Basic Objects of Geometry

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Undefined Terms

- Point
- Line
- Plane

There is no definition of point, line and plane, but there are some characteristics of that terms

Point (1)

- A Point has no size and no dimension
- There are countless points on any one line
- A point has an exact location on a graph, shape or in "space"

Point (2)

- A point is indicated by a pencil scratch marks on a paper
- A point is named with an italicized uppercase letter placed next to it

A •

- A series of points are make up lines, segments, rays, and planes

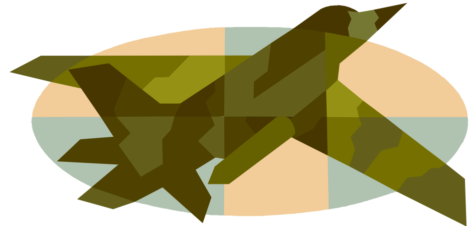
Line (1)

- A line goes in opposite directions and never, never, never ends.
- A line has an infinite set of points that extends in both directions.
- A line is perfect straight and has no thickness.

Line (2)

- A line is named by any one italicized lowercase letter or by naming any two points on the line.
- If the line is named by using two points on the line, a small symbol of a line (with two arrows) is written above the two letters. For example, this line could be referred to as line $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$ or g

Plane (1)



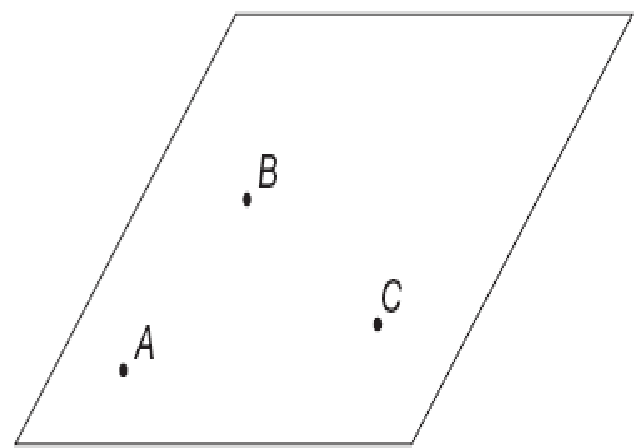
- A Plane (no, not the one that flies!) is a flat surface that goes on forever in all directions.
- Imagine sitting on a row boat in the middle of the ocean. No matter which way you look…all you see is water…forever.
- Or, imagine a floor that extends in all directions as far as you can see

Plane (2)

- A *plane* is an infinite set of points extending in all directions along a perfectly flat surface. It is infinitely long and infinitely wide.
- A *plane* is a flat surface that has no thickness or boundaries.

Plane (3)

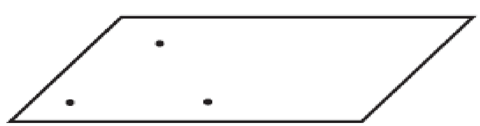
A plane is named by a single uppercase letter and is often represented as a four-sided figure, as in planes *U* or named by three uppercase letters, plane ABC.

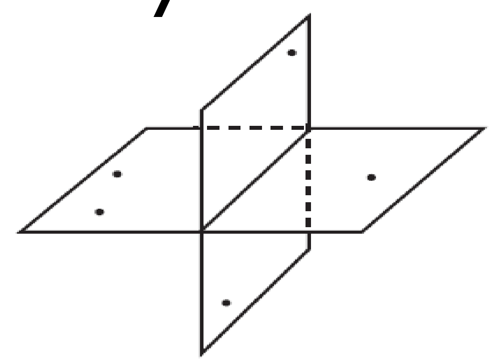


Collinear and Coplanar

- Some points are collinear if they are on a same line

- Some points are coplanar if they are on a same plane





Relation Between Two Objects

Two Points ?

Point and Line ?

Point and Plane ?

Two Lines ?

Line and Plane ?

Two Planes ?

