The Fission Products (FP) group constant treatment for long-life Pb-Bi-cooled fast power reactors

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Abstract: In this study we focus on the Fission Products (FP) group constant treatment by considering around 50 of the most important nuclides. We then calculate the fission product effective yield for each of the modified chains and also generate one group constant using the System for Reactor Analysis Code (SRAC) and other methods (Origen, *etc.*). We use two approaches for investigating the important FP nuclides: the equilibrium model and the numerical solution for the time-dependent model. Based on the results we obtained three global patterns of the time-dependent atomic density change during burn-up for the considered nuclides. The first pattern is about nuclides that soon reach the asymptotic value, which can be grouped together by weight that may depend on parameters such as flux and power density. The second pattern includes nuclides that change during burn-up, with a nonlinear pattern, which can be combined into one group or more by nonlinear weight (quadratic, cubic, *etc.*). The third pattern is about nuclides that change in an almost linear way during burn-up, which can be grouped into two or more group constants by flux level, power level and time.

Keywords: asymptotic; flux; power density; group; weight; burn-up; nuclear energy.

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1 Introduction

Small and very small nuclear power plants with a moderate economic aspect are important candidates for electric power generation in many parts of the Third World, including outside the Java-Bali area in Indonesia. A nuclear energy system in the range 5–50 MWe matches the requirements of many cities and provinces outside the Java-Bali islands. In addition to electricity, a desalination plant or a cogeneration plant is a good candidate for nuclear energy applications. Madura Island is one place where the Indonesian government has planned to install a desalination plant for a clean water source. Owing to the difference in load between afternoon and night, the use of fast reactors is a better choice. Lead (Pb) and lead-bismuth (Pb-Bi)-cooled nuclear power reactors are now considered potential candidates for the next generation of nuclear power reactors. Various versions of lead-cooled nuclear power reactors have been analysed and safety analysis has also been applied to them. The results are generally satisfactory, as mentioned in Su'ud (2008a–b), Su'ud *et al.* (2005), Su'ud (1998) and Su'ud *et al.* (2007).

One important feature of lead/lead-bismuth cooled fast reactors is the zero burn-up core capability, which can eliminate possible superprompt critical accidents and make possible an inherent safety feature based on a reactivity feedback mechanism. The new design and safety approach, however, needs a high-quality system analysis as well as nuclear and material data to reduce calculation errors, so that their influence on the key design and safety parameters can be negligible. In the present research, benchmarking will be performed using various calculation systems and some experimental results. It is expected that the results will contribute to the achievement of the above goal.

2 Calculation methods

The current research focuses on solving the Fission Products (FP) treatment group constant with the following methods:

Method 1 Rigorous treatment: We cover 165 nuclides with other relevant FP nuclides in direct individual burn-up calculations. This method will give rigorous results but with considerable calculation time. However, this method is important to test other simpler methods.

Method 2 Lumped FP treatment: We build the best FP lumped cross-section for many general conditions and use this FP group constant in the burn-up calculation. This method can give accurate results if the spectrum is the same or near the spectrum to build the lumped FP cross-section.

Method 3 Combination method: We treat some of the most important nuclides individually and treat the rest of the FP using a lumped FP cross-section. This method seems to be good for general usage.

Method 4 Lumped FP cross-section with many interpolable parameters: We develop a concept similar to the background cross-section in the Bondanrenko-based cell calculation libraries. This will improve lumped FP cross-section results for general usage.

Method 5 We develop a few group-effective FP similar to those in the reactor kinetics problem. If we can get a few reasonably good group-effective FP then we can generally solve for all types of the core.

In general, our methodology consists of the following six steps:

Step 1 The important FP nuclides that have a strong influence on the overall FP cross-section are identified. Based on the study of Tabuchi and Aoyama (2000), we select the 50 most important nuclides for fast reactors. Based on this selection we then identify the relevant and important decay chains which should be considered.

Step 2 The important FP decay chains relevant to the important nuclides are identified.

Step 3 The contribution of each FP nuclide to the overall FP cross-section based on the equilibrium model is analysed.

Step 4 The contribution of each FP nuclide to the overall FP cross-section based on the time-dependent model is analysed.

Step 5 Based on the relevant and important decay chains, a differential equation for the model can be derived, and using the equilibrium approximation model, we can obtain the formula for the contribution of each nuclide to a certain flux level. The detailed process will be discussed in the next section.

Step 6 To see the process towards equilibrium in Section 4, the time-dependent change of each important nuclide is calculated. The calculation is performed based on the most important equation using analytical or numerical methods. A detailed explanation is given in Section 4.

The mathematical equations used in this study are as follows:

$$\frac{dN_{Kr5m}}{dt} = y85 * F\phi - \lambda_{Kr5m} N_{Kr5m} \tag{1}$$

$$\frac{dN_{Kr5}}{dt} = f_2 \lambda_{Kr5m} N_{Kr5m} - \lambda_{Kr5} N_{Kr5} \tag{2}$$

$$\frac{dN_{Kr5}}{dt} = (1 - f_2) \lambda_{Kr5m} N_{Kr5m} + \lambda_{Kr5} N_{Kr5} - \sigma_{aRb5} \phi N_{Rb5} \tag{3}$$

$$\frac{dN_{Y1}}{dt} = y91 * F\phi - \lambda_{Y1} N_{Y1} \tag{4}$$

$$\frac{dN_{Zr1}}{dt} = \lambda_{Y1} N_{Y1} - \sigma_{aZr1}\phi N_{Zr1} \tag{5}$$

$$\frac{dN_{Zr2}}{dt} = y92 * F\phi - \sigma_{aZr2}\phi N_{Zr2} + \sigma_{cZr1}\phi N_{Zr1} \tag{6}$$

$$\frac{dN_{Zr3}}{dt} = y93 * F\phi - \lambda_{Zr3} N_{Zr3} - \sigma_{aZr3}\phi N_{Zr3} + \sigma_{cZr2}\phi N_{Zr2} \tag{7}$$

$$\frac{dN_{Nb3}}{dt} = \lambda_{Zr3} N_{Zr3} - \sigma_{aNb3}\phi N_{Nb3} \tag{8}$$

$$\frac{dN_{Zr4}}{dt} = y94 * F\phi - \sigma_{aZr4}\phi N_{Zr4} + \sigma_{cZr3}\phi N_{Zr3} \tag{9}$$

