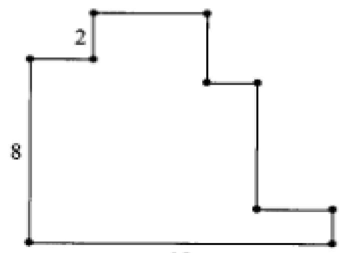
Questions for Central Mathematics Competition 2010

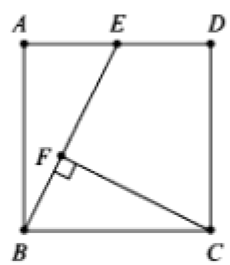
1. Suppose $A = 543 + 544 + \cdots + 765$, $B = 545 + 546 + \cdots + 767$, $C = 544 + 545 + \cdots + 766$, $D = 542 + 543 + \cdots + 764$, and $E = 546 + 547 + \cdots + 768$. The biggest value is
2. The digits 2, 4, 5, 6, 8 and 9 can be distributed among the lettered squares in the array so that the sum of the entries on each of the rows and columns is the same number X. What is X?

7	*a*	*b*	1
c			*d*
3	*e*	*f*	10

3. The adjacent sides of the decagon shown meet at right angles. Its perimeter is …

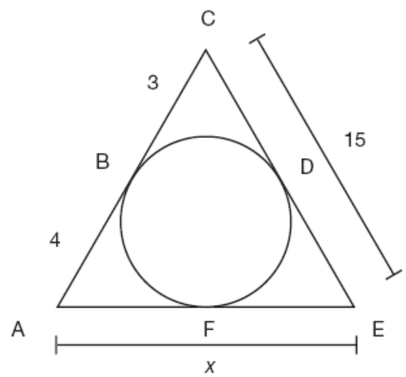


4. If x students can solve the problem in c hours, then x + y students can solve the problem in …
5. The 2012th digit from the last of 2010^{2010} is …
6. Four girls, Hima, Ulfa, Nita and Ucik sang songs in a concert as trios, with one girl sitting out each time. Hima sang seven songs, which was more than any other girl, and Ulfa sang four songs, which was fewer than any other girl. How many songs did these trios sing?
7. If $x + 3y + 2z = 48$ and $2x + 3y + 4z = 69$, then the value of $3x + 3y + 6z$ is …
8. The number of prime factor of $(4^4)^2 - 1$ is …
9. In the figure, ABCD is a 2 x 2 square, E is the midpoint AD and F is on BE. If CF is perpendicular to BE, then the are of quadrilateral CDEF is …



10. For any positive integer *n*, let $n! = (n)(n-1)(n-2) \ldots (2)(1)$. The last digit of
$$(1 \cdot 1!) + (2 \cdot 2!) + (3 \cdot 3!) + (4 \cdot 4!) + \cdots + (2009 \cdot 2009!) + (2010 \cdot 2010!)$$
is …
11. Ani has 20 coins in her wallet, Rp 100, Rp 200 and Rp 500 coins. The total value is Rp 5000. If she has Rp 500 coins more than Rp 200 coins, how many Rp 100 coins that she has?

12. Triangle AEC is circumscribed. Find the value of *x*.

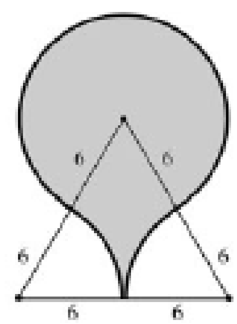


13. Let f be a linear function of real numbers with the properties that $f(0) = 2, f(0) \leq f(1)$ and $f(1) \geq f(2)$. What is the value of $f(2010)$?
14. If a, b and c are three (not necessarily different) numbers chosen randomly and with replacement from the set $\{1,2,3,4,5\}$, then the probability that $ab + c$ is even is …
15. How many three-element subsets of the set {88, 95, 99, 132, 166, 173} have the property that the sum of the three elements is even?
16. If x, y and z are positive number satisfying

