



PERSAMAAN DIFERENSIAL

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Silabus Materi

- Definisi PD dan Peny. PD
- Penggolongan PD
- PD linear order satu

PD Terpisah.

Fungsi Homogen

PD Homogen.

PD Eksak

Faktor Integral fungsi x saja.

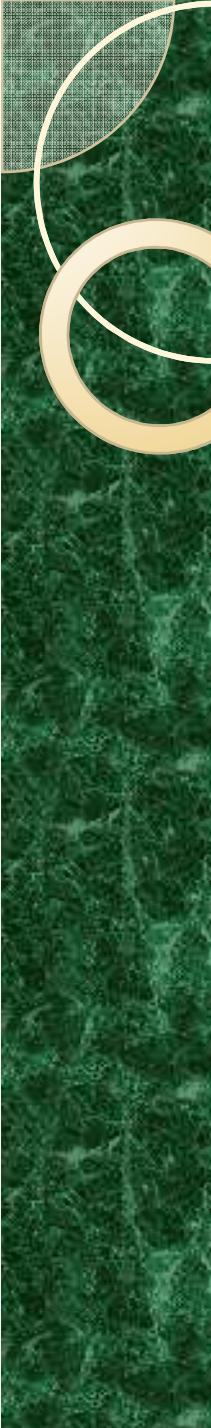
Faktor integral fungsi y saja.



Lanjutan silabus

Faktor integral fungsi x dan y. PD Non Eksak

- Bentuk Umum PD Linear Tingkat Satu
- PD Bernoulli
- Aplikasi PD order satu
- UTS
- PD Linear Order tinggi
- Bentuk Umum
- Penyelesaian umum Persamaan Cauchy-Euler
- Aplikasi PD linear order tinggi



Evaluasi

- Tugas-tugas 25%
- Kuis/Kehadiran 10%
- UTS 30%
- UAS 35%
- Jumlah 100%



Referensi

- **A.Wajib :**

[A] Boyce, E.W. & Richard C. DiPrima. 2004. *Elementary Differential Equation and Boundary Value Problems, Eight Edition.* New York: John Wiley&Sons,Inc.

[B] Ross, S.L. 1984. *Differential Equations, Third Edition.* New York: John Wiley&Sons,Inc.

- **B.Anjuran :**

[C] Tenenbaum, M. & Harry Pollard. 1963. *Ordinary Differential Equations.* New York: Dover Publication, Inc.

[D] Ayres, F. 1999. *Differential Equations.* Schaum's Outline series. Mc Graw-Hill Company.

[E] Kreyszig, E.2006. *Advanced Engineering Mathematics, 9th ed.* New York: John Wiley & Sons, Inc.



Pendahuluan (Pretes)

- Apa yang dimaksud dengan PD? Berikan contohnya.
- Apa yang Anda ketahui tentang order atau tingkat pada PD?



Lanjutan:

- Sebutkan aplikasi PD yang Anda ketahui.
- Apa yang anda harapkan dari perkuliahan PD semester ini?
- Tulis: Nama, NIM, HP



Pengertian PD

- suatu bentuk persamaan yang memuat derivatif (turunan) satu atau lebih variabel tak bebas terhadap satu atau lebih variabel bebas suatu fungsi.
- Notasi PD:
 $y' = dy/dx;$
 $x' = dx/dt$



Contoh PD:

$$1. \frac{d^2y}{dx^2} + xy\left(\frac{dy}{dx}\right)^2 = 0$$

$$2. \frac{d^4x}{dt^4} + 5\frac{d^2x}{dt^2} + 3x = \sin t$$

$$3. \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} = v$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$



Klasifikasi PD:

1. PD Biasa :

sebuah bentuk persamaan yang memuat turunan satu atau lebih variabel tak bebas terhadap satu variabel bebas suatu fungsi.

Berdasarkan turunan tertinggi;

- PDB Orde 1 : turunan tertingginya adalah turunan pertama
- PDB Orde 2 : turunan kedua merupakan turunan tertinggi
- PDB Orde 3 : turunan ketiga merupakan turunan tertingginya.
- Dan seterusnya

2. PD Parsial

Persamaan Differensial yang memiliki lebih dari satu variabel bebas.

PD biasa

$$y' = \sin x + \cos x$$

$$y'' + 7y = 0$$

$$y'' + 3y' - 4y = 0$$

$$y''' - e^x y'' - yy' = (x^2 + 1)y^2$$



PD Parsial

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

$$\frac{\partial v}{\partial x} - \frac{\partial v}{\partial y} + 2v = 0$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial y} = k$$

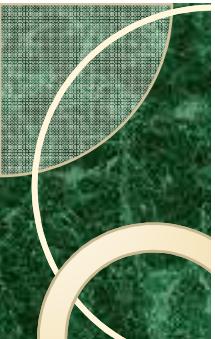
$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = e$$



PDB Linear

Persamaan diferensial biasa linear order n dapat dituliskan sebagai:

$$a_0(x) \frac{d^n y}{dx^n} + a_1(x) \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_n(x) y = b(x)$$



PD LINEAR ORDER TINGGI

DEFINITION

A linear ordinary differential equation of order n in the dependent variable y and the independent variable x is an equation that is in, or can be expressed in, the form

$$a_0(x) \frac{d^n y}{dx^n} + a_1(x) \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1}(x) \frac{dy}{dx} + a_n(x)y = F(x), \quad (4.1)$$

where a_0 is not identically zero. We shall assume that a_0, a_1, \dots, a_n and F are continuous real functions on a real interval $a \leq x \leq b$ and that $a_0(x) \neq 0$ for any x on $a \leq x \leq b$. The right-hand member $F(x)$ is called the nonhomogeneous term. If F is identically zero, Equation (4.1) reduces to

$$a_0(x) \frac{d^n y}{dx^n} + a_1(x) \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1}(x) \frac{dy}{dx} + a_n(x)y = 0 \quad (4.2)$$

and is then called homogeneous.

PD LINEAR ORDER DUA

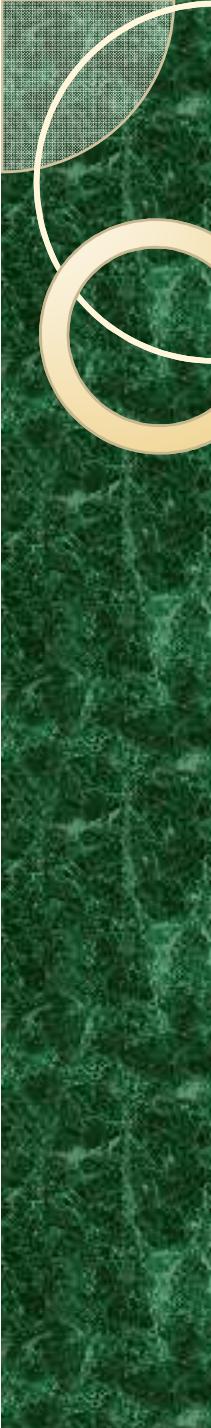
For $n = 2$, Equation (4.1) reduces to the *second-order nonhomogeneous linear differential equation*

$$a_0(x) \frac{d^2y}{dx^2} + a_1(x) \frac{dy}{dx} + a_2(x)y = F(x) \quad (4.3)$$

and (4.2) reduces to the corresponding second-order *homogeneous equation*

$$a_0(x) \frac{d^2y}{dx^2} + a_1(x) \frac{dy}{dx} + a_2(x)y = 0. \quad (4.4)$$

Here we assume that a_0, a_1, a_2 , and F are continuous real functions on a real interval $a \leq x \leq b$ and that $a_0(x) \neq 0$ for any x on $a \leq x \leq b$.



Contoh

► **Example 4.1**

The equation

$$\frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + x^3y = e^x$$

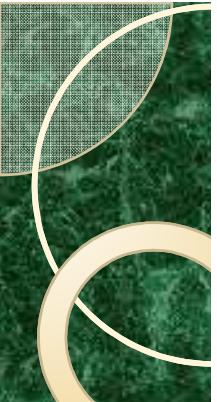
is a linear ordinary differential equation of the second order.