$$\frac{dN_{Zr5}}{dt} = y95 * F\phi - \lambda_{Zr5} N_{Zr5} - \sigma_{aZr5}\phi N_{Zr5} + \sigma_{cZr4}\phi N_{Zr4} \tag{10}$$

$$\frac{dN_{Nb5}}{dt} = \lambda_{Zr5} N_{Zr5} - \sigma_{aNb5}\phi N_{Nb5} - \lambda_{Mo5} N_{Mo5} \tag{11}$$

$$\frac{dN_{Mo5}}{dt} = \lambda_{Nb5} N_{Nb5} - \sigma_{aNb5}\phi N_{Nb5} \tag{12}$$

$$\frac{dN_{Zr6}}{dt} = y96 * F\phi - \sigma_{aZr6}\phi N_{Zr6} + \sigma_{cZr5}\phi N_{Zr5} \tag{13}$$

$$\frac{dN_{Mo7}}{dt} = y97 * F\phi - \sigma_{aMo7}\phi N_{Mo7} + \sigma_{cZr6}\phi N_{Zr6} \tag{14}$$

$$\frac{dN_{Mo8}}{dt} = y98 * F\phi - \sigma_{aMo8}\phi N_{Mo8} + \sigma_{cMo7}\phi N_{Mo7} \tag{15}$$

$$\frac{dN_{Tc9}}{dt} = y99 * F\phi - \lambda_{Tc9} N_{Tc9} - \sigma_{aTc9}\phi N_{Tc9} + \sigma_{cNb8}\phi N_{Nb8} \tag{16}$$

$$\frac{dN_{Ru9}}{dt} = \lambda_{Ru9} N_{Ru9} - \sigma_{aNb5}\phi N_{Nb5} \tag{17}$$

$$\frac{dN_{Mo0}}{dt} = \lambda_{Nb0} N_{Nb0} - \sigma_{aNb0}\phi N_{Nb0} + \sigma_{cTc9}\phi N_{Tc9} \tag{18}$$

$$\frac{dN_{Ru1}}{dt} = y101 * F\phi - \sigma_{aRu1}\phi N_{Ru1} + \sigma_{cNb0}\phi N_{Nb0} \tag{19}$$

$$\frac{dN_{Ru2}}{dt} = y102 * F\phi - \sigma_{aRu2}\phi N_{Ru2} \tag{20}$$

$$\frac{dN_{Ru3}}{dt} = y103 * F\phi - \lambda_{Ru3} N_{Ru3} - \sigma_{aRu3}\phi N_{Ru3} \tag{21}$$

$$\frac{dN_{Rh3}}{dt} = \lambda_{Ru3} N_{Ru3} - \sigma_{aRh3}\phi N_{Rh3} \tag{22}$$

$$\frac{dN_{Ru4}}{dt} = y104 * F\phi - \sigma_{aRu4}\phi N_{Ru4} + \sigma_{cRu3}\phi \mathrm{N}_{Ru3} \tag{23}$$

$$\frac{dN_{Pd5}}{dt} = y105 * F\phi - \sigma_{aPd5}\phi N_{Pd5} + \sigma_{cRu4}\phi \mathrm{N}_{Ru4} \tag{24}$$

$$\frac{dN_{Ru6}}{dt} = y106 * F\phi - \lambda_{Ru6} N_{Ru6} - \sigma_{aRu6}\phi N_{Ru6} \tag{25}$$

$$\frac{dN_{Pd6}}{dt} = \lambda_{Ru6} N_{Ru6} - \sigma_{aPd6}\phi N_{Pd6} + \sigma_{cPd5}\phi \mathrm{N}_{Pd5} \tag{26}$$

$$\frac{dN_{Pd7}}{dt} = y107 * F\phi - \sigma_{aPd7}\phi N_{Pd7} + \sigma_{cPd6}\phi N_{Pd6} \tag{27}$$

$$\frac{dN_{Pd8}}{dt} = y108 * F\phi - \sigma_{aPd8}\phi N_{Pd8} + \sigma_{cPd7}\phi N_{Pd7} \tag{28}$$

$$\frac{dN_{Ag9}}{dt} = y109 * F\phi - \sigma_{aAg9}\phi N_{Ag9} + \sigma_{cPd8}\phi N_{Pd8} \tag{29}$$

$$\frac{dN_{Cd1}}{dt} = y111 * F\phi - \sigma_{aCd1}\phi N_{Cd1} \tag{30}$$

$$\frac{dN_{I7}}{dt} = y127 * F\phi - \sigma_{aI7}\phi N_{I7} \tag{31}$$

$$\frac{dN_{I9}}{dt} = y129 * F\phi - \sigma_{aI9}\phi N_{I9} \tag{32}$$

$$\frac{dN_{Xe1}}{dt} = y131 * F\phi - \sigma_{aXe1}\phi N_{Xe1} \tag{33}$$

$$\frac{dN_{Xe2}}{dt} = y132 * F\phi - \sigma_{aXe2}\phi N_{Xe2} + \sigma_{cXe1}\phi N_{Xe1} \tag{34}$$

$$\frac{dN_{Cs3}}{dt} = y133 * F\phi - \sigma_{aCs3}\phi N_{Cs3} + \sigma_{cXe2}\phi N_{Xe2} \tag{35}$$

$$\frac{dN_{Xe4}}{dt} = y134 * F\phi - \sigma_{aXe4}\phi N_{Xe4} \tag{36}$$

$$\frac{dN_{Cs5}}{dt} = y135 * F\phi - \lambda_{Cs5} N_{Cs5} + \sigma_{cXe4}\phi N_{Xe4} \tag{37}$$

$$\frac{dN_{Ba5}}{dt} = \lambda_{Cs5} N_{Cs5} - \sigma_{aBa5}\phi N_{Ba5} \tag{38}$$

$$\frac{dN_{Cs7}}{dt} = y137 * F\phi - \lambda_{Cs7} N_{Cs7} \tag{39}$$

$$\frac{dN_{Ba7}}{dt} = \lambda_{Cs7} N_{Cs7} - \sigma_{aBa7}\phi N_{Ba7} \tag{40}$$

$$\frac{dN_{La9}}{dt} = y139 * F\phi - \sigma_{aLa9}\phi N_{La9} \tag{41}$$

$$\frac{dN_{Ce1}}{dt} = y141 * F\phi - \lambda_{Ce1} N_{Ce1} \tag{42}$$

$$\frac{dN_{\mathrm{Pr}1}}{dt} = \lambda_{Ce1} N_{Ce1} - \sigma_{a\mathrm{Pr}1}\phi N_{\mathrm{Pr}1} \tag{43}$$

