

Problem Solving in Mathematics

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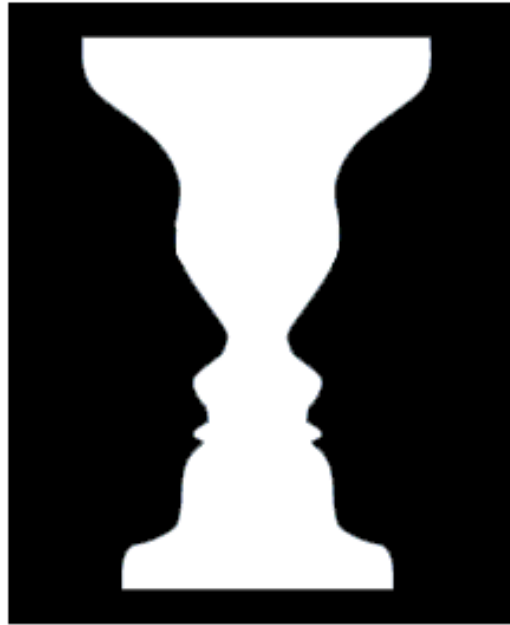
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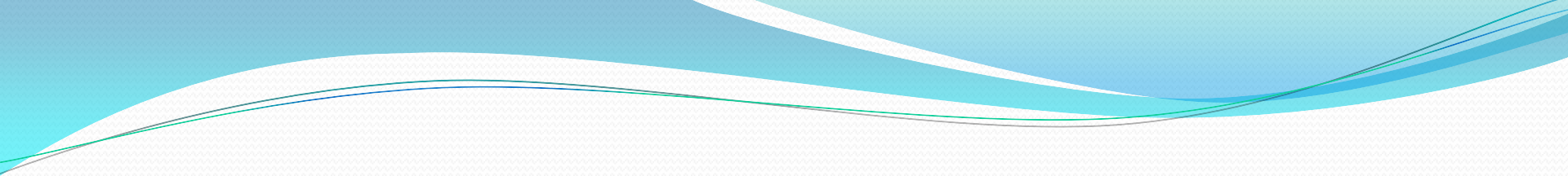
Day – 2 on 9 October 2014. QITEP in Mathematics, PPPPTK Yogyakarta
Differentiated Instruction For Senior High School Mathematics Teachers

Outline

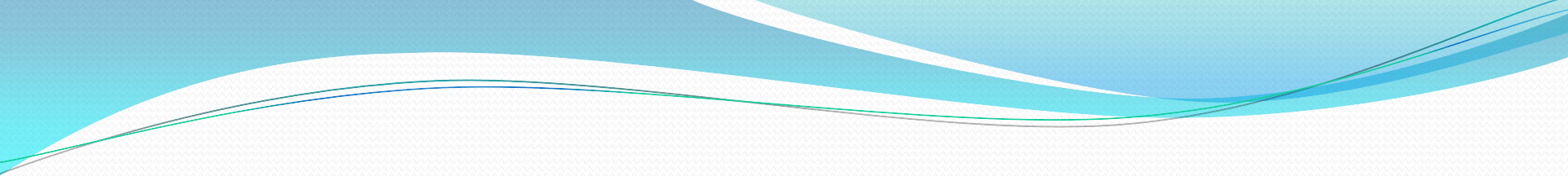
- Recall last discussions
- Worked example based learning
- Expertise reversal effect
- Example of effective worked examples
- Learning WE in group-works
- EXERCISE – PBL for new concepts in mathematics

Try to rehearse what we have learned yesterday!





Suppose five days after the day before yesterday is Friday.
What day of the week is tomorrow?



