TUGAS 1 KALKULUS LANJUT

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LOGISTIC SEQUENCES

(Taken from Calculus: Concepts and Contexts by James Stewart)

A sequences that arises in ecology as a model for population growth is defined by the logistic equation

$$P_{n+1} = k.P_n(1 - P_n),$$

where P_n measures the size of the population of the n-th generation of a single species. To keep the numbers manageable, P_n is a fraction of the maximal size of the population, so $0 \leq P_n \leq 1$. Notice that the form of this equation is similar to the logistic differential equation. The discrete -model with sequences instead of continuous functions- is preferable for modeling insect populations, where mating and death occur in a periodic fashion.

An ecologist is interested in predicting the size of the population as time goes on, and ask the question: Will it stabilize at a limiting value? Will it change in a cyclical fashion? Or will it exhibit random behaviour?

Write a program to compute the first n terms of this sequence starting with an initial population P_0, where $0 < P_0 < 1$. Use this program to do the following

1. Calculate 20 or 30 terms of the sequence for $P_0 = \frac{1}{2}$ and for two values of k such that $1 < k < 3$. Graph the sequences. Do they appear to converge? Repeat for a different value of P_0 between 0 and 1. Does the limit depend on the choice of P_0? Does it depend on the choice of k?
2. Calculate terms of the sequence for a value of k between 3 and 3,4 and plot them. What do you notice about the behaviour of the terms?
3. Experiment with values of k between 3,4 and 3,5. What happens to the terms?
4. For values of k between 3,6 and 4, compute and plot at least 100 terms and comment on the behaviour of the sequence. What happens if you change P_0 by 0,001? This type of behaviour is called chaotic and is exhibited by insect populations under certain conditions.