

2.6. TEOREMA LIMIT UTAMA

Andaikan n bil. bulat positif, k konstanta, f & g fungsi-fungsi yg mempunyai limit di c . Maka:

$$1. \lim_{x \rightarrow c} k = k$$

$$2. \lim_{x \rightarrow c} x = x$$

$$3. \lim_{x \rightarrow c} kf(x) = k \lim_{x \rightarrow c} f(x)$$

$$4. \lim_{x \rightarrow c} [f(x) \pm g(x)] = \lim_{x \rightarrow c} f(x) \pm \lim_{x \rightarrow c} g(x)$$

$$5. \lim_{x \rightarrow c} [f(x) \cdot g(x)] = \lim_{x \rightarrow c} f(x) \cdot \lim_{x \rightarrow c} g(x)$$

$$6. \lim_{x \rightarrow c} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)}, \text{ asal } \lim_{x \rightarrow c} g(x) \neq 0$$

$$7. \lim_{x \rightarrow c} [f(x)]^n = \left[\lim_{x \rightarrow c} f(x) \right]^n$$

$$8. \lim_{x \rightarrow c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow c} f(x)},$$

asal $\lim_{x \rightarrow c} f(x) > 0$ jk n genap.

Contoh:

$$1. \lim_{x \rightarrow -1} 3x^2 - 1 = \lim_{x \rightarrow -1} 3x^2 - \lim_{x \rightarrow -1} 1$$

$$\begin{aligned}
 &= 3 \lim_{x \rightarrow -1} x^2 - 1 = 3 \left[\lim_{x \rightarrow -1} x \right]^2 - 1 \\
 &= 3(-1)^2 - 1 = 2
 \end{aligned}$$

$$\begin{aligned}
 2. \quad &\lim_{x \rightarrow -2} \sqrt[4]{x^3 + 6x} = \sqrt[4]{\lim_{x \rightarrow -2} x^3 + 6x} \\
 &= \sqrt[4]{\lim_{x \rightarrow -2} x^3 + \lim_{x \rightarrow -2} 6x} = \sqrt[4]{-8 - 12} \\
 &= \sqrt[4]{-20} \text{ (tidak ada).} \\
 \therefore &\lim_{x \rightarrow c} f(x) \text{ harus } > 0 \text{ jk n genap (sifat 8).}
 \end{aligned}$$

Teorema Substitusi

Jika suatu f fungsi polinom atau fungsi rasional, maka

$$\lim_{x \rightarrow c} f(x) = f(c)$$

asalkan dlm kasus fs. rasional, penyebut $\neq 0$.

Contoh:

$$\begin{aligned}
 1. \quad &\lim_{x \rightarrow 1} x^3 + 2x^2 + x + 6 \\
 &= (1)^3 + 2(1)^2 + 1 + 6 = 10
 \end{aligned}$$

Soal Tambahan

$$1. \lim_{x \rightarrow 0} \frac{5x^3 + 8x^2}{3x^4 - 16x^2}$$

$$2. \lim_{y \rightarrow 1} \frac{y^3 - 1}{y - 1}$$

$$3. \lim_{y \rightarrow 4} \frac{y - 4}{\sqrt{y} - 2}$$

$$4. \lim_{x \rightarrow 2} \frac{1}{x-2} - \frac{4}{x^2-4}$$

$$5. \lim_{x \rightarrow 0^-} \frac{x}{|x|}$$

$$6. \lim_{x \rightarrow -2^-} \frac{x}{|x+2|}$$