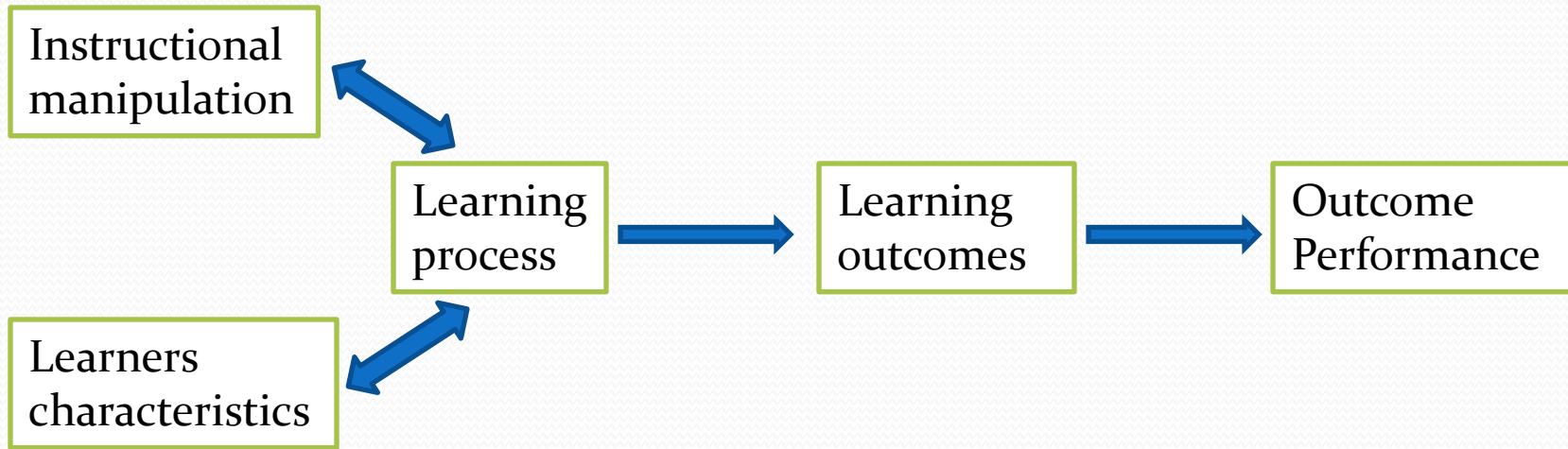
Two points on the surface of the unit sphere (in 3-space) are connected by an arc A which passes through the interior of the sphere. Prove that if the length of A is less than 2 , then there is a hemisphere H which does not intersect A .

HEURISTIC (RULE OF THUMBS)

“General problem solving strategy”

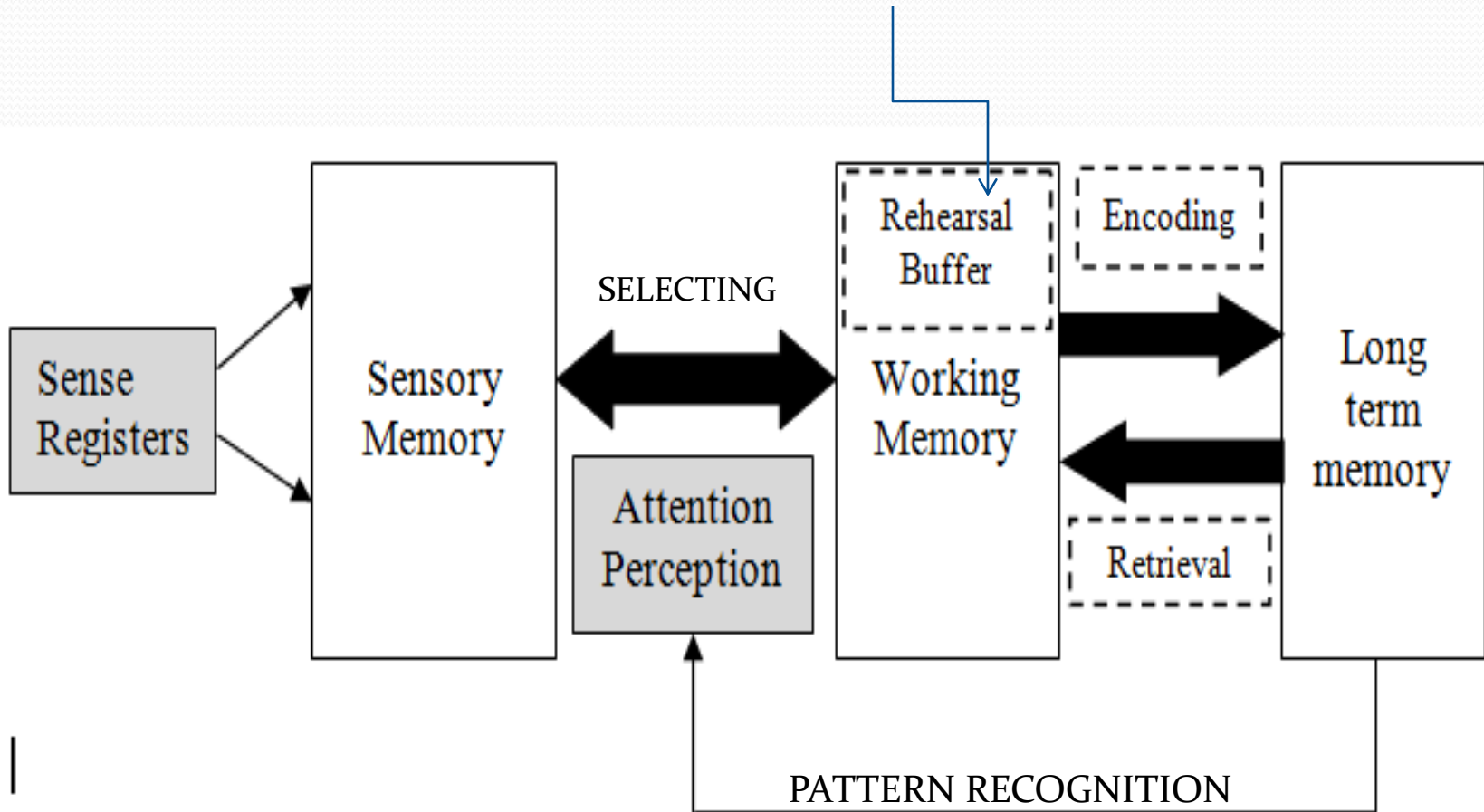
- Pattern recognition
- Working backwards
- Guess and test
- Simulation or experimentation
- Reduction/expansion
- Organised listing/exhaustive listing
- Logical deduction
- Divide and conquer