► **Example 4.2**

The equation

$$\frac{d^3y}{dx^3} + x \frac{d^2y}{dx^2} + 3x^2 \frac{dy}{dx} - 5y = \sin x$$

is a linear ordinary differential equation of the third order.



Prinsip Superposisi

$$a_0(x) \frac{d^n y}{dx^n} + a_1(x) \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1}(x) \frac{dy}{dx} + a_n(x)y = 0. \quad (4.2)$$



THEOREM 4.2 BASIC THEOREM ON LINEAR HOMOGENEOUS DIFFERENTIAL EQUATIONS

Hypothesis. Let f_1, f_2, \dots, f_m be any m solutions of the homogeneous linear differential equation (4.2).

Conclusion. Then $c_1 f_1 + c_2 f_2 + \cdots + c_m f_m$ is also a solution of (4.2), where c_1, c_2, \dots, c_m are m arbitrary constants.





Contoh

► **Example 4.6**

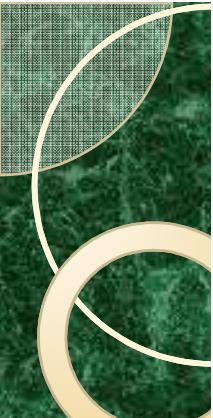
The student will readily verify that $\sin x$ and $\cos x$ are solutions of

$$\frac{d^2y}{dx^2} + y = 0.$$

Theorem 4.2 states that the linear combination $c_1 \sin x + c_2 \cos x$ is also a solution for any constants c_1 and c_2 . For example, the particular linear combination

$$5 \sin x + 6 \cos x$$

is a solution.



Contoh: lanjutan

► Example 4.7

The student may verify that e^x , e^{-x} , and e^{2x} are solutions of

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0.$$

Theorem 4.2 states that the linear combination $c_1 e^x + c_2 e^{-x} + c_3 e^{2x}$ is also a solution for any constants c_1 , c_2 , and c_3 . For example, the particular linear combination

$$2e^x - 3e^{-x} + \frac{2}{3}e^{2x}$$

is a solution.



Latihan

4. Consider the differential equation

$$\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 3y = 0. \quad (\text{A})$$

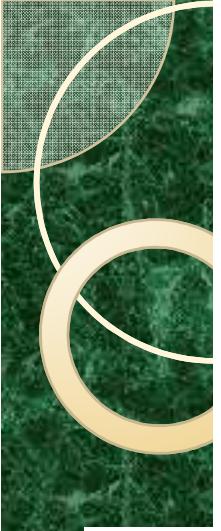
- (a) Show that each of the functions e^x and e^{3x} is a solution of differential equation (A) on the interval $a \leq x \leq b$, where a and b are arbitrary real numbers such that $a < b$.
- (b) What theorem enables us to conclude at once that each of the functions

$$5e^x + 2e^{3x}, \quad 6e^x - 4e^{3x}, \quad \text{and} \quad -7e^x + 5e^{3x}$$

is also a solution of differential equation (A) on $a \leq x \leq b$?

- (c) Each of the functions

$$3e^x, \quad -4e^x, \quad 5e^x, \quad \text{and} \quad 6e^x$$



13. Given that x , x^2 , and x^4 are all solutions of

$$x^3 \frac{d^3y}{dx^3} - 4x^2 \frac{d^2y}{dx^2} + 8x \frac{dy}{dx} - 8y = 0,$$

show that they are linearly independent on the interval $0 < x < \infty$ and write the general solution.



Metode Reduksi Order

THEOREM 4.6

Hypothesis. Let f be a nontrivial solution of the n th-order homogeneous linear differential equation

$$a_0(x) \frac{d^n y}{dx^n} + a_1(x) \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1}(x) \frac{dy}{dx} + a_n(x)y = 0. \quad (4.2)$$

Conclusion. The transformation $y = f(x)v$ reduces Equation (4.2) to an $(n - 1)$ st-order homogeneous linear differential equation in the dependent variable $w = dv/dx$.