PERSAMAAN DIFERENSIAL

Pertemuan 9, PD Bernouli

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PRESENTATION PARTS

1. Review PD Homogen
2. Review PD Linear
3. PD Bernoulli

Review PD Homogen

Definisi homogen

A function f of two variables x and y is said homogeneous if it satisfies

$$f(tx, ty) = f(x, y)$$

for any number t.

Definition

A function M of two variables x and y is said to be homogenous of degree-n if it satisfies

$$M(tx, ty) = t^n M(x, y)$$

Definition

A first differential equation

$$M(x,y)dx + N(x,y)dy = 0 \text{ or } \frac{dy}{dx} = f(x,y)$$

is called **homogeneous differential equation** if *M* and *N* are homogeneous functions of the same degree *n* or *f* is a homogenous function.

Menyelesaikan PD Homogen

- PD Homogen (dalam x dan y)
- Transform y = vx
- PD separabel (dalam v dan x)
- Selesaikan PD separabel
- Inverse transform v = y/x

PD Linear

Definition

A first-order ordinary differential equation is linear in the dependent variable y and the independent variable x if it is, or can be, written in the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Menyelesaikan PD Linear

Suppose the integrating factor

$$\mu(x) = e^{\int P(x)\,dx}$$

The general solution of linear differential equation as follows

$$y = \mu^{-1}(x)\left[\int \mu(x)\,Q(x)\;dx + c\right]$$

PD Bernoulli

Definition

An equation of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

is called a **Bernoulli differential equation.**

Theorem

Suppose n ≠ 0 or n ≠ 1, then the transformation $v = y^{1-n}$ reduces the Bernoulli equation

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

into a linear equation in v.

Example

Solve the differential equation below

$$x\frac{dy}{dx} + y = x^2y^2$$

Answer :

To solve the differential equations above, first we have to divided both side with xy^2. Here we get,

$$y^{-2}\frac{dy}{dx}+x^{-1}y^{-1}=x$$

Suppose $v=y^{-1}$ then $\frac{dv}{dy}=-y^{-2}$ or else $\frac{dv}{dx}=-y^{-2}\frac{dy}{dx}$. By here, we get

$$\frac{dv}{dx}+x^{-1}v=x$$

To solve the equation, we use integrating factor

$$\mu(x) = e^{\int \frac{1}{x} dx} = e^{\ln|x|} = x$$

Then the solution is

$$v = \mu^{-1}(x)\left[\int \mu(x)\, Q(x)\ dx + c\right]$$

$$v = x^{-1}\left[\int x^2\ dx + c\right] \rightarrow v = x^{-1}\left[\frac{1}{3}x^3 + c\right]$$

$$= \frac{1}{3}x^2 + cx^{-1}$$

$$y^{-1} = \frac{1}{3}x^2 + cx^{-1}$$

Solve the following differential equations.

1. $\frac{dy}{dx} - \frac{1}{x}y = xy^2$
2. $\frac{dy}{dx} + \frac{y}{x} = y^2$
3. $\frac{dy}{dx} + \frac{1}{3}y = e^x y^4$
4. $x\frac{dy}{dx} + y = xy^3$

Referensi :

1. Ross, L. Differential Equations. John Wiley & Sons.

THANK YOU

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Karangmalang Sleman Yogyakarta