Classroom activity



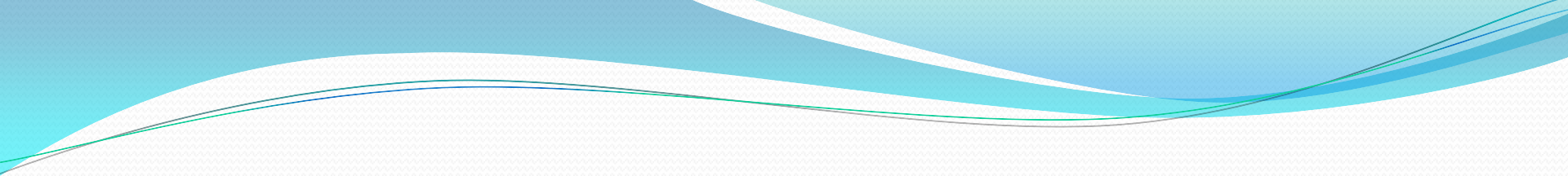
LEARNING PROCESS:

organise information, build connection among information and integration with prior knowledge, and eventually construct knowledge, encode knowledge to LTM

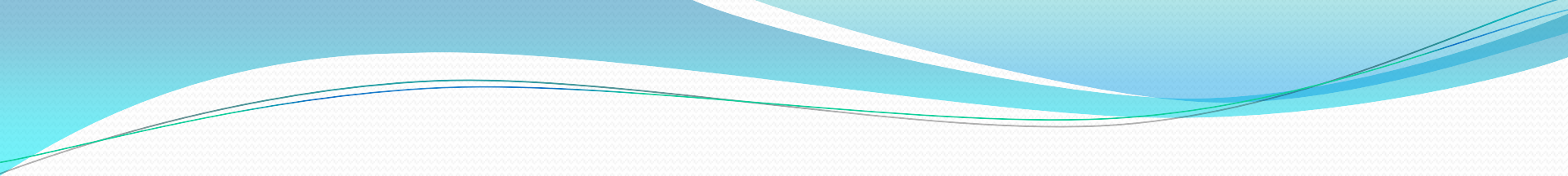


Tôi đã được học về cách học sinh học trong khi làm giải quyết vấn đề . Học tập được xác định là xây dựng kiến thức và tự động hóa các nội dung toán học; mặc dù học có thể xây dựng các kiến thức về kỹ năng giao tiếp và kỹ năng tự điều chỉnh.

