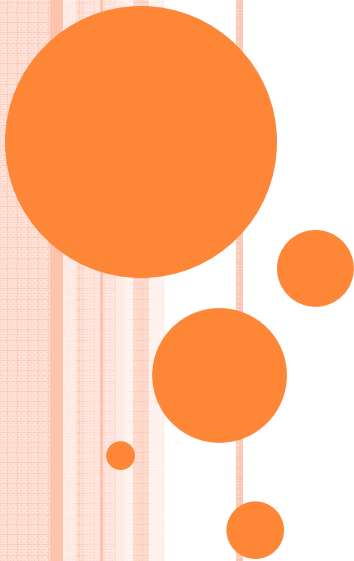


PERSAMAAN & PERTIDAKSAMAAN



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WHAT IS WRONG??

$$x = 2 \quad (1)$$

$$3x - 2x = 2 \quad (2)$$

$$3x = 2x + 2 \quad (3)$$

$$x^2 + 3x = x^2 + 2x + 2 \quad (4)$$

$$x^2 + 3x - 10 = x^2 + 2x - 8 \quad (5)$$

$$(x - 2)(x + 5) = (x - 2)(x + 4) \quad (6)$$

$$x + 5 = x + 4 \quad (7)$$

$$1 = 0 \quad (8)$$



REVIEW

Work with Properties of Real Numbers

The equal sign is used to mean that one expression is equivalent to another. Four important properties of equality are listed next. In this list, a , b , and c represent real numbers.

1. The **reflexive property** states that a number always equals itself; that is, $a = a$.
2. The **symmetric property** states that if $a = b$ then $b = a$.
3. The **transitive property** states that if $a = b$ and $b = c$ then $a = c$.
4. The **principle of substitution** states that if $a = b$ then we may substitute b for a in any expression containing a .



Commutative Properties

$$(a) \quad 3 + 5 = 8$$

$$5 + 3 = 8$$

$$3 + 5 = 5 + 3$$

$$(b) \quad 2 \cdot 3 = 6$$

$$3 \cdot 2 = 6$$

$$2 \cdot 3 = 3 \cdot 2$$

Commutative Properties

$$a + b = b + a \quad (1a)$$

$$a \cdot b = b \cdot a \quad (1b)$$

NB:

Penjumlahan dan perkalian bilangan real dengan sifat komutatif tidak mempengaruhi hasil akhir/solusi.



Associative Properties

$$(a) \quad 2 + (3 + 4) = 2 + 7 = 9$$

$$(2 + 3) + 4 = 5 + 4 = 9$$

$$2 + (3 + 4) = (2 + 3) + 4$$

$$(b) \quad 2 \cdot (3 \cdot 4) = 2 \cdot 12 = 24$$

$$(2 \cdot 3) \cdot 4 = 6 \cdot 4 = 24$$

$$2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4$$

Associative Properties

$$a + (b + c) = (a + b) + c = a + b + c \quad (2a)$$

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c = a \cdot b \cdot c \quad (2b)$$

NB:

Penjumlahan dan perkalian bilangan real dengan sifat asosiatif tidak mempengaruhi hasil akhir/solusi.



Distributive Property

(a) $2 \cdot (x + 3) = 2 \cdot x + 2 \cdot 3 = 2x + 6$ *Use to remove parentheses.*

(b) $3x + 5x = (3 + 5)x = 8x$ *Use to combine two expressions.*

(c) $(x + 2)(x + 3) = x(x + 3) + 2(x + 3) = (x^2 + 3x) + (2x + 6)$
 $= x^2 + (3x + 2x) + 6 = x^2 + 5x + 6$

Distributive Property

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad (3a)$$

$$(a + b) \cdot c = a \cdot c + b \cdot c \quad (3b)$$



Identity Properties

$$0 + a = a + 0 = a \quad (4a)$$

$$a \cdot 1 = 1 \cdot a = a \quad (4b)$$

Additive Inverse Property

$$a + (-a) = -a + a = 0 \quad (5a)$$

Multiplicative Inverse Property

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1 \quad \text{if } a \neq 0 \quad (5b)$$

DEFINITION: DOMAIN

The set of values that a variable may assume is called the **domain of the variable**.

Finding the Domain of a Variable

The domain of the variable x in the expression

$$\frac{5}{x - 2}$$

is $\{x|x \neq 2\}$, since, if $x = 2$, the denominator becomes 0, which is not defined.

PERSAMAAN LINER:

1.1 Linear Equations

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Properties of Real Numbers (Section R.1, pp. 9–13)
- Domain of a Variable (Section R.2, p. 21)



Now Work the 'Are You Prepared?' problems on page 90.

- OBJECTIVES**
- 1 Solve a Linear Equation (p. 84)
 - 2 Solve Equations That Lead to Linear Equations (p. 86)
 - 3 Solve Problems That Can Be Modeled by Linear Equations (p. 87)



PERSAMAAN SATU VARIABEL

- PSV :

pernyataan (bisa bernilai benar atau salah) yang memuat satu variabel dan dinyatakan dengan tanda “=”.