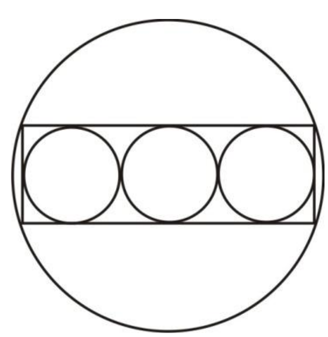
$$x + \frac{1}{y} = 4, y + \frac{1}{z} = 1 \text{ and } z + \frac{1}{x} = \frac{7}{3}$$

then the value of $xyz =$

17. Find the circumference of the shaded area.



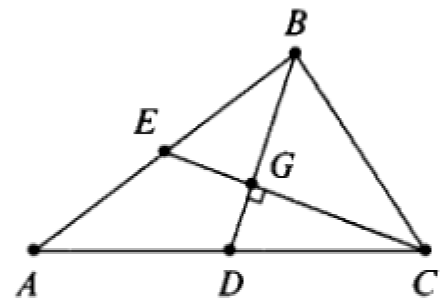
18. The number of ordered pairs of digits (a, b, c) such that $\overline{abcabc}$ is divisible by 13 is
19. The greatest prime factor less than 10000 of 1001001001 is …
20. How many two-digit positive integers N have the property that the sum of N and the number obtained by reversing the order of the digits of N is a perfect square?
21. A speaker talked for sixty minutes to a full auditorium. Twenty percent of the audience heard the entire talk and ten percent slept through the entire talk. Half of the remainder heard one third of the talk and the other half heard two thirds of the talk. The average number of minutes of the talk heard by members of the audience is …
22. Three circles, each with a radius of 10 cm, intersect with each other so that the center of three circles are all located on one line to another. The circles are in a circle within the rectangle, while the rectangle is located in a large circle as shown. What is the area of the large circle?



23. The number of integer between 1 and 100 which isn't square or cubic number is …

24. How many people do we need to have in a room to make it a favorable bet (probability of success greater than $\frac{1}{2}$) that two people in the room will have the same birthday?

25. Medians BD and CE of triangle ABC are perpendicular, BD = 8, and CE = 12. The area of triangle ABC is …



Exploration

1. Explain how you would determine whether $10\sqrt{26}$-50 is a positive or negative number without carrying out the actual computation.

2. Figures 0, 1, 2 and 3 consist of 1, 5, 13 and 25 nonoverlapping unit squares, respectively. If the pattern were continued, how many nonoverlapping unit squares would there be in figure 100?

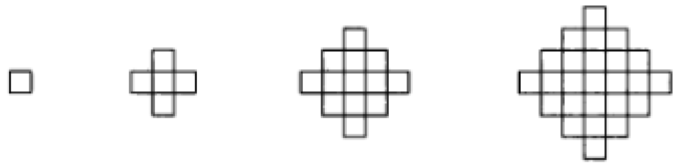
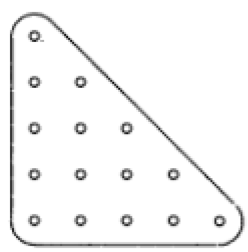


figure 0 figure 1 figure 2 figure 3

3. Given right triangle ABC with $\angle$A is the right angle. Point P is the midpoint of side AC. Point Q lies on AB. Points R and S both lie on BC such that PQRS is a rectangle. If the area of APQ is 24, calculate the area of PQRS.

4. There are 5 yellow pegs, 4 red pegs, 3 green pegs, 2 blue pegs and 1 orange peg to be placed on a triangular peg board. In how many ways can the pegs be placed so that no (horizontal) row or (vertical) column contains two pegs of the same color?



5. Determine all paired number (x,y) such that $x+y$, xy dan $\frac{x}{y}$ have the same value

6. Find a number of each letter that satisfying these conditions

$$\begin{array}{r} DONALD \\ +\ GERALD \\ \hline ROBERT \end{array}$$

Where

1) D = 5
2) Every number from 0 to 9 has its corresponding letter
3) Each letter must be assigned a number different from that given for any other letter