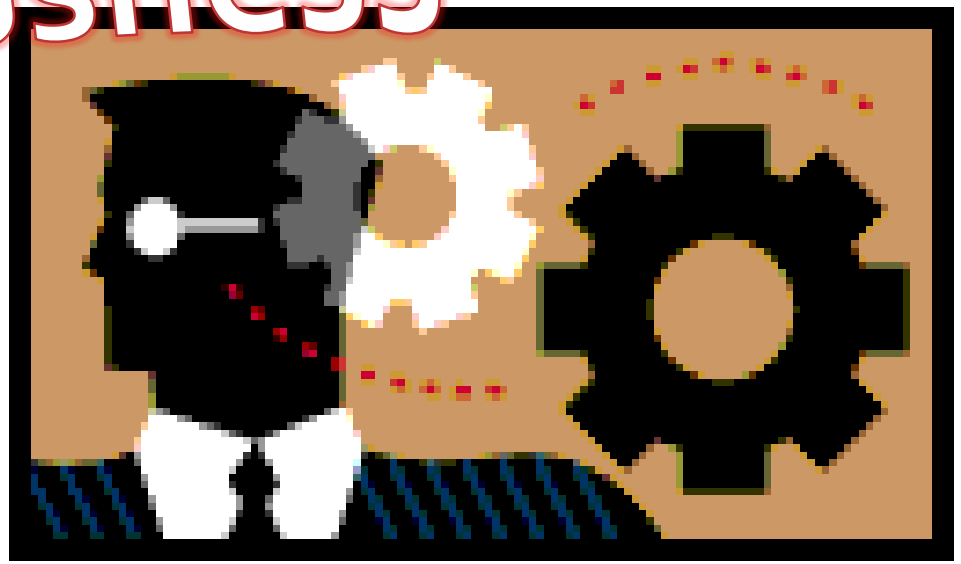
LEARNING PROCESS:
organise information, build connection among information and integration with prior knowledge, and eventually construct knowledge, encode knowledge to LTM



Working memory?

- What is in your mind now?

Conciousness



Working memory

- Repeat a telephone number
- Repeat an unfamiliar foreign word
- 12 + 13 = ?
- 314343543 + 89786592 = ?

Working memory

- What have you been doing just before this?
- Close your eyes and pick up an object in front of you.
- How many windows in your house?

Working Memory

- Limited in capacity
  - Miller's research: the magic number of seven (7±2 chunks of new meaningful information)
  - Cowan's research: 4±1 chunks of new information to be processed
- Limited in duration
  - Recalls decay over time unless actively rehearsal occurs
  - Information lost very rapidly when people are distracted from rehearsing
  - Forgetting occurs due to interference (of new information) rather than time

Instructional implication of working memory properties?