- Contoh:

a. $x + 5 = 9$

c. $\frac{x^2 - 4}{x + 1} = 0$

b. $x^2 + 5x = 2x - 2$

d. $\sqrt{x^2 + 9} = 5$



SOLUSI DAN HIMPUNAN SOLUSI

- Solusi atau akar:
nilai dari suatu variabel yang menghasilkan pernyataan benar pada persamaan.
- Coba tentukan solusi PSV pada contoh dan tuliskan himpunan solusinya.

One method for solving an equation is to replace the original equation by a succession of *equivalent equations* until an equation with an obvious solution is obtained.



CONTOH:

$$2x + 3 = 13$$

$$2x = 10$$

$$x = 5$$

Solving an Equation

Solve the equation: $3x - 5 = 4$



Procedures That Result in Equivalent Equations

1. Interchange the two sides of the equation:

$$\text{Replace } 3 = x \text{ by } x = 3$$

2. Simplify the sides of the equation by combining like terms, eliminating parentheses, and so on:

$$\begin{aligned} &\text{Replace } (x + 2) + 6 = 2x + (x + 1) \\ &\text{by } x + 8 = 3x + 1 \end{aligned}$$

3. Add or subtract the same expression on both sides of the equation:

$$\begin{aligned} &\text{Replace } 3x - 5 = 4 \\ &\text{by } (3x - 5) + 5 = 4 + 5 \end{aligned}$$

4. Multiply or divide both sides of the equation by the same nonzero expression:

$$\begin{aligned} &\text{Replace } \frac{3x}{x-1} = \frac{6}{x-1} \quad x \neq 1 \\ &\text{by } \frac{3x}{x-1} \cdot (x-1) = \frac{6}{x-1} \cdot (x-1) \end{aligned}$$

5. If one side of the equation is 0 and the other side can be factored, then we may use the Zero-Product Property* and set each factor equal to 0:

$$\begin{aligned} &\text{Replace } x(x - 3) = 0 \\ &\text{by } x = 0 \text{ or } x - 3 = 0 \end{aligned}$$



PENYELESAIAN:

Replace the original equation by a succession of equivalent equations.

$$3x - 5 = 4$$

$$(3x - 5) + 5 = 4 + 5 \quad \text{Add 5 to both sides.}$$

$$3x = 9 \quad \text{Simplify.}$$

$$\frac{3x}{3} = \frac{9}{3} \quad \text{Divide both sides by 3.}$$

$$x = 3 \quad \text{Simplify.}$$

✓ **Check:** It is a good practice to check the solution by substituting 3 for x in the original equation.

$$3x - 5 = 4$$

$$3(3) - 5 \stackrel{?}{=} 4$$

$$9 - 5 \stackrel{?}{=} 4$$

$$4 = 4$$

The solution checks. The solution set is $\{3\}$.



Steps for Solving Equations

- STEP 1:** List any restrictions on the domain of the variable.
- STEP 2:** Simplify the equation by replacing the original equation by a succession of equivalent equations following the procedures listed earlier.
- STEP 3:** If the result of Step 2 is a product of factors equal to 0, use the Zero-Product Property and set each factor equal to 0 (procedure 5).
- STEP 4:** Check your solution(s).

PERSAMAAN LINEAR

A linear equation in one variable is equivalent to an equation of the form

$$ax + b = 0$$

where a and b are real numbers and $a \neq 0$.

○ Contoh:

$$3x + 12 = 0 \quad -2x + 5 = 0 \quad \frac{1}{2}x - \sqrt{3} = 0$$



LATIHAN:

Solving a Linear Equation

Solve the equation: $\frac{1}{2}(x + 5) - 4 = \frac{1}{3}(2x - 1)$



PENYELESAIAN

To clear the equation of fractions, multiply both sides by 6, the least common multiple of the denominators of the fractions $\frac{1}{2}$ and $\frac{1}{3}$.

$$\frac{1}{2}(x + 5) - 4 = \frac{1}{3}(2x - 1)$$

$$6\left[\frac{1}{2}(x + 5) - 4\right] = 6\left[\frac{1}{3}(2x - 1)\right] \quad \text{Multiply both sides by 6, the LCM of 2 and 3.}$$

$$3(x + 5) - 6 \cdot 4 = 2(2x - 1) \quad \text{Use the Distributive Property on the left and the Associative Property on the right.}$$

$$3x + 15 - 24 = 4x - 2 \quad \text{Use the Distributive Property.}$$

$$3x - 9 = 4x - 2 \quad \text{Combine like terms.}$$

$$3x - 9 + 9 = 4x - 2 + 9 \quad \text{Add 9 to each side.}$$

$$3x = 4x + 7 \quad \text{Simplify.}$$

$$3x - 4x = 4x + 7 - 4x \quad \text{Subtract 4x from each side.}$$

$$-x = 7 \quad \text{Simplify.}$$

$$x = -7 \quad \text{Multiply both sides by } -1.$$

LATIHAN

Solving an Equation That Leads to a Linear Equation

Solve the equation: $(2y + 1)(y - 1) = (y + 5)(2y - 5)$

Solving an Equation That Leads to a Linear Equation

Solve the equation: $\frac{3}{x - 2} = \frac{1}{x - 1} + \frac{7}{(x - 1)(x - 2)}$



