7.7. FUNGSI TRIGONOMETRI & BALIKANNYA

Ingat kembali,

$\tan x = \frac{\sin x}{\cos x}$ $\qquad$ $\cot x = \frac{\cos x}{\sin x}$

$\sec x = \frac{1}{\cos x}$ $\qquad$ $\text{cosec } x = \frac{1}{\sin x}$

Turunan dari masing-masing fungsi tsb dpt dicari, yaitu:

Contoh: $D_x \cot x = D_x \left(\frac{\cos x}{\sin x}\right) = \frac{-\sin x . \sin x - \cos x . \cos x}{\sin^2 x}$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\csc^2 x.$$

- **Rumus Turunan**

$D_x (\sin x) = \cos x$ $\qquad$ $D_x (\sec x) = \tan x$

$D_x (\cos x) = -\sin x$ $\qquad$ $D_x (\text{cosec } x) = \csc x \cot x$

$D_x (\tan x) = \sec^2 x$ $\qquad$ $D_x (\cot x) = -\csc^2 x$

Jika $u = f(x)$, maka :

$$D_x(\sin u) = \cos u \,.\, D_x u$$

Hal ini berlaku pula pd fungsi-fungsi lainnya.

Contoh:

1. $D_x(\cos(3x^2 + 4))$

2. $D_x\left(\frac{x^2}{1-tan^2 x}\right)$

➢ Integral Fungsi Trigonometri

$\int \sin x \; dx = -\cos x + C$	$\int cosec^2 x \, dx = -\cot x + C$
$\int \cos x \; dx = \sin x + C$	$\int cosec\, x . \cot x \; dx$ $= -\operatorname{cosec} \; x + C$
$\int sec^2 x \, dx = \tan x + C$	$\int \tan x \, dx = -ln\lvert\cos x\rvert + C$
$\int \sec x . \tan x \; dx = \sec x + C$	$\int \cot x \; dx = ln\lvert\sin x\rvert + C$

➢ Fungsi Balikan Trigonometri

Ingat kembali fungsi balikan trigonometri:

$$y = \sin x \; \Leftrightarrow x = sin^{-1}y \; \Leftrightarrow x = arc \sin y$$

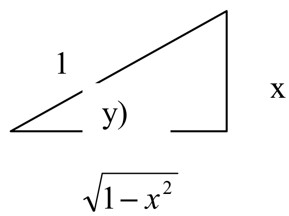
$$y = \cos x \; \Leftrightarrow x = arc \cos y \quad , o < x < \pi$$

$$y = \tan x \;\Leftrightarrow\; x = arc \tan y \quad , -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$y = \sec x \;\Leftrightarrow\; x = arc \sec y \quad , o < x < \frac{\pi}{2}, x \neq \frac{\pi}{2}$$

➢ **Turunan Fungsi Balikan Trigonometri**

➢ $y = \text{arc} \sin x \;\Leftrightarrow\; x = \sin y$

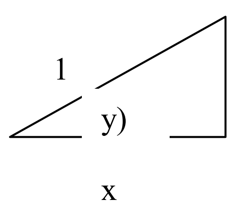


$$x = \sin y \rightarrow \frac{dx}{dy} = \cos \text{y} = \frac{\sqrt{1-x^2}}{1}$$

$$\rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$\therefore\; y = \text{arc} \sin x \;\rightarrow D_x\,(\text{arc} \sin \text{x}) = \frac{1}{\sqrt{1-x^2}}\;, -1 < x < 1$

➢ $y = \text{arc} \cos\; x \;\Leftrightarrow\; x = \cos y$



$$x = \cos y \rightarrow \frac{dx}{dy} = \ldots$$

$$\rightarrow \frac{dy}{dx} = \cdots$$

$\therefore\; y = \text{arc} \cos x \;\rightarrow D_x\,(\text{arc} \cos\; x) = \;\ldots \ldots . . , -1 < x < 1$

- $y = \text{arc tan } x \rightarrow D_x \ (\text{arc tan } x) = \dots\dots\dots\dots....$

- $y = \text{arc sec } x \rightarrow D_x \ (\text{arc sec } x) = \dots\dots\dots\dots.....$, $|x| > 1$

<u>Contoh</u>

1. $D_x \ (\text{arc cos } (x^2))$
2. $D_x \ \left(e^x . \text{arc sin } \left(\sqrt{x^3 + 1}\right)\right)$

Dari sebelumnya, diperoleh:

i. $\int \frac{1}{\sqrt{1-u^2}} \ du = arc \sin u + C$

ii. $\int \frac{-1}{\sqrt{1-u^2}} \ du = arc \cos u + C$

iii. $\int \frac{1}{1+u^2} \ du = arc \tan u + C$

iv. $\int \frac{1}{u\sqrt{u^2-1}} \ du = arc \sec u + C$

Jika $u = f(x)$, maka :

i. $\int \frac{1}{\sqrt{a^2-u^2}} \ du = arc \sin \left(\frac{u}{a}\right) + C$

ii. $\int \frac{1}{a^2+u^2}\, du = \frac{1}{a}\, arc\tan\left(\frac{u}{a}\right) + C$

iii. $\int \frac{1}{u\sqrt{u^2-1}}\, du = \frac{1}{a}\, arc\sec\left(\frac{|u|}{a}\right) + C$

Contoh:

1. $\int \frac{1}{1+4x^2} dx$
2. $\int \frac{1}{\sqrt{4-x^2}} dx$
3. $\int \frac{e^x}{1+e^{2x}} dx$
4. $\int \frac{5}{x^2-8x+25} dx$
5. $\int \frac{y}{\sqrt{16-9y^4}} dy$

8.3. SUBSTITUSI YANG MERASIONALKAN

❖ **Integral yg Melibatkan $\sqrt[n]{ax+b}$**

→ Substitusi $u = \sqrt[m]{ax+b}$ utk menghilangkan akar

Contoh:

1. $\int x.\sqrt[3]{x-4}\ dx$
2. $\int t(3t+2)^{3/2}\ dt$
3. $\int \frac{dx}{x-\sqrt{x}}$

❖ **Integral yg Melibatkan Bentuk $\sqrt{a^2 - x^2}$, $\sqrt{a^2 + x^2}$, $\sqrt{x^2 - a^2}$**

→ Substitusi *trigonometri* utk merasionalkan integral.

i. $\sqrt{a^2 - x^2}$, → Substitusi $x = a\sin t$, akan diperoleh:

$$\sqrt{a^2 - x^2} = \sqrt{a^2 - a^2 sin^2 t} = \sqrt{a^2(1 - sin^2 t)}$$

$$= a\cos t$$

ii. $\sqrt{a^2 + x^2}$, → Substitusi $x = a\tan t$, akan diperoleh:

$$\sqrt{a^2 + x^2} = \cdots \ldots \ldots \ldots.$$

iii. $\sqrt{x^2 - a^2}$ → Substitusi $x = a\sec t$, akan diperoleh:

$$\sqrt{x^2 - a^2} = \cdots \ldots \ldots \ldots$$

Contoh:

1. $\int \frac{1}{\sqrt{4-x^2}} dx$
2. $\int \frac{1}{\sqrt{x^2+4x+5}} dx$
3. $\int_2^3 \frac{1}{x^2\sqrt{x^2-1}} dx$