$$\frac{dN_{Ce2}}{dt} = y142 * F\phi - \sigma_{aCe2}\phi N_{Ce2} + \sigma_{cCe1}\phi N_{Ce1} \tag{44}$$

$$\frac{dN_{Nd3}}{dt} = y143 * F\phi + \sigma_{cCe2}\phi N_{Ce2} - \sigma_{aNd3}\phi N_{Nd3} \tag{45}$$

$$\frac{dN_{Nd5}}{dt} = y145 * F\phi - \sigma_{aNd5}\phi N_{Nd5} \tag{46}$$

$$\frac{dN_{Nd6}}{dt} = y146 * F\phi - \lambda_{Nd6} N_{Nd6} + \sigma_{cNd5}\phi N_{Nd5} \tag{47}$$

$$\frac{dN_{Pm6}}{dt} = \lambda_{Nd6} N_{Nd6} - \sigma_{aPm6}\phi N_{Pm6} \tag{48}$$

$$\frac{dN_{Nd7}}{dt} = y147 * F\phi - \lambda_{Nd7} N_{Nd7} + \sigma_{cNd6}\phi N_{Nd6} \tag{49}$$

$$\frac{dN_{Pm7}}{dt} = \lambda_{Nd7} N_{Nd7} - \sigma_{aPm7}\phi N_{Pm7} \text{-} \lambda_{Pm7} \mathrm{N}_{Pm7} \tag{50}$$

$$\frac{dN_{Sm7}}{dt} = \lambda_{Pm7} N_{Pm7} - \sigma_{aSm7}\phi N_{Sm7} \tag{51}$$

$$\frac{dN_{Nd8}}{dt} = y148 * F\phi - \sigma_{aNd8}\phi N_{Nd8} + \sigma_{cNd7}\phi N_{Nd7} \tag{52}$$

$$\frac{dN_{Sm9}}{dt} = y149 * F\phi - \sigma_{aSm9}\phi N_{Sm9} + \sigma_{cNd8}\phi N_{Nd8} \tag{53}$$

$$\frac{dN_{Nd0}}{dt} = y150 * F\phi - \sigma_{aNd0}\phi N_{Nd0} \tag{54}$$

$$\frac{dN_{Sm1}}{dt} = y151 * F\phi - \lambda_{Sm1} N_{Sm1} + \sigma_{cNd0}\phi N_{Nd0} \tag{55}$$

$$\frac{dN_{Eu1}}{dt} = \lambda_{Sm1} N_{Sm1} - \sigma_{aEu1}\phi N_{Eu1} \tag{56}$$

$$\frac{dN_{Sm2}}{dt} = y152 * F\phi - \sigma_{aSm2}\phi N_{Sm2} + \sigma_{cSm1}\phi N_{Sm1} \tag{57}$$

$$\frac{dN_{Eu3}}{dt} = y153 * F\phi - \sigma_{aEu3}\phi N_{Eu3} + \sigma_{cSm2}\phi N_{Sm2} \tag{58}$$

$$\frac{dN_{Eu5}}{dt} = y155 * F\phi - \lambda_{Eu5}N_{Eu5} - \sigma_{aEu5}\phi \mathrm{N}_{Eu5} \tag{59}$$

$$\frac{dN_{Gd5}}{dt} = \lambda_{Eu5}N_{Eu5} - \sigma_{aGd5}\phi N_{Gd5}. \tag{60}$$

We calculate the atomic density during long-life burn-up using the equilibrium approach and by directly solving the above equations using numerical methods (finite difference and analytical methods).

3 Calculation results and discussion

For the data for calculation, we use the fission yield from Japanese Nuclear Data Committee (JNDC) nuclear data, and for one group microscopic cross-section data we calculated them based on RBEC benchmark model using System for Reactor Analysis Code (SRAC) (Okumura *et al.*, 2002).

Figures 1–8 show the effective/cumulative yield from JNDC.

The first pattern is about nuclides that soon reach the asymptotic value, such as Nb-95, Y-91, Zr-95, Ru-103, Ru-106, Ce-141, Nd-147 and Sm-151. Such nuclides can be grouped together by weight which may depend on some parameters such as flux and power density. These results are also in line with the equilibrium model. Ru-106 may be on the boundary between the first pattern and the second pattern.

The second pattern includes nuclides that change during burn-up nonlinearly. Such nuclides include Kr-85, Pd-106, Cs-137, Ce-142, Pm-147, Sm-147 and Eu-155. They can be combined into one group or more by weight (quadratic, cubic, quartic, *etc.*).

The third pattern is about nuclides that change almost linearly during burn-up. Such nuclides include Rb-85, Zr-91, Zr-92, Zr-93, Zr-94, Zr-96, Mo-95, Mo-97, Mo-98, Mo-100, Tc-99, Ru-101, Ru-102, Ru-104, Rh-103, Pd-105, Pd-107, Pd-108, Ag-109, Cd-111, I-127, I-129, Xe-131, Xe-132, Xe-134, Cs-133, Cs-135, La-139, Pr-141, Nd-143, Nd-145, Nd-146, Nd-148, Nd-150, Sm149, Sm152 and Eu-153. Such nuclides can be grouped into two or more group constants by flux level, power level and time.