LATIHAN:

Solve the equation: $\frac{3x}{x-1} + 2 = \frac{3}{x-1}$



PENYELESAIAN:

$$(2y + 1)(y - 1) = (y + 5)(2y - 5)$$

$$2y^2 - y - 1 = 2y^2 + 5y - 25$$

$$-y - 1 = 5y - 25$$

$$-y = 5y - 24$$

$$-6y = -24$$

$$y = 4$$

Multiply and combine like terms.

Subtract $2y^2$ from each side.

Add 1 to each side.

Subtract $5y$ from each side.

Divide both sides by -6 .

✓**Check:** $(2y + 1)(y - 1) = [2(4) + 1](4 - 1) = (8 + 1)(3) = (9)(3) = 27$

$$(y + 5)(2y - 5) = (4 + 5)[2(4) - 5] = (9)(8 - 5) = (9)(3) = 27$$

Since the two expressions are equal, the solution $y = 4$ checks.

The solution set is $\{4\}$.

PENYELESAIAN:

$$\frac{3}{x-2} = \frac{1}{x-1} + \frac{7}{(x-1)(x-2)}$$

$$(x-1)(x-2) \frac{3}{x-2} = (x-1)(x-2) \left[\frac{1}{x-1} + \frac{7}{(x-1)(x-2)} \right]$$

$$3x - 3 = (x-1)(x-2) \frac{1}{x-1} + (x-1)(x-2) \frac{7}{(x-1)(x-2)}$$

$$3x - 3 = (x-2) + 7$$

$$3x - 3 = x + 5$$

$$2x = 8$$

$$x = 4$$

Multiply both sides by $(x-1)(x-2)$. Cancel on the left.

Use the Distributive Property on each side; cancel on the right.

Combine like terms.

Add 3 to each side.

Subtract x from each side.

Divide by 2.

✓**Check:** $\frac{3}{x-2} = \frac{3}{4-2} = \frac{3}{2}$

$$\frac{1}{x-1} + \frac{7}{(x-1)(x-2)} = \frac{1}{4-1} + \frac{7}{(4-1)(4-2)} = \frac{1}{3} + \frac{7}{3 \cdot 2} = \frac{2}{6} + \frac{7}{6} = \frac{9}{6} = \frac{3}{2}$$

Since the two expressions are equal, the solution $x = 4$ checks.

The solution set is $\{4\}$.



PENYELESAIAN:

- Tentukan domain dulu.

$$\begin{aligned}\frac{3x}{x-1} + 2 &= \frac{3}{x-1} \\ \left(\frac{3x}{x-1} + 2\right) \cdot (x-1) &= \frac{3}{\cancel{x-1}} \cdot \cancel{(x-1)} \\ \frac{3x}{\cancel{x-1}} \cdot \cancel{(x-1)} + 2 \cdot (x-1) &= 3 \\ 3x + 2x - 2 &= 3 \\ 5x - 2 &= 3 \\ 5x &= 5 \\ x &= 1\end{aligned}$$

Multiply both sides by $x - 1$; cancel on the right.

Use the Distributive Property on the left side; cancel on the left.

Simplify.

Combine like terms.

Add 2 to each side.

Divide both sides by 5.

The solution appears to be 1. But recall that $x = 1$ is not in the domain of the variable. The equation has no solution.



SOAL APLIKASI

Converting to Fahrenheit from Celsius

In the United States we measure temperature in both degrees Fahrenheit ($^{\circ}\text{F}$) and degrees Celsius ($^{\circ}\text{C}$), which are related by the formula $C = \frac{5}{9}(F - 32)$. What are the Fahrenheit temperatures corresponding to Celsius temperatures of 0° , 10° , 20° , and 30°C ?

Investments

A total of \$18,000 is invested, some in stocks and some in bonds. If the amount invested in bonds is half that invested in stocks, how much is invested in each category?



TUGAS:

- Tentukan himpunan penyelesaian persamaan berikut:

$$25. 2(3 + 2x) = 3(x - 4)$$

$$33. \frac{2}{3}p = \frac{1}{2}p + \frac{1}{3}$$

$$34. \frac{1}{2} - \frac{1}{3}p = \frac{4}{3}$$

$$37. \frac{x + 1}{3} + \frac{x + 2}{7} = 2$$

$$40. \frac{4}{y} - 5 = \frac{5}{2y}$$

$$43. (x + 7)(x - 1) = (x + 1)^2$$

$$50. \frac{2x}{x + 3} = \frac{-6}{x + 3} - 2$$

$$57. \frac{6t + 7}{4t - 1} = \frac{3t + 8}{2t - 4}$$

TUGAS:

