SYSTEM REPRESENTATION

In this topic, we will learn:

- Important definitions of block diagrams and signal flow graphs: signals, system, summing junctions, pickoff points, series, cascade and feedback forms.

- Techniques of simplifying block diagrams.

- Signal flow graphs.

- Changing block diagrams to signal flow graphs and vice versa.

- Mason's Rule and example questions.

INTRODUCTION

- A control system consists of the inter-connection of subsystems.
- A more complicated system will have many inter-connected subsystems.
- For the purpose of analysis, we want to represent the multiple subsystems as a single transfer function.
- A system with multiple subsystems can be represented in two ways:
  - Block diagrams
  - Signal flow graphs

BLOCK DIAGRAMS

- The basic components in a block diagram are:

<u>Signals</u>

$$R(s) \longrightarrow \qquad C(s) \longrightarrow$$

The direction of signal flow is shown by the arrow.

<u>System blocks</u>

$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$$
Input   System   Output
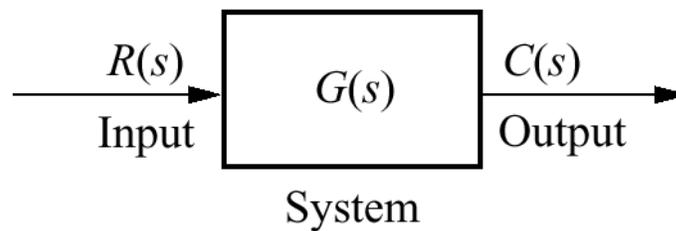
The system block represented by a transfer function.

<u>Summing junctions</u>

$$R_1(s) \; + \; \bigotimes \; C(s) = $$
$$R_2(s) \quad + \quad -$$
$$R_3(s)$$
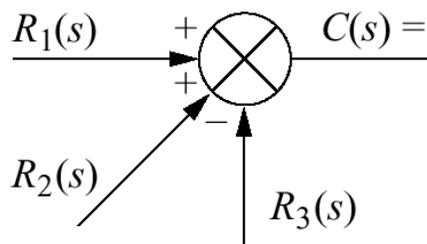
The signals are added/subtracted algebraically

Pickoff points
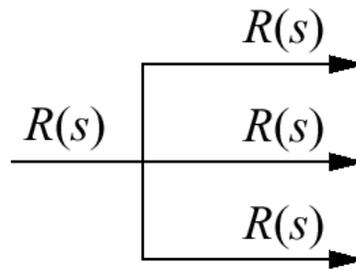
The same signal is distributed to other subsystems

- The subsystems in a block diagram are normally connected in three forms:

Cascade form
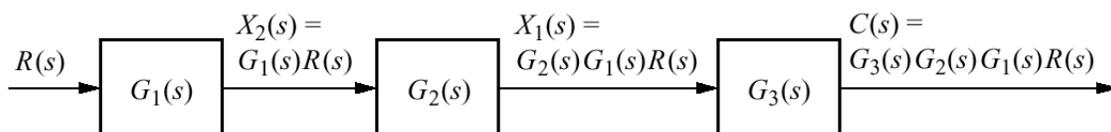
The block diagram can be reduced into a single block by multiplying every block to give:

Parallel form
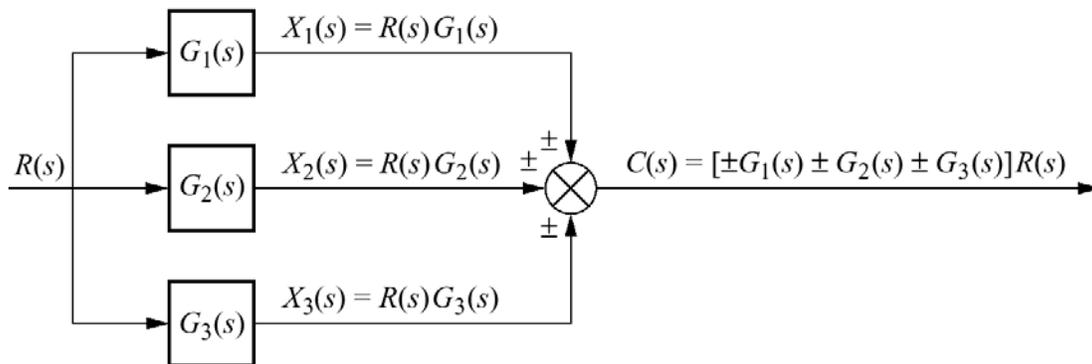
The block diagram can be reduced into a single block by summing every block to give:

Feedback form
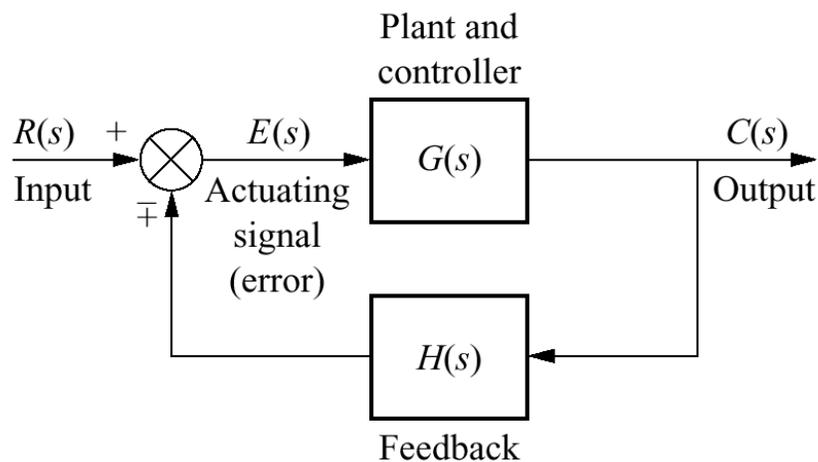
- closed-loop transfer function:

- open-loop transfer function:

Moving blocks to create familiar forms

- It is not always apparent to get block diagrams in the familiar forms.

- We have to move blocks to get the familiar forms in order to be able to reduce the block diagram into single transfer function.

- Moving the summing junction to the front of a block

(*a*)
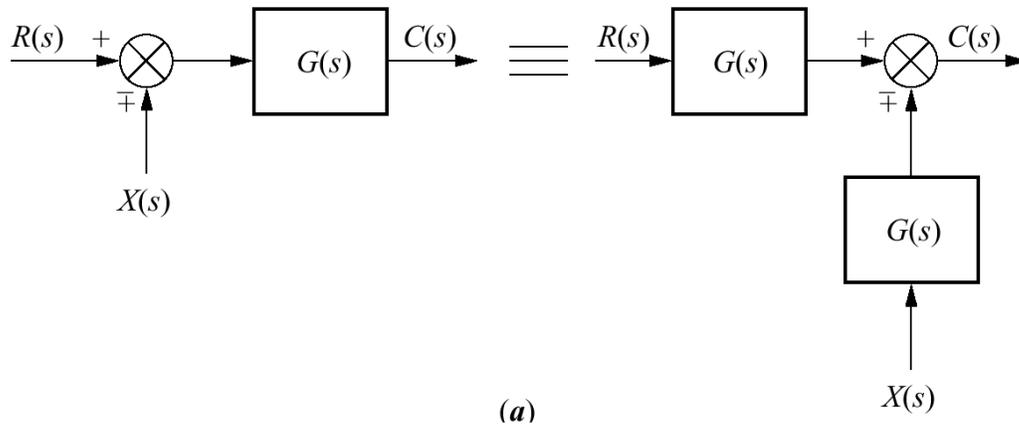
● Moving the summing junction to the back of a block

