PERSAMAAN DIFERENSIAL

Pertemuan 5, PD Eksak

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PRESENTATION PARTS

1

- Solusi PD Eksak
  - Metode Standar
  - Metode Grouping

2

- Solusi PD Eksak - Metode Alternatif

Solusi PD Eksak

Diberikan PD eksak

$$M(x,y)\,dx + N(x,y)\,dy = 0.$$

terdapat $F(x,y)$ sedemikian sehingga

$$\frac{\partial F(x,y)}{\partial x} = M(x,y) \text{ dan } \frac{\partial F(x,y)}{\partial y} = N(x,y)$$

Akibatnya

$$\frac{\partial F(x,y)}{\partial x}dx + \frac{\partial F(x,y)}{\partial y}dy = 0.$$

Jadi,

$$dF(x, y) = 0$$

Sehingga diperoleh solusi umumnya

$$F(x, y) = C$$

Pertanyaannya...

Which $F(x, y)$??

Diketahui PD bersifat eksak maka

$$\frac{\partial F(x,y)}{\partial x} = M(x,y) \quad (I)$$

$$\frac{\partial F(x,y)}{\partial y} = N(x,y) \quad (II)$$

Dari persamaan I maka diperoleh

$$F(x,y) = \int M(x,y)\,\partial x + \phi(y)$$

Dari persamaan II,

$$\frac{\partial}{\partial y}\left[\int M(x,y)\ \partial x + \phi(y)\right] = N(x,y)$$

Jadi,

$$\phi(y) = \int N(x,y) - \left[\int \frac{\partial}{\partial y} M(x,y)\ \partial x\right] \partial y$$

Jadi,

$$F(x,y) = \int M(x,y)\, \partial x + \phi(y)$$

dengan

$$\phi(y) = \int N(x,y) - \left[\int \frac{\partial}{\partial y} M(x,y)\ \partial x \right] \partial y$$

Contoh : Tentukan solusi umum dari

$$(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$$

Jawab :

$$x^3 + 2x^2y + y^2 = c$$

Latihan :

$$(2x \cos y + 3x^2y)\, dx + (x^3 - x^2 \sin y - y)\, dy = 0$$

Metode Grouping

Syarat :

Persamaan Diferensial merupakan jumlahan dari persamaan diferensial eksak.

Contoh :

1. $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$

2. $(2x\cos y + 3x^2y)\,dx + (x^3 - x^2\sin y - y)\,dy = 0$

Metode Alternatif

Jika

$$\int M(x,y)\,\partial x = S(x,y) + g_1(x) + h_1(y)$$

Dan

$$\int N(x,y)\,\partial x = S(x,y) + g_2(x) + h_2(y)$$

maka

$$F(x,y) = S(x,y) + g_1(x) + h_2(y)$$

Contoh :

1. $(x^2 + y^2)dx + (2xy + \cos y)dy = 0$

2. $(1 - 2xy)dx + (4y^3 - x^2)dy = 0$

Referensi :

1. Ross, L. Differential Equations. John Wiley & Sons.

2. Oswaldo Gonz´alez-Gaxiola1 and S. Hern´andez Linares. *An Alternative Method to Solve Exact Differential Equations*. International Mathematical Forum, 5, 2010, no. 54, 2689 - 2697

3. http://math.stackexchange.com/questions/215324/proof-for-exact-differential-equations-shortcut diakses tanggal 5 oktober 2016

THANK YOU

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Karangmalang Sleman Yogyakarta

Exercise 5.1

Solve the following equations.

1. $(3x + 2y)\,dx + (2x + y)dy = 0$
2. $(y^2 + 3)\,dx + (2xy - 4)dy = 0$
3. $(6xy + 2y^2 - 5)dx - (x^3 + y)\,dy = 0$
4. $(a^2 + 1)\cos r\,dr + 2a \sin r\;\,da = 0$
5. $(y \sec^2 x + \sec x \tan x)\,dx + (\tan x + 2y)dy = 0$

Exercises 5.2

1. Consider the differential equation

$$(4x + 3y^2)\,dx + 2xy\,dy = 0$$

   a. Show that this equation is not exact
   b. Find the integratiiing factor of the form x^n, where n is a positive integer.
   c. Multiply the given equation through by the integrating factor found in (b) and solve the resulting exact equation.

2. Consider the differential equation

$$(y^2 + 2xy)dx - x^2\,dy = 0$$

   a. Show that this equation is not exact.
   b. Multiply the given equatttion through by y^n, where n is an integer and then determine n so that y^n is an integrating factor of the given equation.
   c. Multiply the given equation through by the integrating factor found in (b) and solve the resulting exact equation.
   d. Show that y = 0 is a solution of the original nonexact equation but is not a solution of the essentially equivalent exact equation found in step (c).
   e. Graph several integral curves of the original equation, including all those whose equations are (or can be writen) in some special form.