Figure 1 Time-dependent nuclide density for Kr-85 and Rb-85 during the burn-up process (see online version for colours)

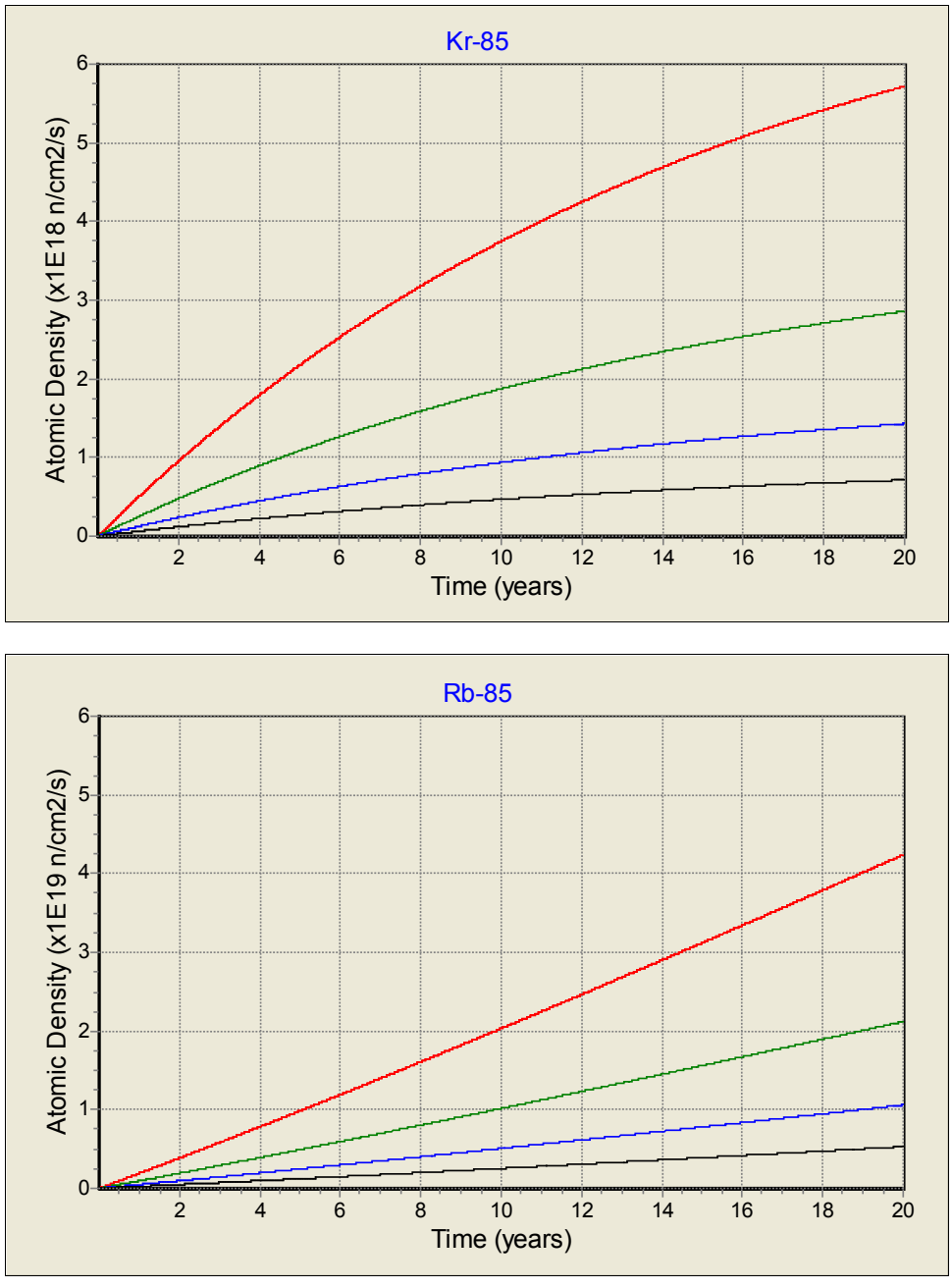


Figure 2 Time-dependent nuclide density for Nb-95 and Y-91 during the burn-up process (see online version for colours)

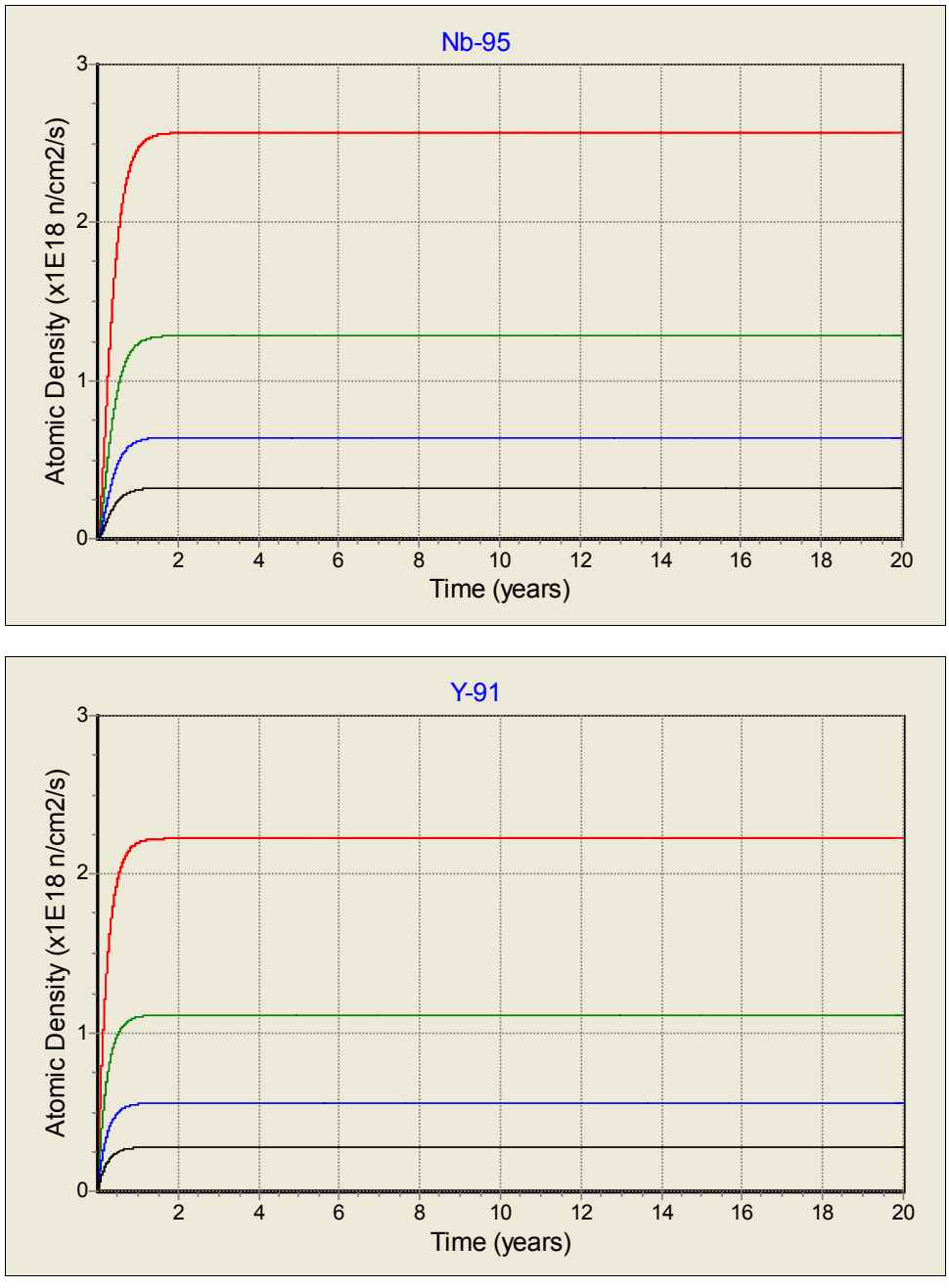


Figure 3 Time-dependent nuclide density for Zr-91 and Zr-95 during the burn-up process (see online version for colours)