- 私は、問題解決をしながら、生徒が学習方法について学習されています。学習は、知識の構築と自動化数学コンテンツとして定義されます。学習は、コミュニケーション能力や自主規制スキルに知識を構築するかも知れません。
- *Watashi wa, mondaikaiketsu o shinagara, seito ga gakushū hōhō ni tsuite gakushū sa rete imasu. Gakushū wa, chishiki no kōchiku to jidō-ka sūgaku kontentsu to shite teigi sa remasu. Gakushū wa, komyunikēshon nōryoku ya jishu kisei sukiru ni chishiki o kōchiku suru kamo shiremasen.*



Saya telah belajar tentang bagaimana siswa belajar sambil melakukan pemecahan masalah . Belajar didefinisikan sebagai konstruksi pengetahuan dan otomatisasi isi matematika; meskipun belajar bisa membangun pengetahuan tentang keterampilan komunikasi atau keterampilan mengatur diri untuk belajar.



I have been learning about how student learns while doing problem solving. Learning is defined as knowledge construction and automation the mathematics content; though learning can be constructing knowledge on communication skills or self-regulatory skills.

PRIOR KNOWLEDGE

- Guide our perception, attention and pattern recognition
- Determine which information to be assigned in working memory (short term memory)
- To be re-constructed using the attended information in working memory
- To be automated in working memory through rehearsal and practice
- To be stored permanently in long term memory and
- To be used for further problem solving

Geary (2002)

- Biologically primary-knowledge
- Biologically secondary-knowledge

Alternative of PBL for novice learners

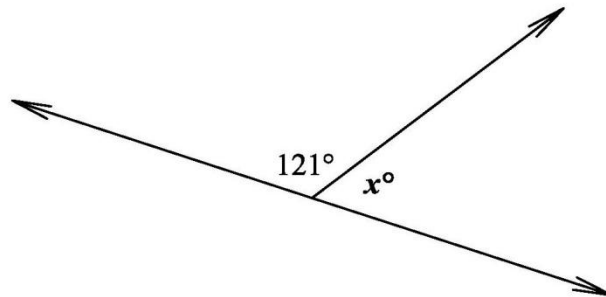
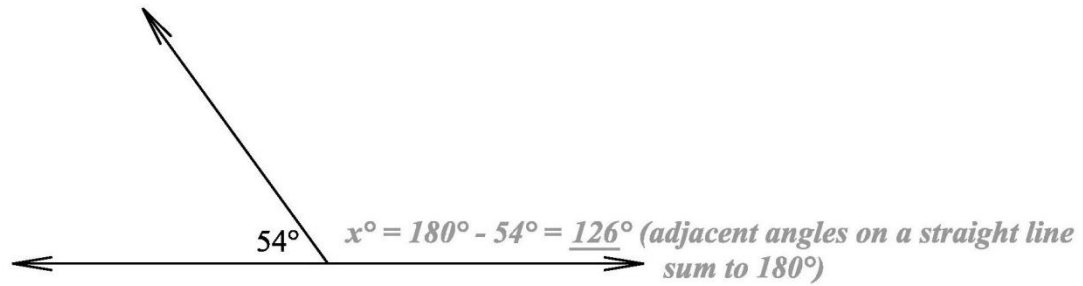
- Goal free problem
- Worked example based learning
 - WE only
 - A block of WE and then a block of PS
 - Pairing WE and similar PS
 - Faded WE

How To Create Effective WE?

- Take what research has showed:
 - Self-explanation effect
 - Split attention effect
 - Redundancy effect
 - Modality effect
 - Expertise reversal effect
- Teacher can consider:
 - Process oriented or product oriented
 - Multiple or uniform solutions
 - Individual or group learning

Problem: Find the value of x and give reasons for each step.

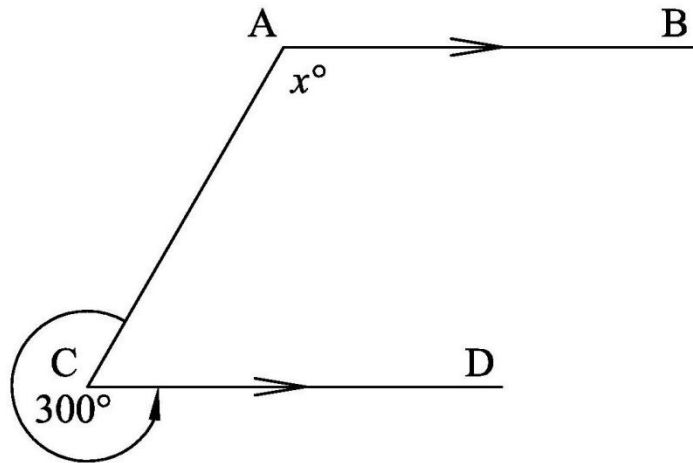
Solution:



Problem: Find the value of x and give reasons for each step.