Relation Between Two Objects (1)

Two Points

- Coincide/same
- Different

Point and Line

- Point lies on the line or the line passes through the point
- Point is outside of the line

Relation Between Two Objects (2)

Point and Plane

- Point lies on the plane
- Point doesn't lie on the plane

Two Lines

- Lies on the same plane: coincides, parallel, cuts each other
- Doesn't lie on the same plane (in space geometry)

Relation Between Two Objects (3)

Line and Plane

- The line lies on the plane
- The line is parallel to the plane
- The line cuts the plane

Two Planes (in a space geometry)

- Two planes are coincide
- The planes are parallel
- Two planes cut each other

Postulates and Theorems (1)

- Geometry begins with assumptions about certain things that are very difficult, if not, it is impossible to prove and to flow on to things that can be proven.
- The assumptions that geometry's logic is based upon are called *postulates*. Sometimes, they referred as *axioms*. The two words mean essentially the same thing.

Postulates and Theorems (2)

Postulate 1: A line contains at least two points.

Postulate 2: A plane contains a minimum of three non-collinear points.

Postulate 3: Through any two points there can be exactly one line.

Postulate 4: Through any three non-collinear points there can be exactly one plane.

Postulate 5: If a line contains two points lie in a plane, then the line lie on the same plane.

Postulate 6: When two planes intersect, their intersection is a line.

Postulates and Theorems (3)

From the six postulates it is possible to prove these **theorems.**

Theorem 1: If two lines intersect, then they intersect in exactly one point.

Theorem 2: If a line intersect an outside plane, then their intersection is a point.

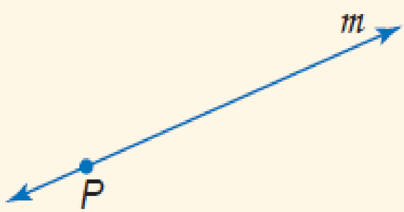
Postulates and Theorems (4)

From the six postulates it is possible to prove these **theorems.**

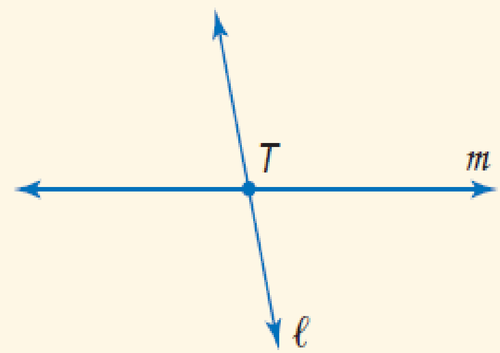
Theorem 3: If a point lies outside a line, then exactly one plane contains the line and the point.

Theorem 4: If two lines intersect, then exactly one plane contains both lines.

Describing What You See

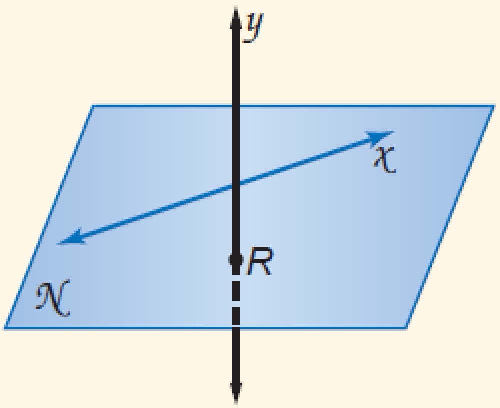


Point P is on line m.
Line m contains P.
Line m passes through P.

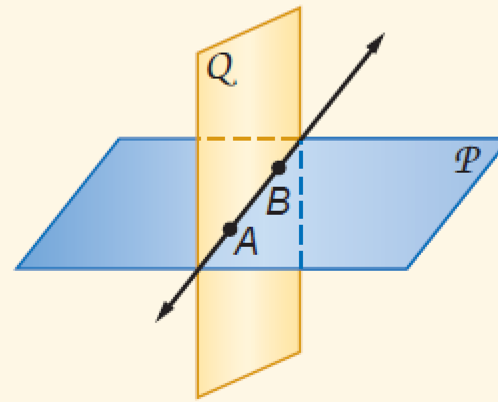


Lines ℓ and m intersect in T.
Point T is the interesection of lines ℓ and m.
Point T is on line m. Point T is on line ℓ.