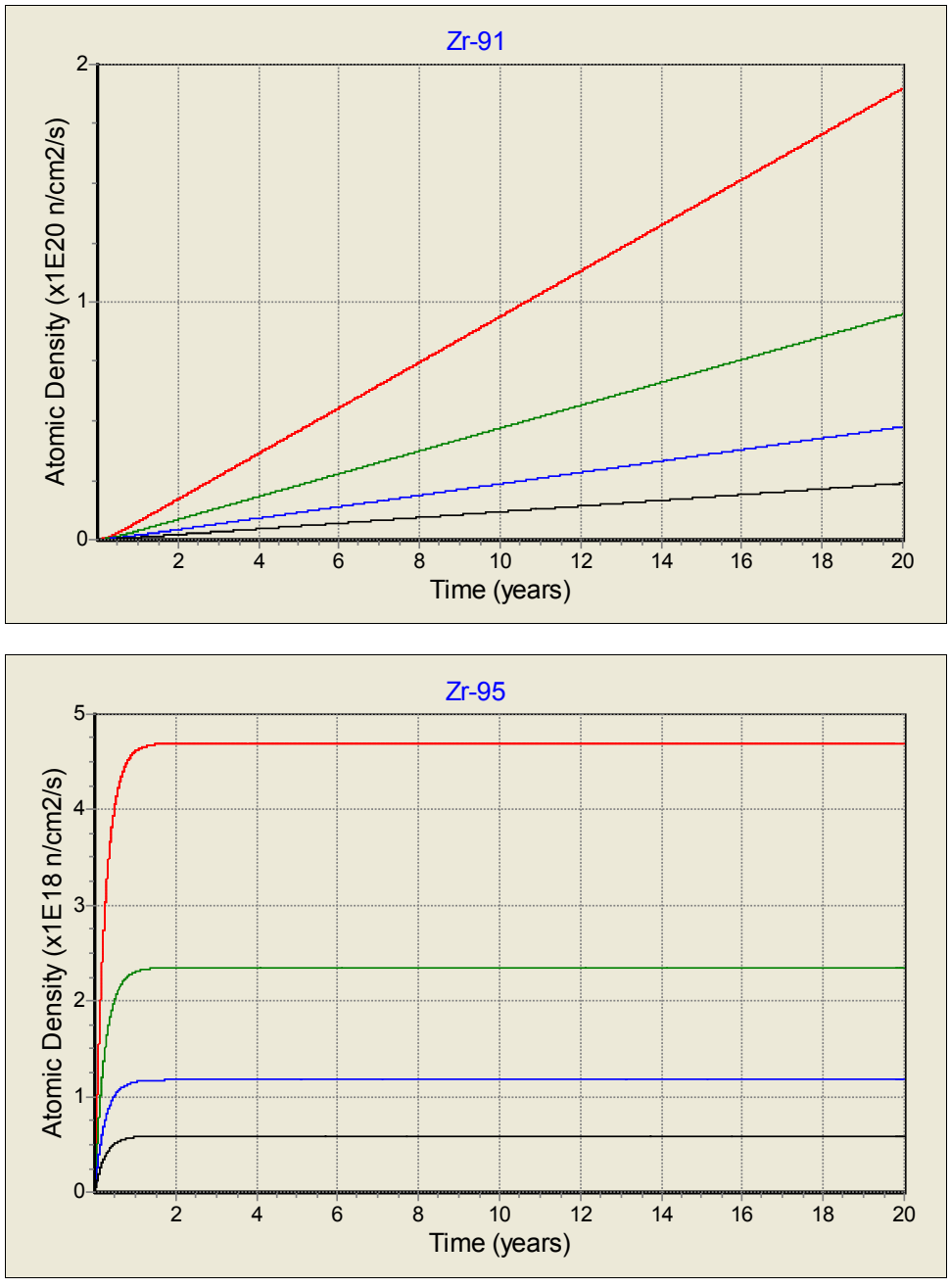


Figure 4 Time-dependent nuclide density for Tc-99 and Ru-106 during the burn-up process (see online version for colours)