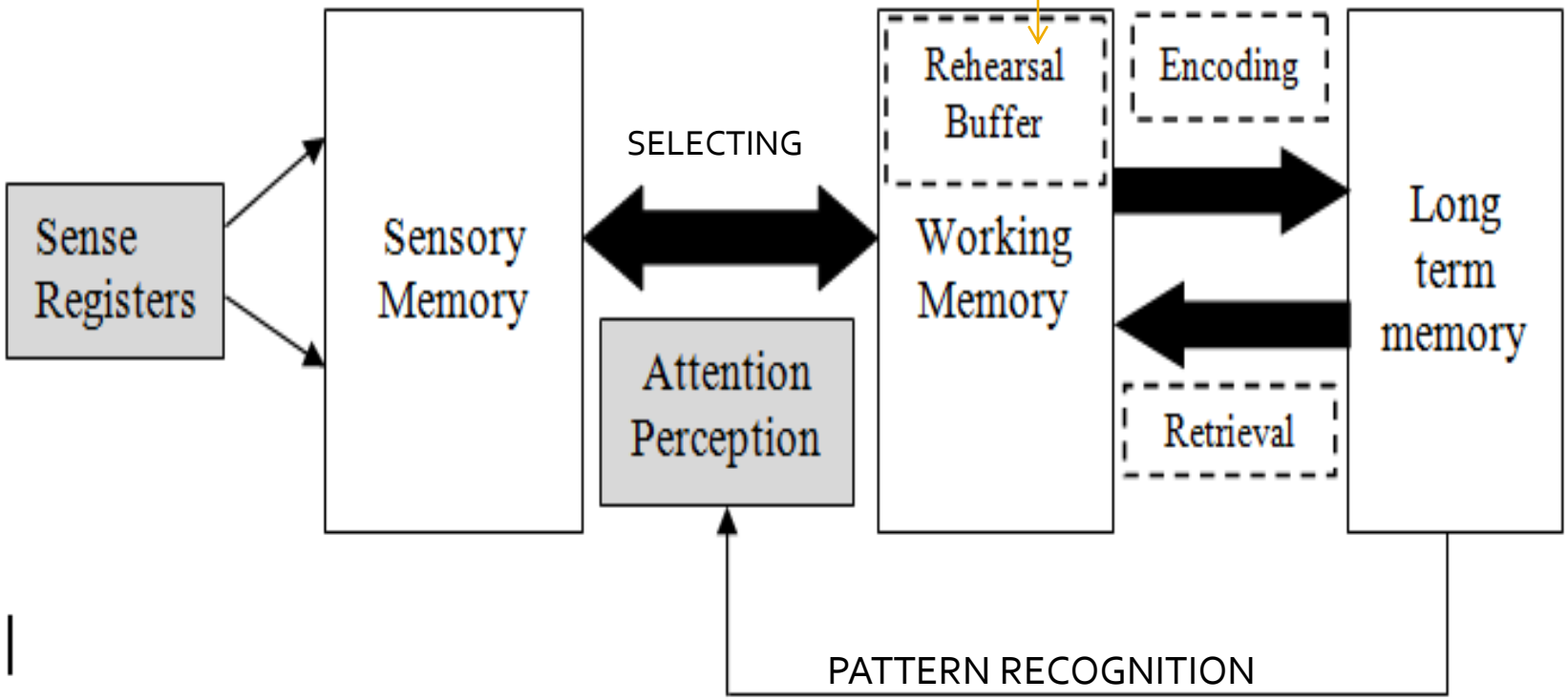
- Keep it simple, do not overload working memory
- Draw attention to most important points
- Present information using different modalities (e.g., visual and auditory)

Working memory can be

- Processing "unlimited" amount of information that is already familiar.

- Give example!

LEARNING PROCESS:
organise information, build connection among information and integration with prior knowledge, and eventually construct knowledge, encode knowledge to LTM



Long term memory

- Unconcious component of our memory
- Unlimited capacity and duration
- Where cognitive structures are organised

- Explicit memory
  - Conscious recall, recognition of previous knowledge/information/experience
- Implicit memory
  - No record of previously remembering events
  - Retention without remembering

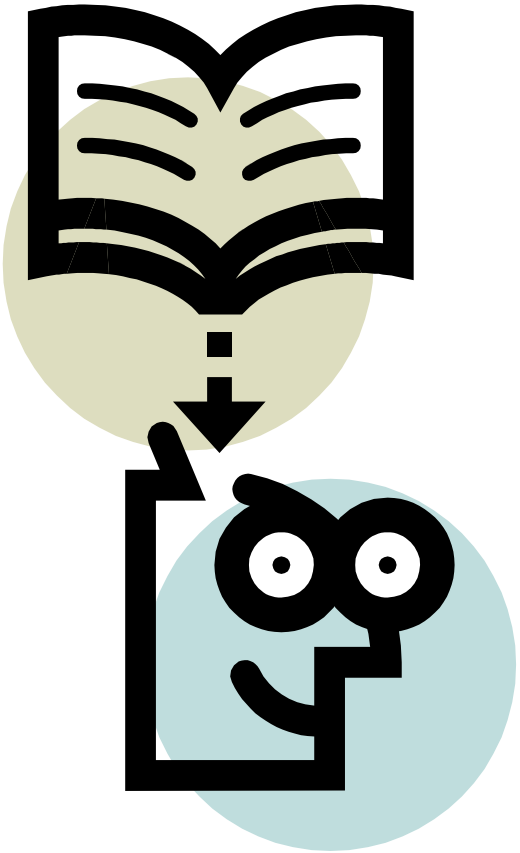
Schemata

- LTM is actively constructed using schemata
- Activated schemata determine what incoming information is relevant
- Schemata are continually reconstructed through learning