Describing What You See

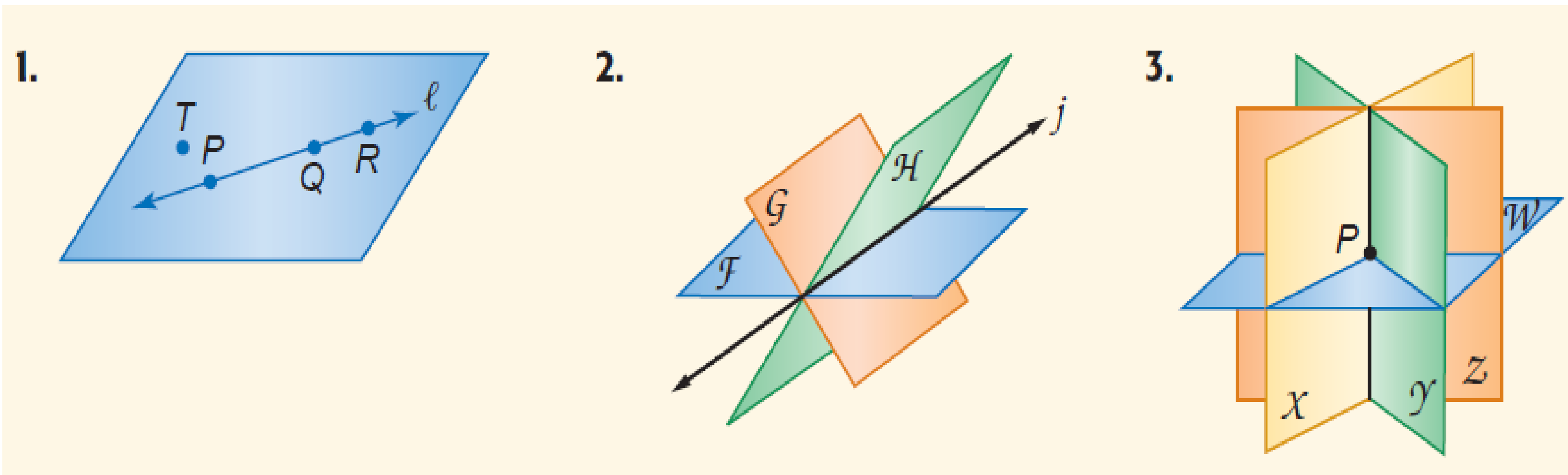


Line x and point R are in $\mathcal{N}$.
Point R lies in $\mathcal{N}$.
Plane $\mathcal{N}$ contains R and line x.
Line y intersects $\mathcal{N}$ at R.
Point R is the intersection of line y with $\mathcal{N}$.
Lines y and x do not intersect.



$\overleftrightarrow{AB}$ is in $\mathcal{P}$ and $\mathcal{Q}$.
Points A and B lie in both $\mathcal{P}$ and $\mathcal{Q}$.
Planes $\mathcal{P}$ and $\mathcal{Q}$ both contain $\overleftrightarrow{AB}$.
Planes $\mathcal{P}$ and $\mathcal{Q}$ intersect in $\overleftrightarrow{AB}$.
$\overleftrightarrow{AB}$ is the intersection of $\mathcal{P}$ and $\mathcal{Q}$.

Describe the figure below !



Segments and Rays

Much of geometry deals with parts of lines, that are segment and ray

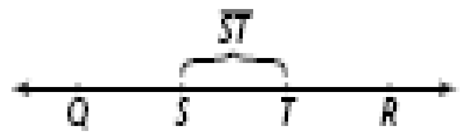
Ray

- A ray is part of a line, but it has one endpoint and the other end keeps going. A ray has an infinite number of points on it.
- Laser beams is a good example of rays.
- When you refer to a ray, you always name the endpoint first.

Raaaaaaaaaaaaaaaaaaaaaaaayyyyyyyyyyyyyyyyyyyyyyyyyyy

Line Segment

- A *line segment* is a finite portion of a line and is named for its two endpoints. In the preceding diagram is segment. It has an infinite number of points on it.