(*b*)
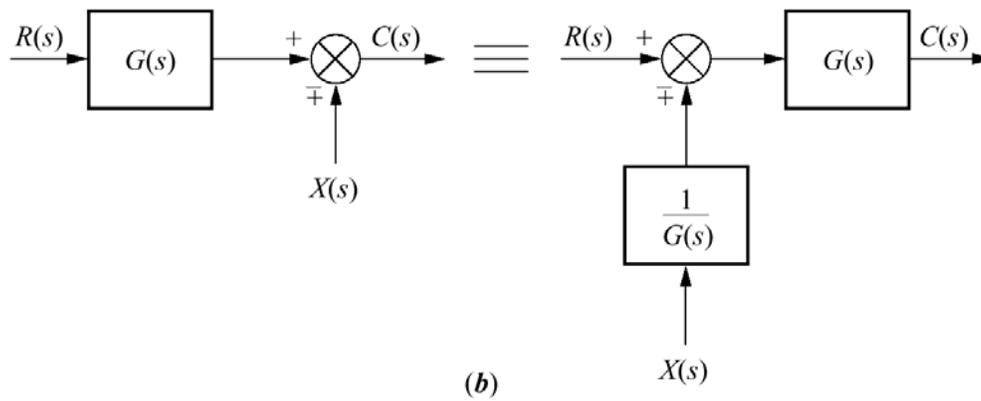
- Moving pick-off point to the front of a block
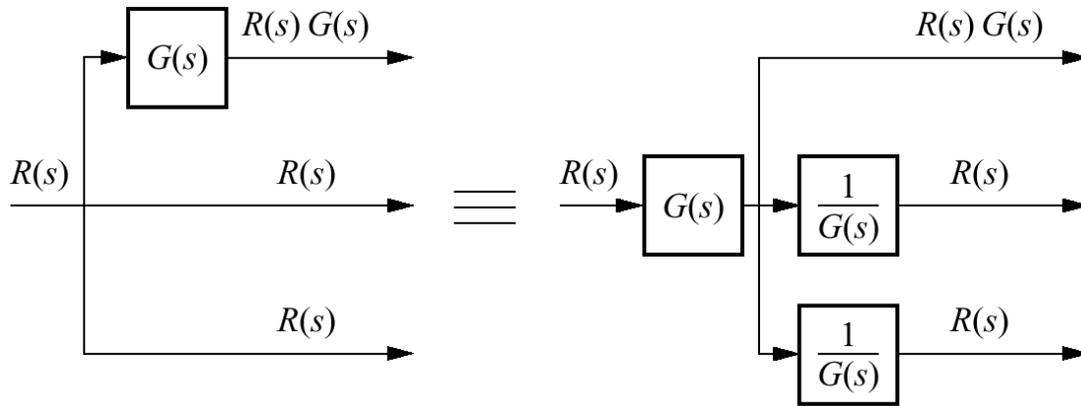
- Moving pick-off point to the back of a block
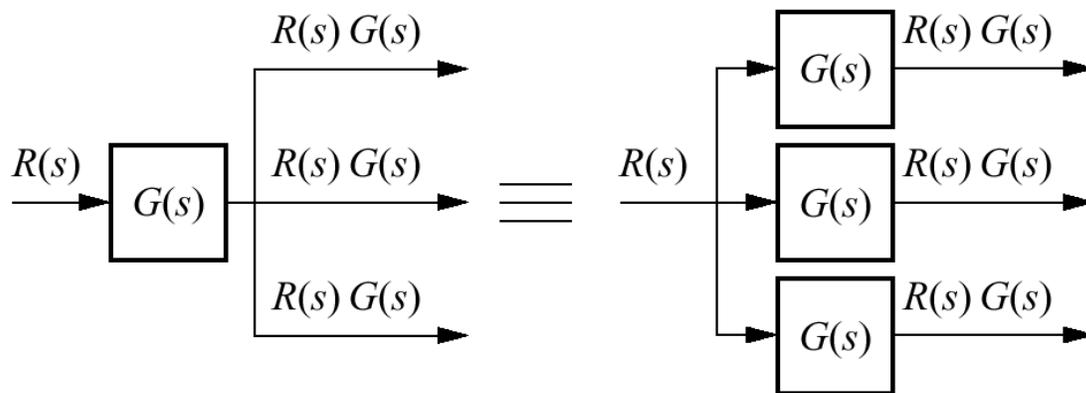