PIAGET'S THEORY

- ASSIMILATION
  - New information that fits into an existing schema is added
- ACCOMODATION
  - Existing schemata are modified in the face of new, confliting information

Things to reactivate before problem solving?



The main purpose of learning through problem solving

- Schema construction and automation
- Transfer knowledge to new/advanced problem solving

AUTOMATED PROCESSING

- Occurs without intension and conciousness
- Less cognitive effort
- Less error
- Performance is quicker
- Automated performance
- Develop learning – to more difficult task
- Skilled learners

Expert vs. Novice problem solver

- The amount of schema constructed in LTM
- The amount of automated schemas

Goal-free problem

Goal free problems discourage students from creating sub-goals and separate the problem state and the problem goal because the problem goal is not given.

Instead, students are required to work forward from given information in order to assist schema construction.

- Can you please give an example of goal-free problems?

Worked example approach

- Anybody knows what it is?
- And, what research says about this?

Zhu and Simon (1997)

Learning from Examples

Examples:

(1) $x^2 + 5x + 6 = (x + 2)(x + 3)$
(2) $x^2 + 7x + 6 = (x + 1)(x + 6)$
(3) $x^2 + 8x + 12 = (x + 2)(x + 6)$
(4) $x^2 + 7x + 12 = (x + 3)(x + 4)$
(5) $x^2 + 13x + 12 = (x + 1)(x + 12)$

Exercises:

(1) $x^2 + 11x + 18 = (\quad)(\quad)$
(2) $x^2 + 9x + 18 = (\quad)(\quad)$
(3) $x^2 + 19x + 18 = (\quad)(\quad)$

Zhu and Simon (1997)

Learning by Doing

Problems: Identical with examples for learning-from-examples section except that right-hand sides of equations are shown as:

$$= (x + \_\_\_\,)\,(x + \_\_\_\,)$$

Exercises: Identical with exercises for learning-from-example section.

PBL vs. WEBL

- What research says about this comparison?
- Expertise reversal effect?

Various instructions based WE

- WE only
- Pairing WE and similar PS
- Faded WE

How To Create Effective WE?

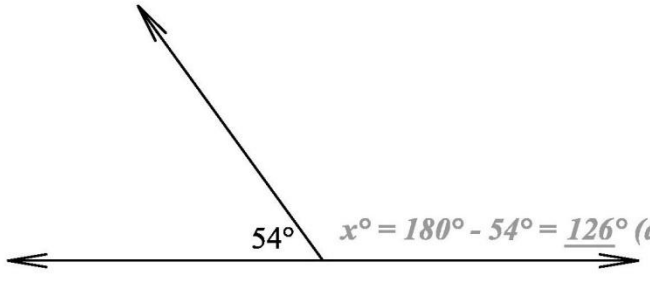
- Self-explanation effect
- Split attention effect
- Redundancy effect
- Modality effect
- Expertise reversal effect

Things to consider:

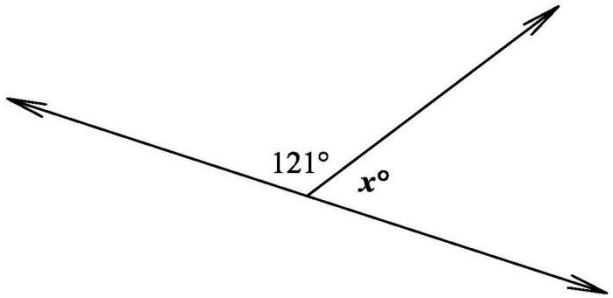
- Process oriented or product oriented
- Mutiple or uniform solutions
- Individual or group learning

Problem: Find the value of x and give reasons for each step.