- A ruler is an example of line segments.
- Line segments are also named with two italicized uppercase letters, but the symbol above the letters has no arrows.
- Notice the bar above the segment's name. Technically, refers to points *S* and *T* and all the points in between them. *ST*, without the bar, refers to the distance from *S* to *T*. You'll notice that is a portion of line QR.

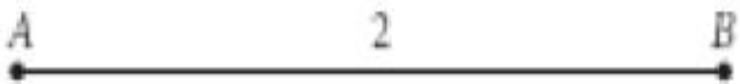
Line Segment

- Each point on a line or a segment can be paired with a single real number, which is known as that point's *coordinate*. The *distance* between two points is the absolute value of the difference of their coordinates.

m $\overleftrightarrow{AB}$ = 2 cm

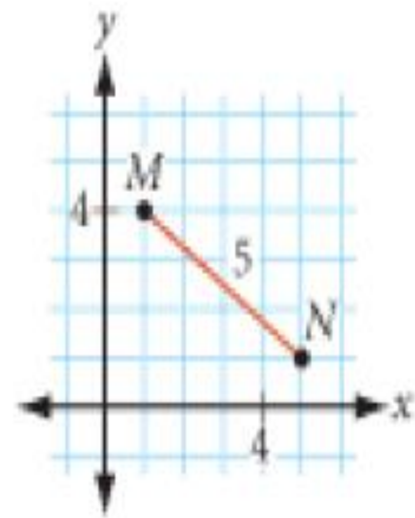
AB = 2 cm

Figure A



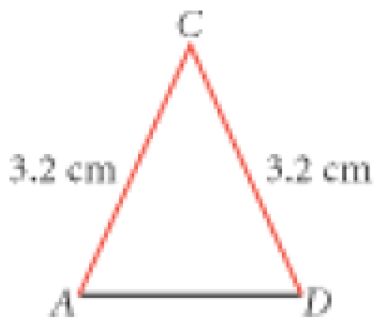
$AB = 2$ in., or $m\overline{AB} = 2$ in.

Figure B



$MN = 5$ units, or $m\overline{MN} = 5$ units

Two segments are **congruent** if and only if they have equal measures, or lengths.



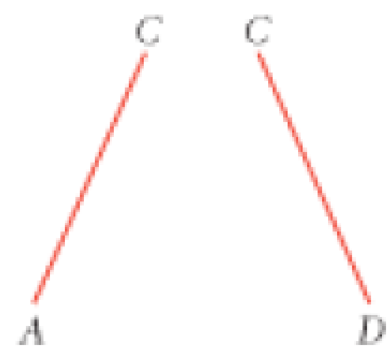
You use "is equal to" with numbers.

$AC = DC$

$3.2 \text{ cm} = 3.2 \text{ cm}$

You use "is congruent to" with figures.

$\overline{AC} \cong \overline{DC}$



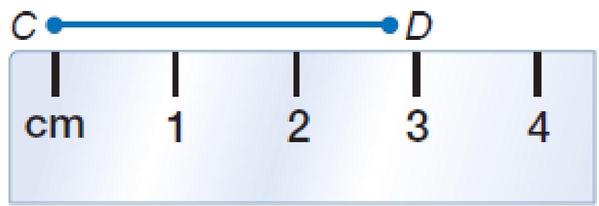
When drawing figures, you show congruent segments by making identical markings.

The Precision of Measurement

- The precision of any measurement depends on the smallest unit available on the measuring tool

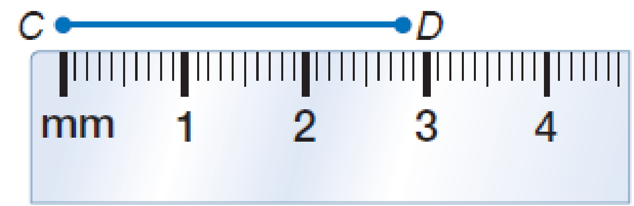
Find the length of $\overline{CD}$ using each ruler.

a.



The ruler is marked in centimeters. Point *D* is closer to the 3-centimeter mark than to 2 centimeters. Thus, $\overline{CD}$ is about 3 centimeters long.