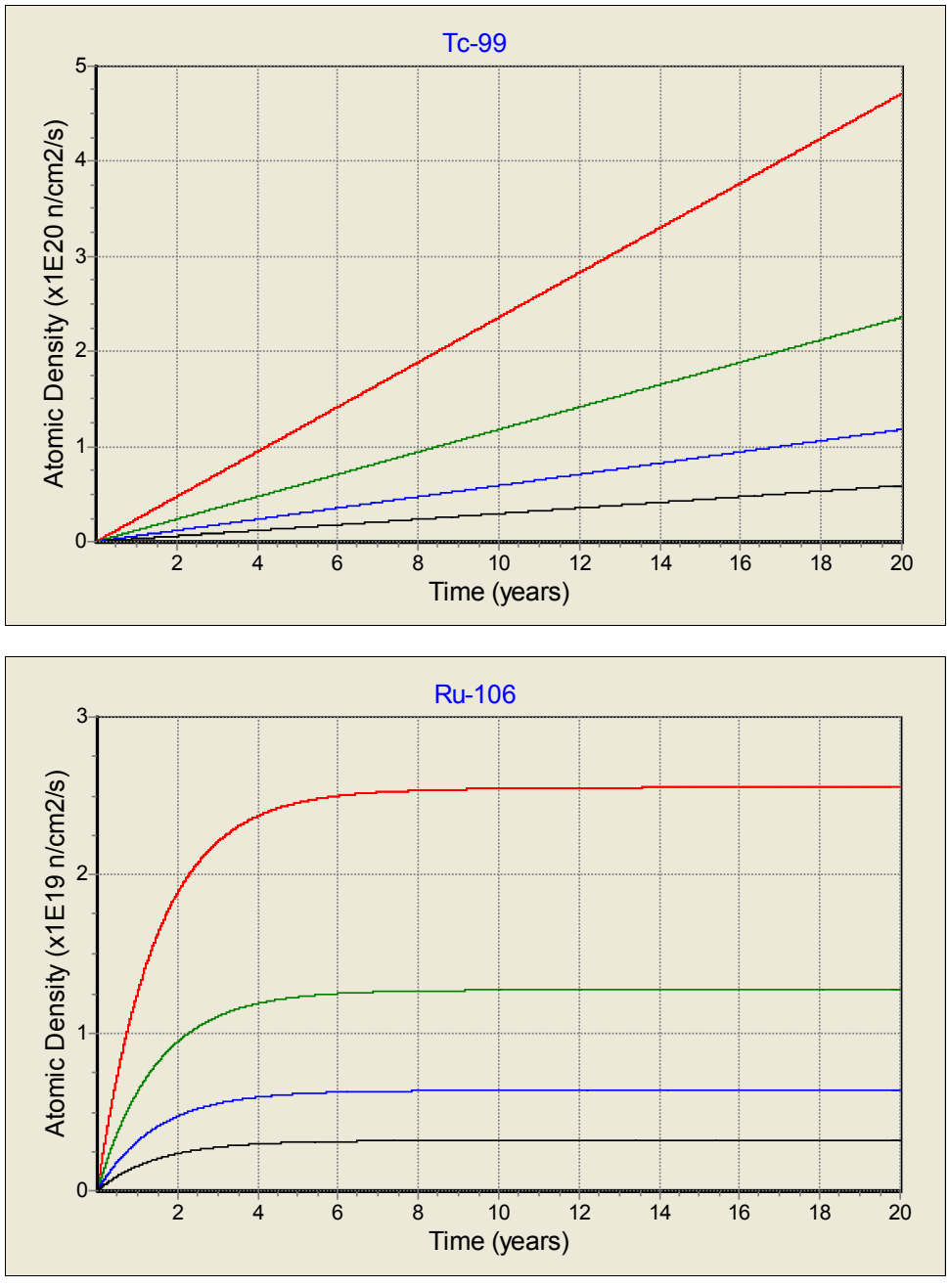


Figure 5 Time-dependent nuclide density for I-129 and Xe-131 during the burn-up process (see online version for colours)

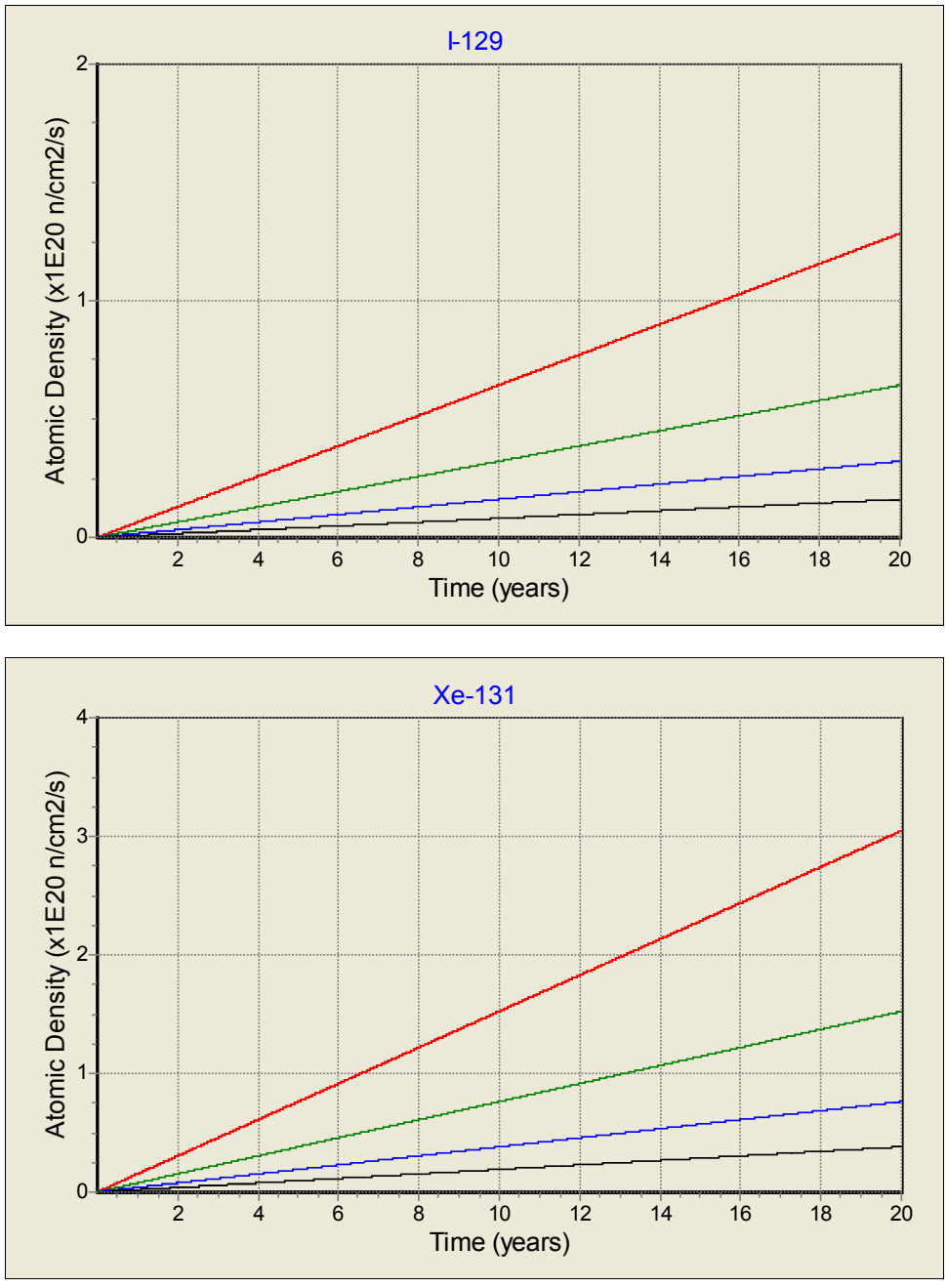


Figure 6 Time-dependent nuclide density for Cs-135 and Cs-137 during the burn-up process (see online version for colours)

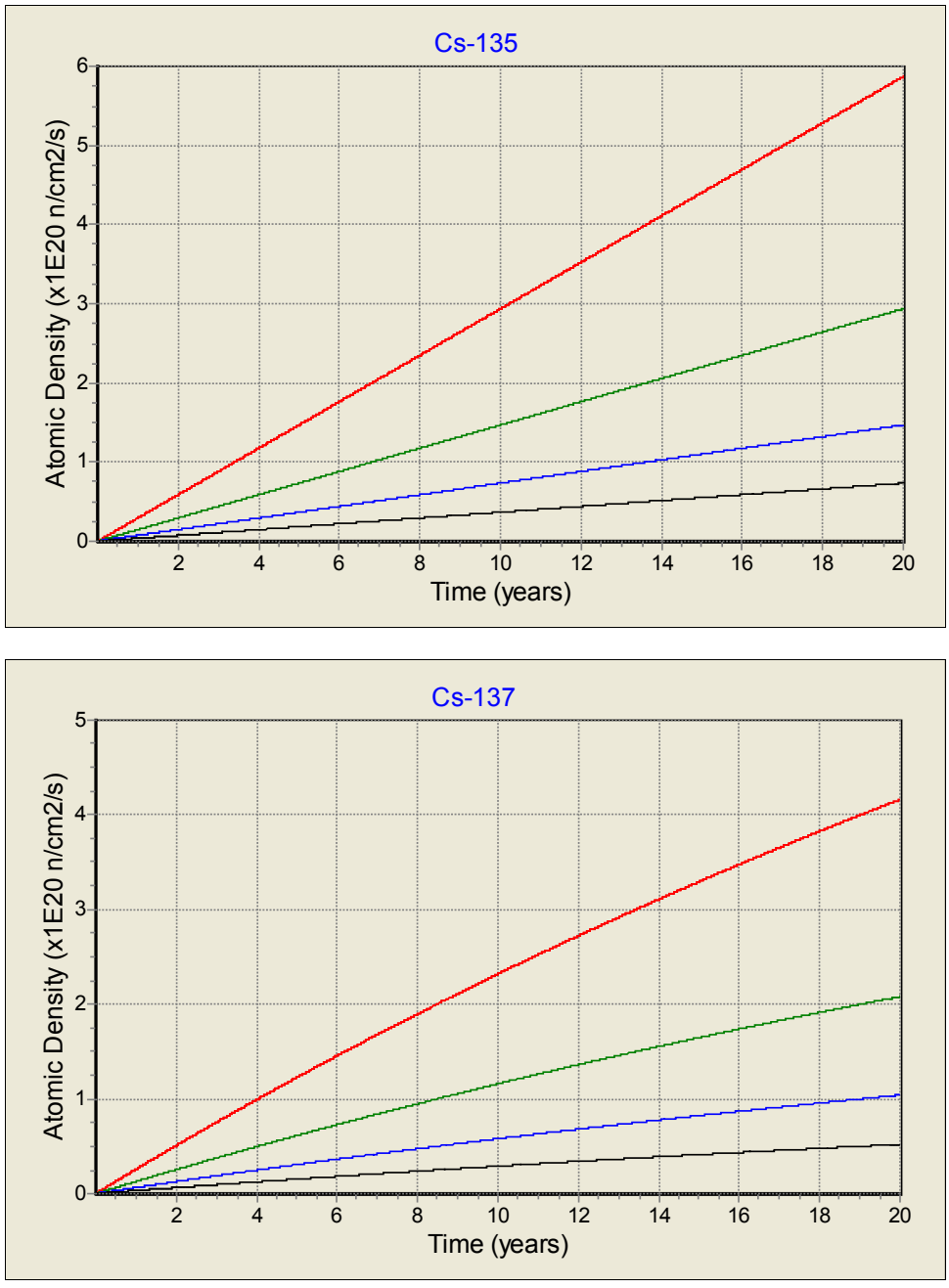


Figure 7 Time-dependent nuclide density for La-139 and Sm-147 during the burn-up process (see online version for colours)

