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SOLVING CAPACITATED VEHICLE ROUTING PROBLEMS WITH TIME WINDOWS BY GOAL PROGRAMMING APPROACH

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Abstract. This paper presents how to build multiobjective linear programming model as solution of Capacitated Vehicle Routing Problem with Time Windows (CVRPTW). We use a goal programming approach to solve the model. We have discussed an objective function for two main goals: the first is to minimize the total number of vehicles and the second is to minimize the travelling time of the used vehicles. The proposed model is applied to a problem distribution of Liquefied Petroleum Gas (LPG). Computational results of the proposed model are discussed.

Key words and Phrases: CVRPTW, goal programming, objective function.

1. Introduction

Several things that related to the distribution problems are how many goods are distributed, how the cost of distribution should be spent , the route of distribution, the travelling time and the number of vehicles are used. Along with the success of the conversion program from kerosene to Liquefied Petroleum Gas (LPG), the needs of LPG are increased. PT Pertamina Persero as the company has maximized distribution of LPG throughout the territory. But LPG shortages still occurs. Consequently, the distribution problems are important to be considered. They are known as Vehicle Routing Problem (VRP).

Vehicle Routing Problem (VRP) is a class of problem in which a set of routes for a fleet of delivery vehicles based at one or several depots must be determined for a number of customers. Main objective of VRP for serving known customer demands by a minimum-cost vehicle routing starting and ending at a depot. Vehicle Routing Problem (VRP) which reducing the cost while delivering on time, namely VRP Time Windows (VRPTW), has become popular issue in optimization research. On the other hand, if the distribution depends on the capacity of vehicle, it is called Capacitated VRP (CVRP) [7]. In this paper, we discuss the CVRP with time windows (CVRPTW). It means that the vehicles must arrive to the customer in a restricted time interval which is given. It can be stated as follows: find a feasible solution of the vehicles, so that to minimize both, the number of vehicles used and the total travel time. To solve the problem, we develop a goal programming model which has more than one goal in the objective function, even its goal contradict each other.

A goal programming approach is an important technique for modelling and helping decision-makers to solve multi objective problems in finding a set of feasible solutions. The purpose of goal programming is to minimize the deviations between the achievement of goals and their aspiration levels [8]. Based on Azi *et.al* [1], Jolai and Aghdaghi [4], if VRP has a time constrains on the periods of the day in which each customer must be visited, it is called Vehicle Routing Problem with Time Window (VRPTW). Belfiore, *et.al* [2], Calvete, *et.al* [3] and Hashimoto, *et al* [5] have developed VRP with time windows. Ombuki, *et.al* [6] have applied a multi objective genetics algorithm to solve VRPTW. Sousa,*et al* [7] have used a multi objective approach to solve a goods distribution by The Just in Time Delivery S.A, a distribution company using Mixed Integer Linear Programming.

In this paper, we present a goal programming model as solution of Capacitated Vehicle Routing Problem with Time Windows (CVRPTW). The objective function are two main goals: the first is to minimize the total number of vehicles and the second is to minimize the travelling time of the used vehicles. Computational results of the proposed model will be discussed. The paper is organized as follows, in Section 2: the mathematical models of Capacitated Vehicle Routing Problem with Time Windows (CVRPTW). In Section 3, Computational results of the proposed model using a set of data obtained. Finally in Section 4, we describe conclusions.

2. Mathematical Models

We formulate CVRPTW model with 5 nodes (customers location) and reasoning as follow:

- A fleet of vehicles with restricted and known capacity which is needed to minimize the number of vehicles.
- One depot with non restricted and known capacity.

- A set of 5 nodes with demand of goods, service time window, travelling time.

We assume that the average velocity of vehicle is constant.

Let $V = \{1, 2, ..., v\}$ represents the fleet of vehicles, $N = \{0, 1, 2, ..., n+1\}$ meaning the set of all nodes including depot, each node representing a customer location. Every vehicle must depart from depot and terminate at node n+1. We formulate the problem as a graph $G(N, A)$ where A is the set of directed arc (the network connections between the customer and the depot) [7]. Each connection is assosiated with a travel time $time_{ij}, \forall i, j \in N, i \neq j$. Time window for customer *i*, $[t_{ia}, t_{ib}]$. It means the vehicle must arrive between that interval time.

The following of mathematical symbols and decision variables used in this paper:

- C : the set of n customers.
- The connection between node *i* to *j* using vehicle *k*.

$$x_{ijk} \begin{cases} 1 & \text{, active} \\ 0 & \text{, not} \end{cases} \qquad , \forall i, j \in N, \forall k \in V \quad , i \neq j, i \neq n+1, j \neq 0.$$

- t_{ik} : time when vehicle *k* come on node *i*, $, \forall i \in N, \forall k \in V$.
- $[t_{ia}, t_{ib}]$: time window for customer i $, \forall i \in C$
- D_i : demand or quantity of goods for customer i $, \forall i \in C$.
- Q_k : vehicle capacity $, \forall k \in V$
- T : constant parameter to consider in the time window constraint $\rightarrow +\infty$.
- d_1^- : negative deviational variable of first goal
- d_2^+ : positive deviational variable of second goal

Goal Programming Model

To generate a goal programming model for the problem, we set the following goals:

Goal 1 : minimize the total number of vehicles/ avoid underutilization of vehicle capacity

Goal 2 : minimize the travelling time of the used vehicles.

So the mathematic formulation of goal programming model is

$$\text{Min } Z = \omega_1 \sum_{k}^{V} d_{1k}^{-} + \omega_2 d_2^{+} \quad (1)$$

subject to

$$\sum_{k}^{V} \sum_{i}^{N} \sum_{j}^{N} x_{ijk} \text{time}_{ij} - d_2^{+} = 0 . \quad (2)$$

$$\sum_{k}^{V} \sum_{j}^{N} x_{ijk} = 1 \quad , \forall i \in C \quad (3)$$

$$\sum_{j}^{N} x_{0jk} = 1 \quad , \forall k \in V \quad (4)$$

$$\sum_{i}^{N} x_{iak} - \sum_{j}^{N} x_{ajk} = 0 \quad , \forall i \in C, \forall k \in V \quad (5)$$

$$\sum_{i}^{N} x_{i,n+1,k} = 1 \quad , \forall k \in V \quad (6)$$

$$\sum_{i}^{C} D_i \sum_{j}^{N} x_{ijk} + d_{1k}^{-} = Q_k \quad , \forall k \in V \quad (7)$$

$$t_{ik} + \text{time}_{ij} - T\left(1 - x_{ijk}\right) \le t_{jk} \quad , \forall i, j \in N, \forall k \in V, T \to +\infty \quad (8)$$

$$t_{ai} \le t_{ik} \le t_{bi} \quad , \forall i \in N, \forall k \in V \quad (9)$$

$$x_{ijk} \in \{0,1\} \quad , \forall i, j \in N, \forall k \in V \quad (10)$$

$$t_{ik} \ge 0 \quad , \forall i \in C, \forall k \in V \quad (11)$$

Based on the model, Eq.(3) is a constraint to ensure that one vehicle visits each customer only. Constrains in Eq.(4), Eq.(5), and Eq(6) to guarantee that each vehicle depart from depot, a vehicle unload the goods after comes to a customer and then go to the next customer . Eq.(7) shows the vehicles only maximum loaded in its capacity. The constraint in Eq.(8) guarantee feasibility of the time schedule of a service time. Eq.(9) is a constraint that represent the time windows for every customer. Decision variables are represented by Eq.(10) and Eq.(11).

3. Computational Results

In this section, we apply the model to the data that obtained from PT Pertamina. The following data includes time distance, demand, and time windows for every customer, see Table.1 and Table.2.

Table 1. Time distance between the nodes (minutes)

Time units	Depot	A	B	C	D	E
Depot	0	7	8	10	3	8
A	5	0	5	6	6	7
B	7	3	0	5	6	8
C	6	2	3	0	7	6
D	4	5	5	7	0	8
E	7	8	8	6	9	0

Table 2. Demand and time windows for each customer

Customer	A	B	C	D	E
Demand (units)	90	200	100	125	100
t_a (time units)	7	8	10	3	8
t_b (time units)	37	75	43	45	41

The customers can be described like shown as follows:

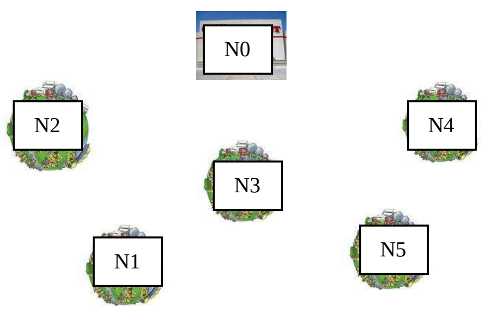

Figure 1. Depot and Customers

A computational process is done using LINGO. The following result obtained routes as Fig. 2.

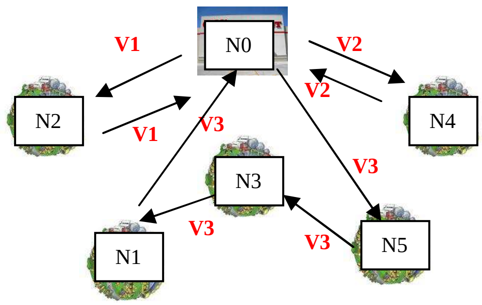

Figure 2. Depot and Customers Route Using Vehicle's Capacity 300 units

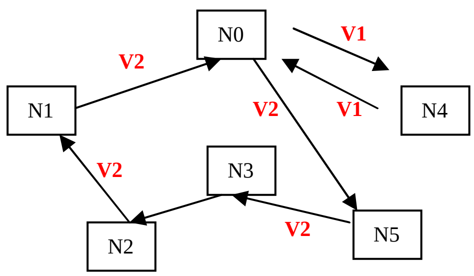

Figure 3. Depot and Customers Route Using Vehicle's Capacity 560 units

Figure 2 Shows that we need three vehicles to satisfy the customer's demand. If we increase the capacity of the vehicles to 560 units, the result can be showed in Fig,3 above. It means, only needs two vehicles to distribute the demands.

4. Conclusion

In this paper we proposed a multi objective linear programming approach to solve a Capacitated Vehicle Routing Problem with Time Windows (CVRPTW). Two goals are to minimize both, the number of vehicles used and the total travel time. The model is applied in the distribution of LPG with five customers. Time windows are very important to be considered. So the travel time can be minimized. As future development we suggest to improve the method to solve CVRPTW.

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