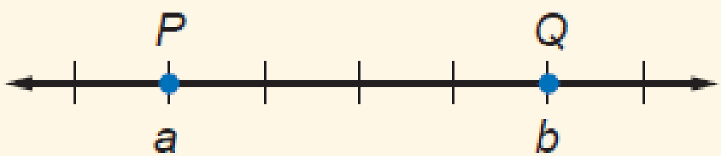
b.



The long marks are centimeters, and the shorter marks are millimeters. There are 10 millimeters for each centimeter. Thus, $\overline{CD}$ is about 28 millimeters long.

Distance Between Two Points

Number Line



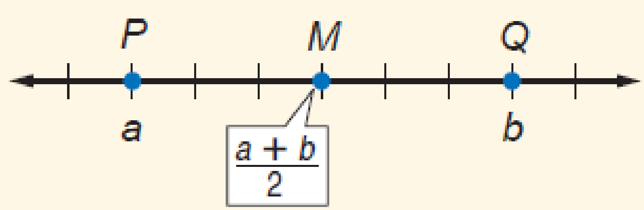
$$PQ = |b - a| \text{ or } |a - b|$$

Coordinate Plane

The distance d between two points with coordinates (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Midpoint of a Segment

<table>
<tr><td>Words</td><td colspan="2">The midpoint M of $\overline{PQ}$ is the point between P and Q such that $PM = MQ$.</td></tr>
<tr><td rowspan="2">Symbols</td><td>Number Line</td><td>Coordinate Plane</td></tr>
<tr><td>The coordinate of the midpoint of a segment with endpoints that have coordinates a and b is $\frac{a+b}{2}$.</td><td>The coordinates of the midpoint of a segment with endpoints that have coordinates (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.</td></tr>
<tr><td>Models</td><td>


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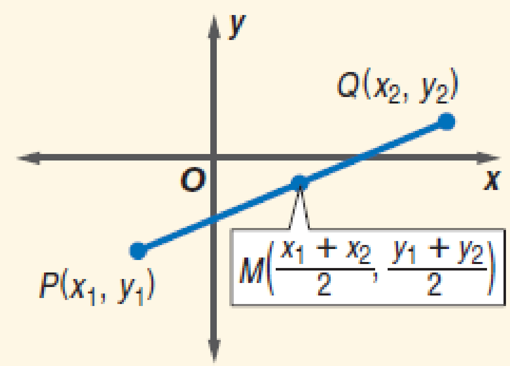

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</table>

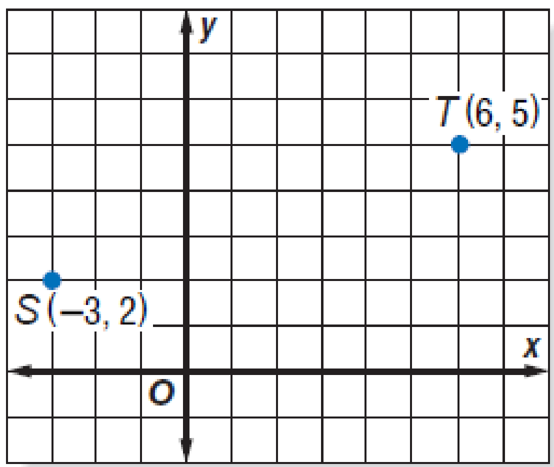
Exercise

Use the Distance Formula to find the distance between each pair of points.

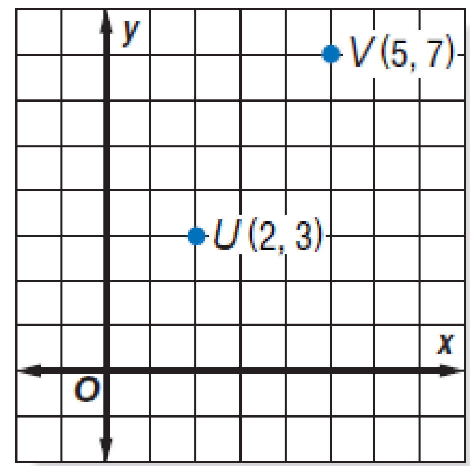
23. $J(0, 0), K(12, 9)$

24. $L(3, 5), M(7, 9)$

25.