Example (5.1): Reduce the following block diagram into a single transfer function
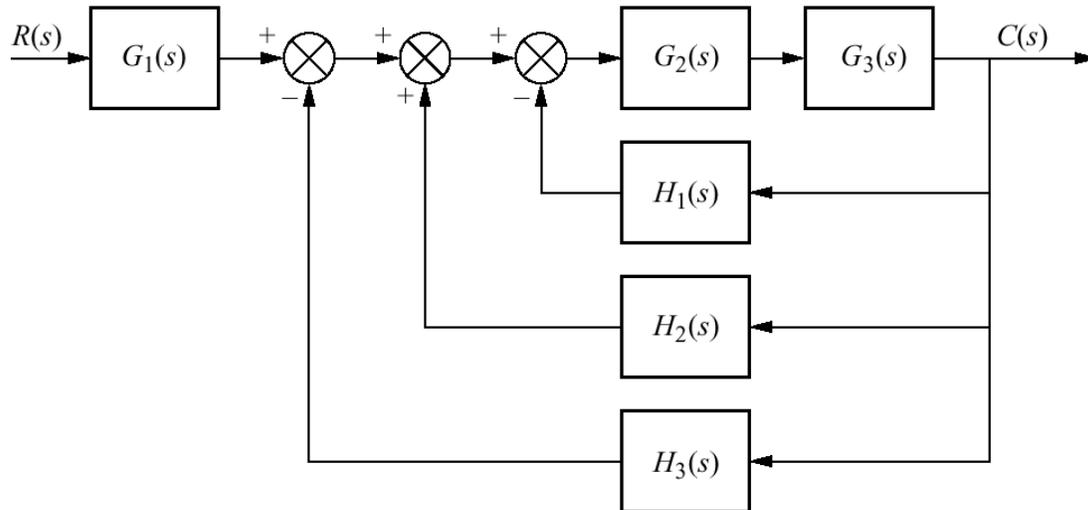
Example (5.2): Reduce the following block diagram into a single transfer function.
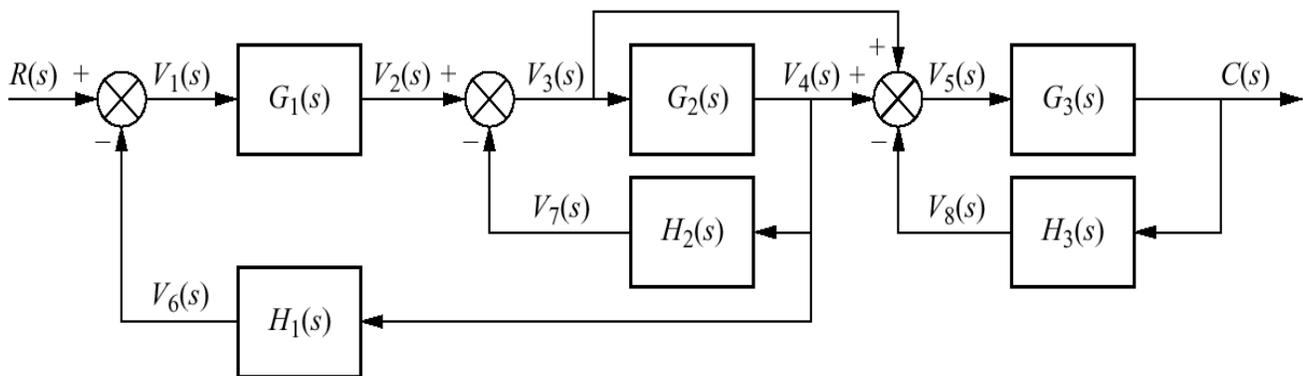
SIGNAL FLOW GRAPHS

- An alternative to block diagrams.
- It consists only of branches to represent systems and nodes to represents signals.

Branches

- Represented by a line with arrow showing the direction of signal flow through the system

$$G(s)$$

- The transfer function is written close to the line and arrow

Nodes

- Represented by a small circle with the signal's name is written adjacent to the node

$$V(s)$$

Example:
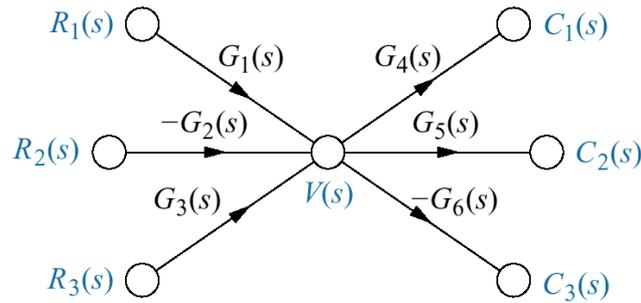
Converting block diagrams into signal-flow graphs

- The block diagrams in cascade, parallel and feedback forms can be converted into signal-flow diagrams.

- We can start with drawing the signal nodes, and then interconnect the signal nodes with system branches.

Cascade form

Parallel form

$$X_1(s) = R(s)G_1(s)$$

$$X_2(s) = R(s)G_2(s)$$

$$X_3(s) = R(s)G_3(s)$$

$$C(s) = [\pm G_1(s) \pm G_2(s) \pm G_3(s)]R(s)$$

Feedback form

Plant and controller

$R(s)$ Input $E(s)$ Actuating signal (error) $G(s)$ $C(s)$ Output

$H(s)$
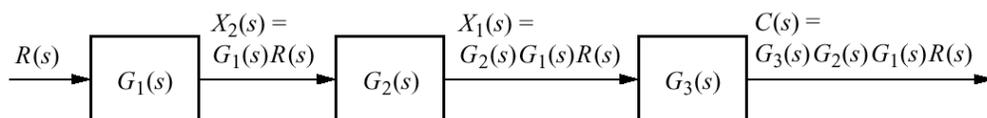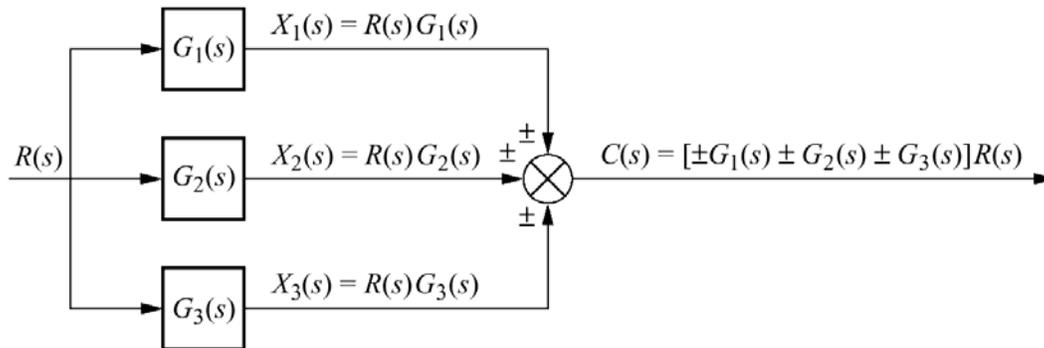
Feedback
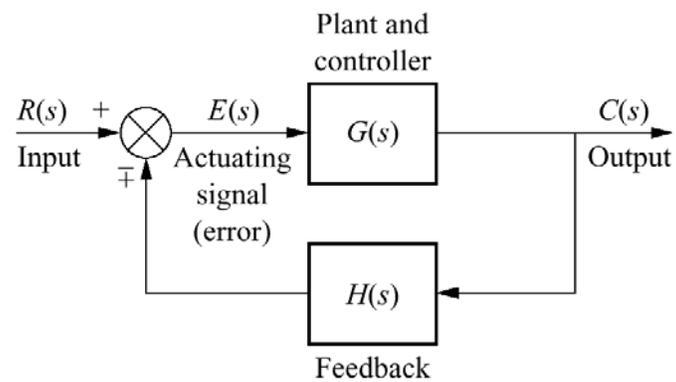
Example (5.6): Convert the following block diagram into signal-flow graph
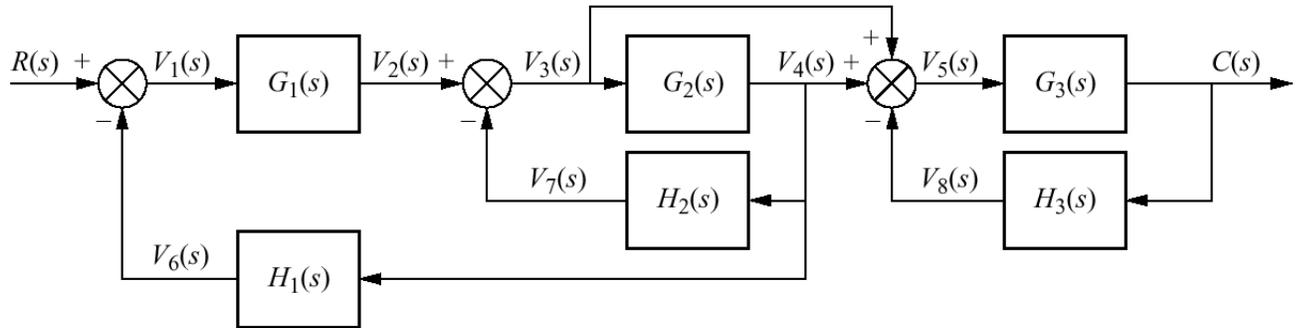
Mason's Rule

- Mason's rule is used to get the single transfer function in the signal-flow graph.

- We need to know some important definitions:
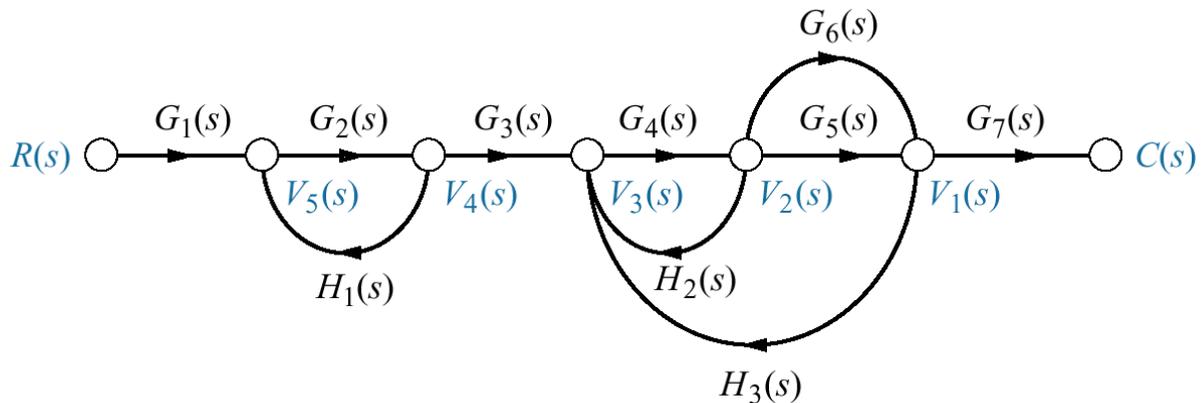
  - Loop – a closed path which starts and ends at the same node.
  - Loop gain – the product of branch gains found by traversing a loop.

  - Forward-path – a path from the input node to the output node of the signal-flow graph in the direction of signal flow.

- ▪ Forward-path gain – the product of gains found by traversing a path from the input node to the output node of the signal-flow graph in the direction of signal flow.

- ▪ Non-touching loops – loops that do not have any nodes in common.

- ▪ Non-touching loops gain – the product of loop gains from non-touching loops taken two, three, four, or more at a time.

Mason's rule:

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum\limits_{k} T_k \Delta_k}{\Delta}$$

k = number of forward path

T_k = the k-th forward-path gain

Δ = 1 - Σ loop gains + Σ non-touching loops gain (taken 2 at a time) - Σ non-touching loops gain (taken 3 at a time) + Σ non-touching loops gain (taken 4 at a time) - ...

Δ_k = Δ - Σ loop gain terms in Δ that touch the k-th forward path. In other words, Δ_k is formed by eliminating from Δ those loop gains that touch the k-th forward path.

Example (5.7): Find the transfer function, C(s)/R(s) of the following signal-flow graph,