1.1 THE REAL NUMBER SYSTEM

1.1 Real Number System

Calculus is based on real number system and its properties.
The simplest number system: **natural numbers**, which are 1, 2, 3,..

- The set of natural numbers is usually denoted as $\mathbb{N}$. So
  $\mathbb{N}$ = {1, 2, 3, 4, …}

If we add all of their negatives and 0, then we get **integer numbers**, which includes …, –3, –2, –1, 0, 1, 2, 3, …

- The set of all integers numbers is with $\mathbb{Z}$. Thus,
  $\mathbb{Z}$ = {…, –3, –2, –1, 0, 1, 2, 3, …}

How about if we want to measure the length or weight of an object? We need **rational numbers**, such as 2.195, 2.4, dan 8.7.

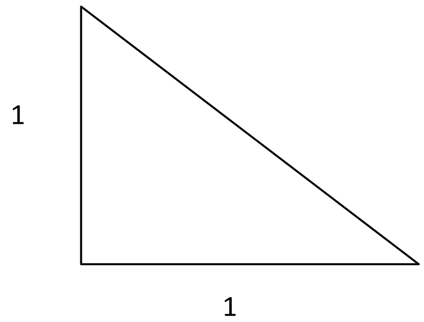
- **Rational number**: *a number that can be written as* $\frac{a}{b}$ *where a and b are integers and b* ≠ 0.

Then, integers are also rational numbers, i.e. 3 is a rational number because it can be written as $\frac{2}{6}$.

- The set of all rational numbers are denoted by $\mathbb{Q}$:

$$\mathbb{Q} = \left\{ \frac{a}{b} \middle| a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0 \right\}$$

What about the length of hypotenuse of this triangle?



Using **irrational numbers** this thing would be easy. Other examples of irrational numbers are $\sqrt{3}$, $\sqrt{7}$, *e* and π.

- The set of all rational and irrational numbers with their negatives and zero is called **real numbers,** & it is denoted as $\mathbb{R}$.

- Relation between those four set $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ an be defined as

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

- Rational/irrational number: can be written as decimal. What's the difference?
    - Rational number:

$$\frac{1}{2} = 0{,}5$$

$$\frac{13}{11} = 1{,}181818\ldots$$

$$\frac{3}{8} = 0{,}375$$

$$\frac{3}{7} = 0{,}428571428571428571\ldots$$

- Irrational number:

$$\sqrt{2} = 1{,}4142135623 \ldots$$
$$\pi = 3{,}1415926535 \ldots$$

Difference: repeated decimal is a rational number. ie. $x = 0.136136136 \ldots$

1.2 Number Operations

Given , $y \in \mathbb{R}$, then we already knew such operations: *addition* (x + y) & *multiplication* (x .y or xy).

- The properties of those two operation in $\mathbb{R}$ are:

1) Commutative: $x + y = y + x$ & $xy = yx$.
2) Assosiative: $x + (y + z) = (x + y) + z$ & $x(yz) = (xy)z$.
3) Distributive: $x(y + z) = xy + xz$.
4) Identity elements:
    - for addtition: 0 because $x + 0 = x$.
    - for multiplication: 1 because $x.1 = x$.
5) Invers:
    - Every $x \in \mathbb{R}$ has an *additive invers* (disebut juga *negatif*) $-x$ such that $x + -x = 0$.
    - Every $x \in \mathbb{R}, x \neq 0$ has a *multiplicative invers* (disebut juga *balikan*) x^{-1} such that $x.x^{-1} = 1$.

1.3 Order

Non zero real numbers can be divided into 2 different sets: positive real and negative real numbers. Based on this fact, we introduce *ordering* relation < (read "less than") defined as:

$$x < y \text{ jika dan hanya jika } y - x \text{ positif.}$$

- $x < y$ has the same meaning with $y > x$.

- Properties of Order:

1) *Trichotomous*: for $\forall x, y \in \mathbb{R}$, exactly one of $x < y$ atau $x = y$ atau $x > y$ *holds*.
2) Transitive: If $x < y$ and $y < z$ then $x < z$.
3) Addition: $x < y \Leftrightarrow x + z < y + z$
4) Multiplication:

$$If\ z\ positif\ then\ x < y \Leftrightarrow xz < yz$$
$$If\ z\ negatif\ then\ x < y \Leftrightarrow xz > yz$$

1.3 Inequalities

- Inequality is an open sentence that uses <, >, ≤ or ≥ relation.
- The solution of an inequality is all real numbers that satisfies the inequality, which usually in the form of an interval atau union of intervals.

Some common intervals:

Penulisan Interval	Penulisan Himpunan	Dalam Garis Bilangan
(a, b)	$\{x \in \mathbb{R} \mid a < x < b\}$	
$[a, b]$	$\{x \in \mathbb{R} \mid a \leq x \leq b\}$	
$[a, b)$	$\{x \in \mathbb{R} \mid a \leq x < b\}$	
$(a, b]$	$\{x \in \mathbb{R} \mid a < x \leq b\}$	
$(-\infty, b)$	$\{x \in \mathbb{R} \mid x < b\}$	
$(-\infty, b]$	$\{x \in \mathbb{R} \mid x \leq b\}$	
(a, ∞)	$\{x \in \mathbb{R} \mid x > a\}$	
$[a, \infty)$	$\{x \in \mathbb{R} \mid x \geq a\}$	
$(-\infty, \infty)$	$\mathbb{R}$	

How to solve the inequalities?

- We can add the same number to both sides of inequality.
- We can multiply a positive number to both sides of inequality.
- We can multiply a negative number to both sides of inequality, and the order relation is inversed.

Example of Inequalities

1) $2x - 7 < 4x - 2$
2) $-5 \leq 2x + 6 < 4$
3) $x^2 - x - 6 < 0$
4) $3x^2 - x - 2 > 0$
5) $\frac{2x-5}{x-2} \leq 1$

Contoh 1

Find the solution of $2x - 7 < 4x - 2$.

Solution:

$$2x - 7 < 4x - 2$$
$$\Leftrightarrow 2x < 4x + 5$$
$$\Leftrightarrow -2x < 5$$
$$\Leftrightarrow x > -\frac{5}{2}$$

Solution: $interval\ (-\frac{5}{2}, \infty) = \left\{x \middle| x > -\frac{5}{2}\right\}$