Solution:

Problem: Find the value of x and give reasons for each step.



Solution:

1. $\angle ACD = 360^\circ - 300^\circ = 60^\circ$

Reason: angles formed by lines running to the same point sum to 360°

2. $\angle BAC = 180^\circ - \angle ACD = 180^\circ - 60^\circ = 120^\circ$

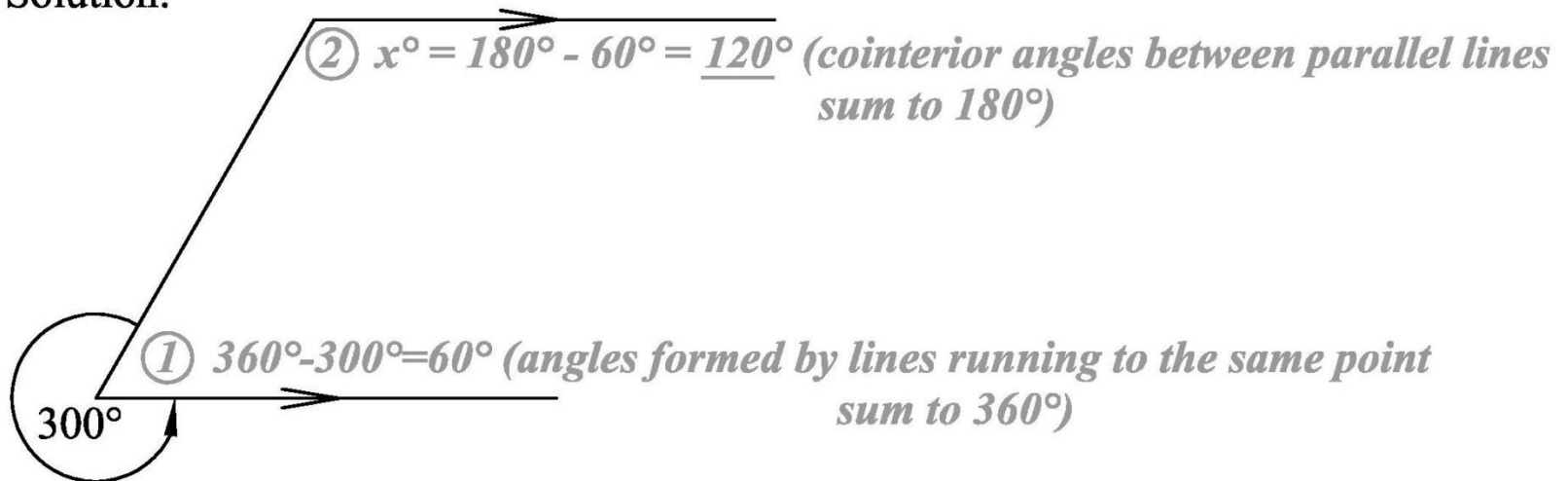
Reason: cointerior angles between parallel lines sum to 180°

3. Therefore, $x = 120^\circ$

Integrated

Problem: Find the value of x and give reasons for each step.

Solution:



A car moving from rest reaches a speed of 20 m/s after 10 seconds. What is the acceleration of the car?

$$u = 0 \text{ m/s}$$

$$v = 20 \text{ m/s}$$

$$t = 10 \text{ s}$$

$$v = u + at$$

$$a = (v - u)/t$$

$$a = (20 - 0)/10$$

$$a = 2 \text{ m/s}^2$$

A car moving from rest (u) reaches a speed of 20 m/s (v) after 10 seconds (t): [$v = u + at$, $a = (v - u)/t = (20 - 0)/10 = 2 \text{ m/s}^2$]. What is the acceleration of the car?

No	Study this example	Practice this problem:	Final answer
1.	$3p + 10 = 85$ $\Leftrightarrow 3p + 10 - 10 = 85 - 10$ [Subtract both sides by 10] $\Leftrightarrow 3p + 0 = 75$ $\Leftrightarrow 3p = 75$ $\Leftrightarrow \frac{3p}{3} = \frac{75}{3}$ [Divide both sides by 3] $\Leftrightarrow p = 25$ Hence, the solution is $p = 25$.	$4a + 13 = 65$	$a = 13$

Individual or group problem solving?



Group work structure where they possess similar prior-knowledge and sharing is simply encouraged.

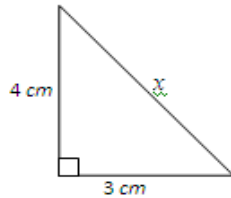


Group work structure where they possess different prior-knowledge and sharing is a must because these prior knowledge are requisited simultaneously to solve and learn the problem.



Study this example.

Find the length of the hypotenuse, x .



Solution:

$$x = \sqrt{4^2 + 3^2}$$

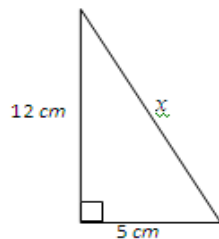
$$x = \sqrt{16 + 9}$$

$$x = \sqrt{25}$$

$$x = 5 \text{ cm}$$

Solve this problem.

Find the length of the hypotenuse, x .

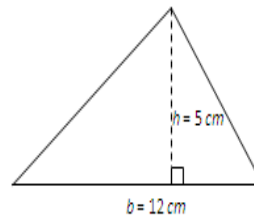


Solution:

D₁

Study this example.

Find the area of an acute triangle with a base of 12 cm and a height of 5 cm.



Solution:

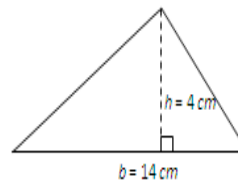
$$A = \frac{1}{2} \times b \times h$$

$$A = \frac{1}{2} \times 12 \text{ cm} \times 5 \text{ cm}$$

$$A = 30 \text{ cm}^2$$

Solve this problem.

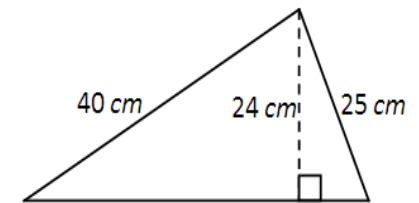
Find the area of an acute triangle with a base of 10 cm and a height of 4 cm.



Solution:

D₂

Find the area of a triangle with dimensions as shown in the figure below.



Solution:

D₁+D₂

Group structure



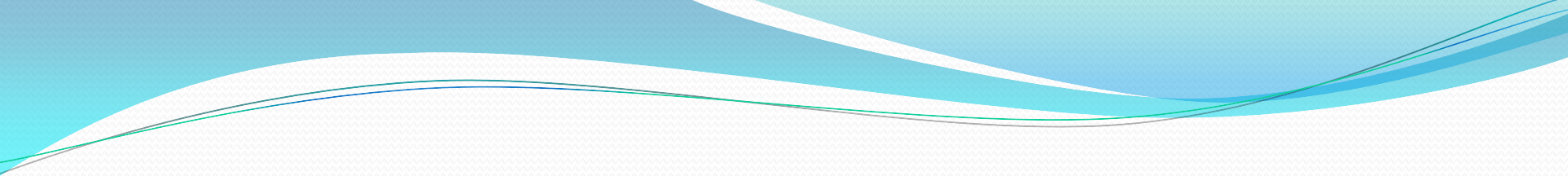
Sharing is encouraged



Sharing is a must

I expect you somewhat
already construct
knowledge about recent
consideration of using
problem solving in
mathematics classroom!





**SOMETIMES WE
EXPECT MORE FROM
OUR STUDENTS
BECAUSE WE WOULD
BE WILLING TO DO
MUCH FOR THEM**



It is now your turn to:

- Re-construct
- Re-learn
- Automate

of what you have learned!

In your group:

Practice designing an instruction using improvised problem based learning

- Decide the learning topic
- We assume it is a new topic for the students
- Identify their prior knowledge
- Then, each group member creates the worked-example/problem solving
- Discuss the instruction from each member and then
- Put them altogether, in order, on POSTER.

Choose topics for Senior High School on:

1. Plane geometry
2. Spatial geometry
3. Statistics
4. Algebra
5. Arithmetics
6. Calculus