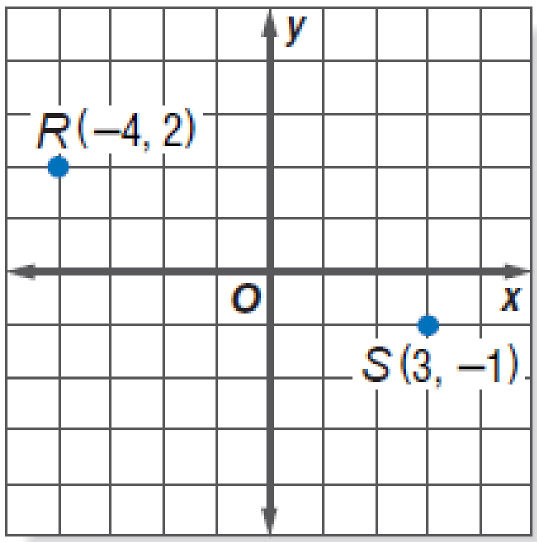
26.



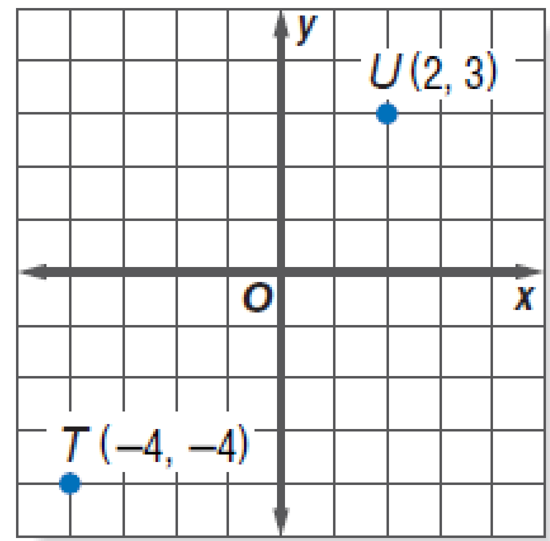
Exercise

Find the coordinates of the midpoint of a segment having the given endpoints.

35.



36.

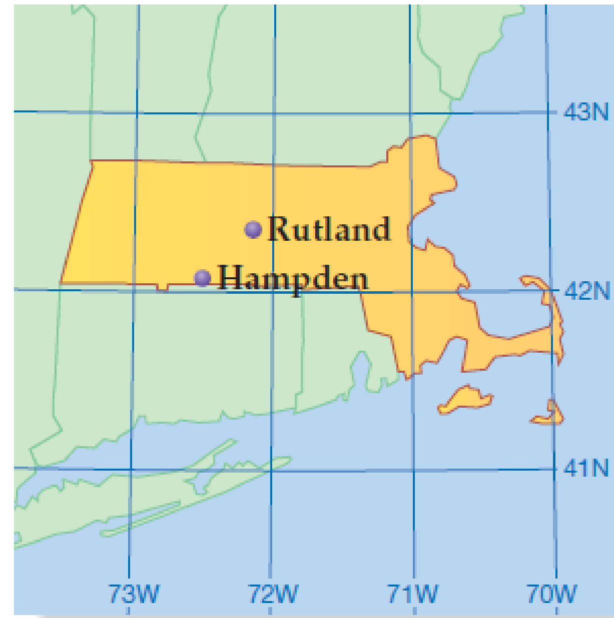


37. $A(8, 4), B(12, 2)$

38. $C(9, 5), D(17, 4)$

Exercise

GEOGRAPHY For Exercises 46–49, use the following information.
The geographic center of Massachusetts is in Rutland at $(42.4°, 71.9°)$, which represents north latitude and west longitude. Hampden is near the southern border of Massachusetts at $(42.1°, 72.4°)$.



46. If Hampden is one endpoint of a segment and Rutland is its midpoint, find the latitude and longitude of the other endpoint.

47. Use an atlas or the Internet to find a city near the location of the other endpoint.

48. If Hampden is the midpoint of a segment with one endpoint at Rutland, find the latitude and longitude of the other endpoint.

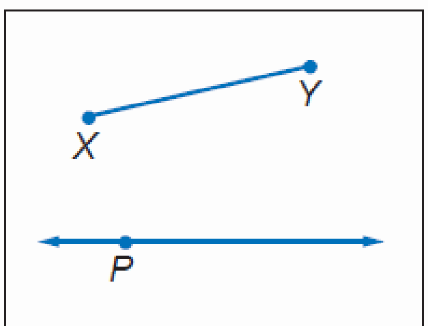
49. Use an atlas or the Internet to find a city near the location of the other endpoint.

Construction

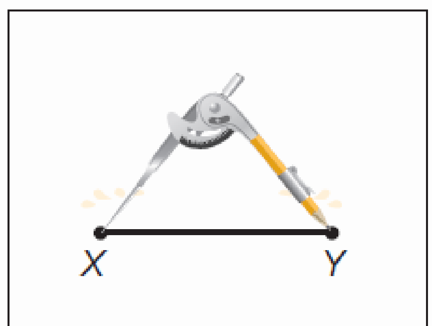
CONSTRUCTION

Copy a Segment

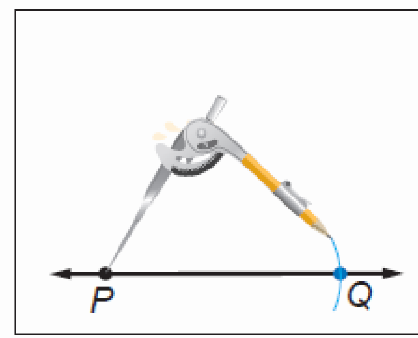
Step 1 Draw a segment $\overline{XY}$. Elsewhere on your paper, draw a line and a point on the line. Label the point P.



Step 2 Place the compass at point X and adjust the compass setting so that the pencil is at point Y.



Step 3 Using that setting, place the compass point at P and draw an arc that intersects the line. Label the point of intersection Q. Because of identical compass settings, $\overline{PQ} \cong \overline{XY}$.



Construction

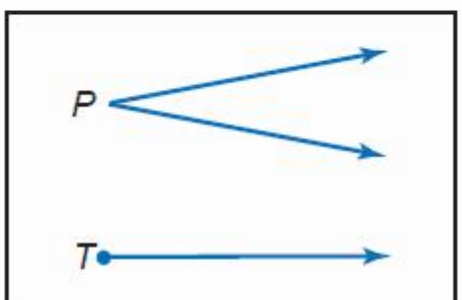
CONSTRUCTION

Concepts in Mot

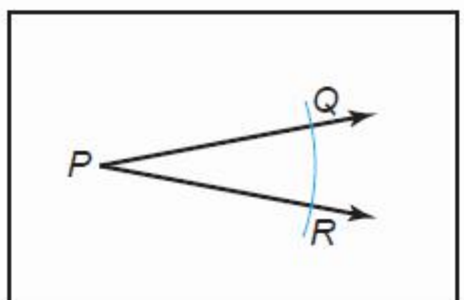
Animation geometryonlin

Copy an Angle

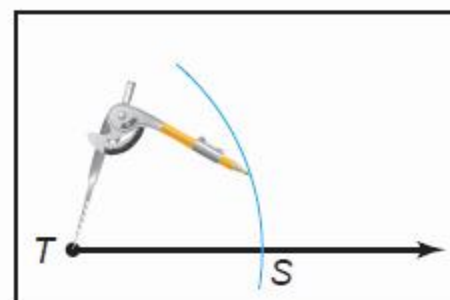
Step 1
Draw an angle like ∠*P* on your paper. Use a straightedge to draw a ray on your paper. Label its endpoint *T*.



Step 2
Place the tip of the compass at point *P* and draw a large arc that intersects both sides of ∠*P*. Label the points of intersection *Q* and *R*.