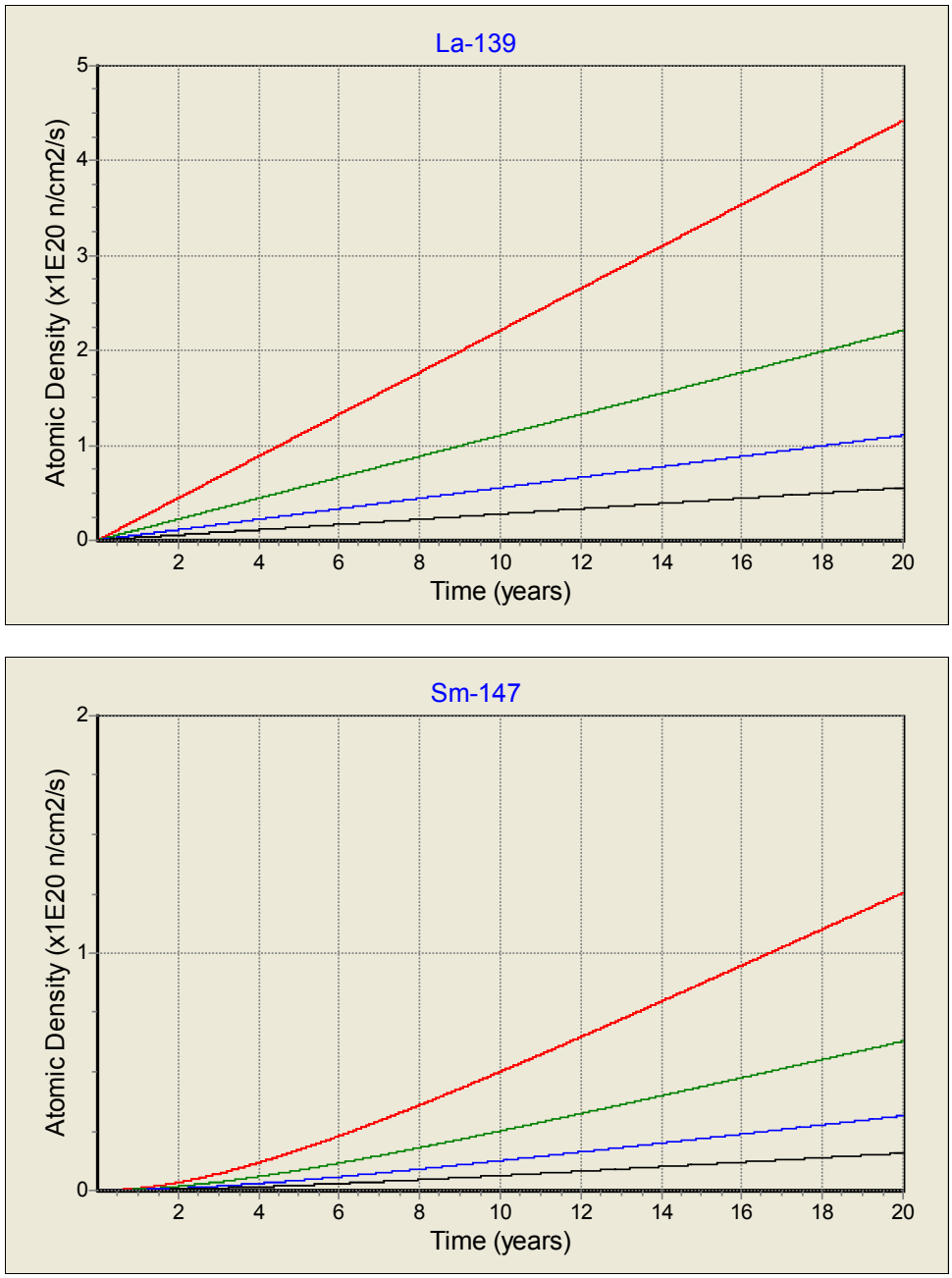
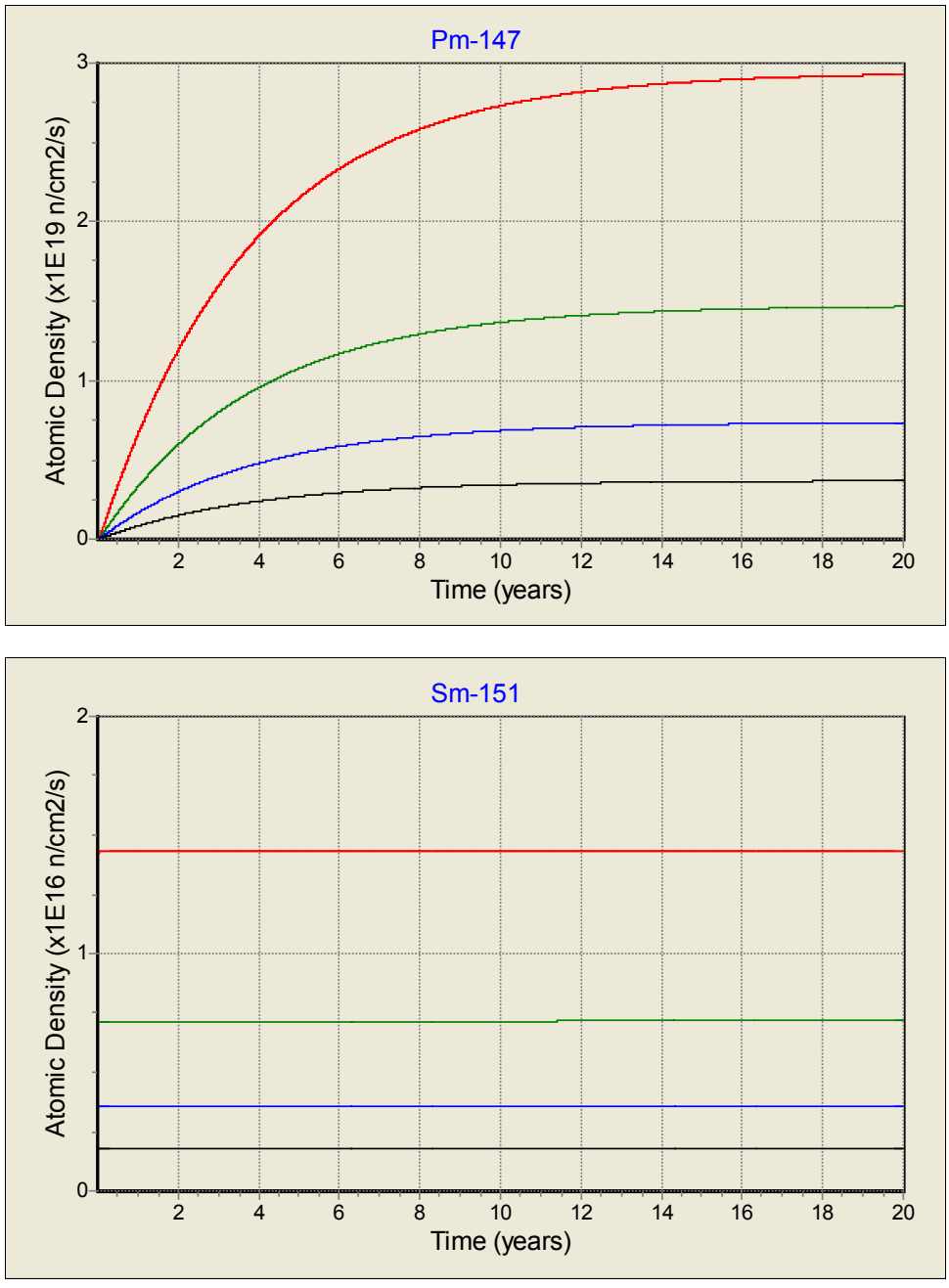


Figure 8 Time-dependent nuclide density for Pm-147 and Sm-151 during the burn-up process (see online version for colours)



4 Conclusion

In this study we focus on the FP group constant treatment by considering around 50 of the most important nuclides. We then calculate the fission product effective yield for each modified chain and generate one group constant using the SRAC and other methods (Origen, *etc.*).

We use two approaches for investigating the important FP nuclides: the equilibrium model and a numerical solution for the time-dependent model. We found that we could separate the FP nuclides into three groups:

1 those that soon reach the asymptotic value

2 those that have a nonlinear pattern

3 those that have a linear pattern.

In future work we will complete the detailed lumped FP model and include this in the full core benchmark calculation.

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