Solution:



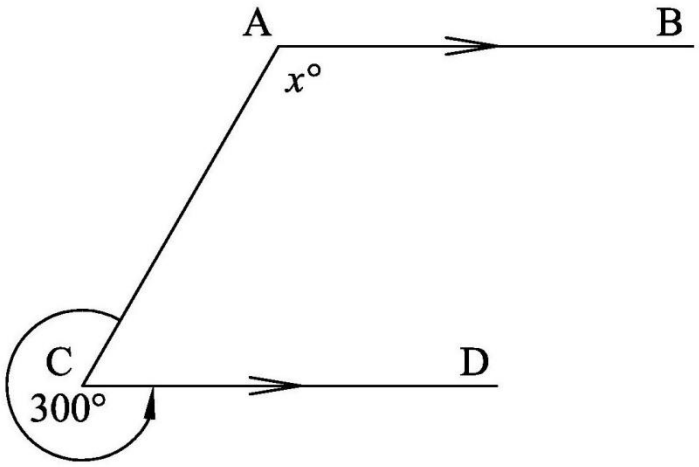
x° = 180° - 54° = 126° (adjacent angles on a straight line sum to 180°)



Problem: Find the value of x and give reasons for each step.

Solution:

Problem: Find the value of x and give reasons for each step.



Solution:

1. $\angle ACD = 360° - 300° = 60°$

   Reason: angles formed by lines running to the same point sum to 360°

2. $\angle BAC = 180° - \angle ACD = 180° - 60° = 120°$

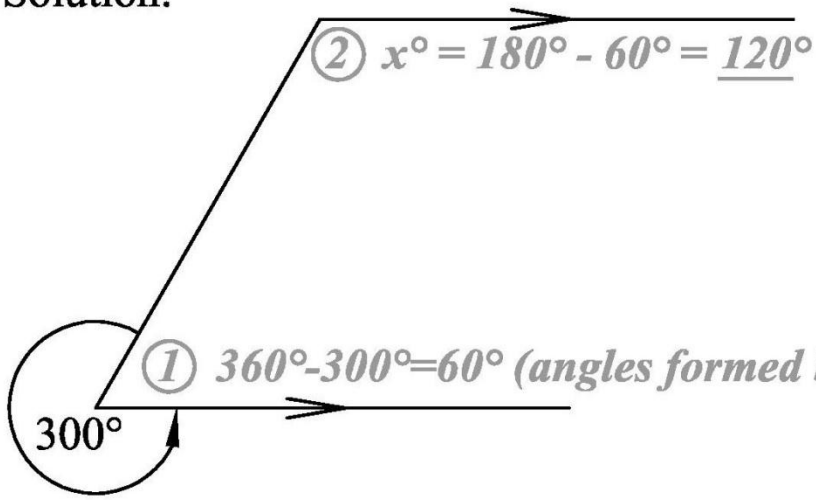
   Reason: cointerior angles between parallel lines sum to 180°

3. Therefore, $x = 120°$

Integrated

Problem: Find the value of x and give reasons for each step.

Solution:



② $x° = 180° - 60° = \underline{120}°$ *(cointerior angles between parallel lines sum to 180°)*

① *360°-300°=60° (angles formed by lines running to the same point sum to 360°)*

A car moving from rest reaches a speed of 20 m/s after 10 seconds. What is the acceleration of the car?

$u = 0$ m/s
$v = 20$ m/s
$t = 10$ s

$v = u + at$
$a = (v - u)/t$
$a = (20 - 0)/10$
$a = 2$ m/s^2

A car moving from rest (u) reaches a speed of 20 m/s (v) after 10 seconds (t): [$v = u + at$, $a = (v - u)/t = (20 - 0)/10 = 2$ m/s^2]. What is the acceleration of the car?

YOUR TASK.

- Practive improving problem solving based instruction