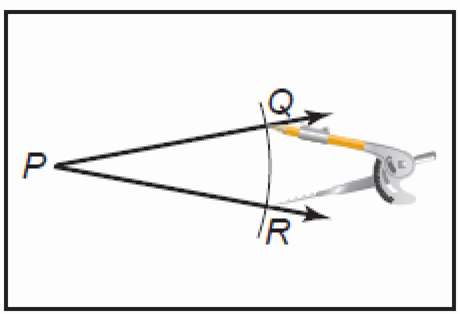


Step 3
Using the same compass setting, put the compass at *T* and draw a large arc that intersects the ray. Label the point of intersection *S*.



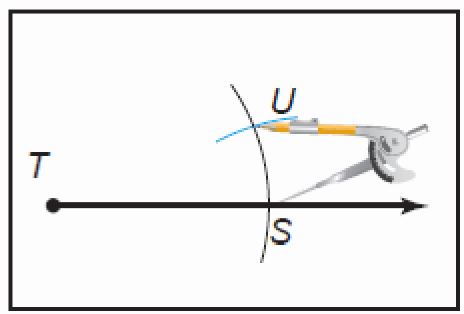
Step 4

Place the point of your compass on *R* and adjust so that the pencil tip is on *Q*.



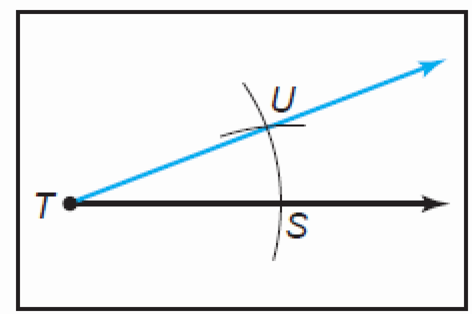
Step 5

Without changing the setting, place the compass at *S* and draw an arc to intersect the larger arc you drew in Step 3. Label the point of intersection *U*.



Step 6

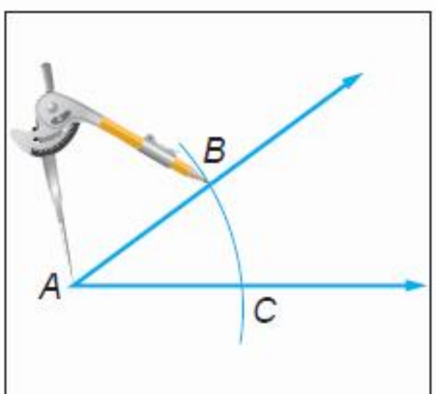
Use a straightedge to draw $\overrightarrow{TU}$.



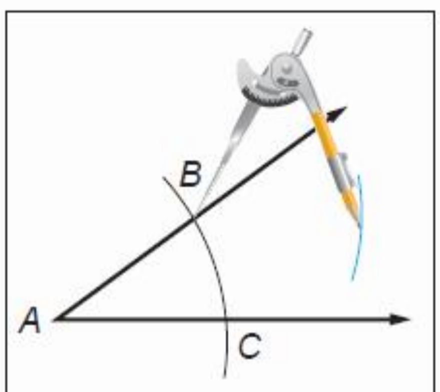
CONSTRUCTION

Bisect an Angle

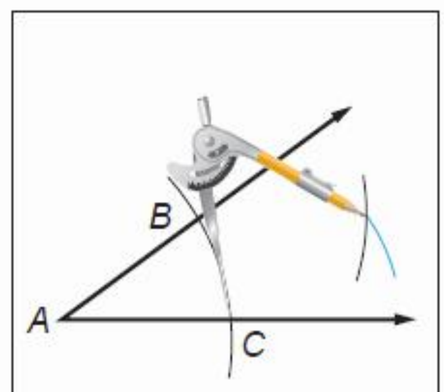
Step 1
Draw an angle and label the vertex as *A*. Put your compass at point *A* and draw a large arc that intersects both sides of $\angle A$. Label the points of intersection *B* and *C*.



Step 2
With the compass at point *B*, draw an arc in the interior of the angle.



Step 3
Keeping the same compass setting, place the compass at point *C* and draw an arc that intersects the arc drawn in Step 2.



Step 4
Label the point of intersection *D*. Draw $\overrightarrow{AD}$. $\overrightarrow{AD}$ is the bisector of $\angle A$. Thus, $m\angle BAD = m\angle DAC$ and $\angle BAD \cong \angle DAC$.