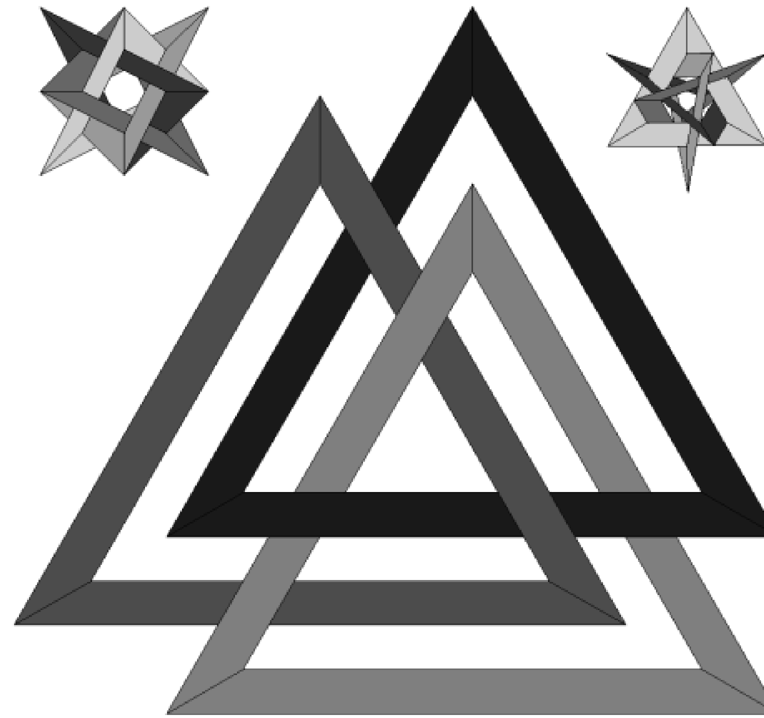
Congruence



Department of Mathematics Education
Faculty of Mathematics and Science YSU
2014

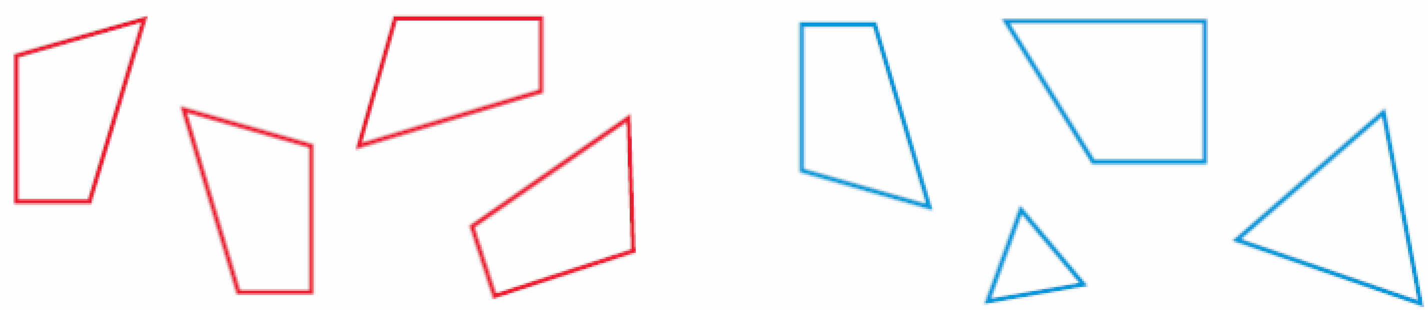
Congruent Polygons

Congruency

Two geometric figures are *congruent* if they have exactly the same size and shape.

Identify Congruent Figure

Each of the red figures is congruent to the other red figures. None of the blue figures is congruent to another blue figure.

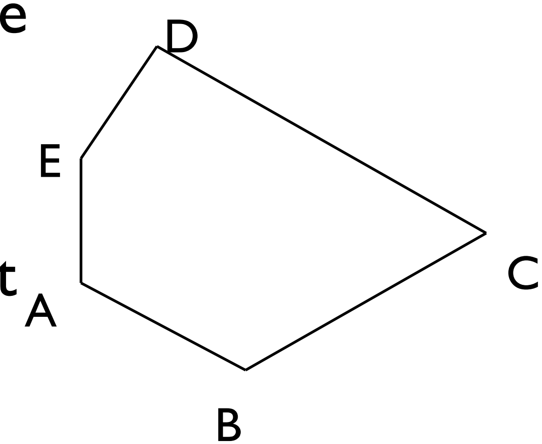


Identify Congruent Figure

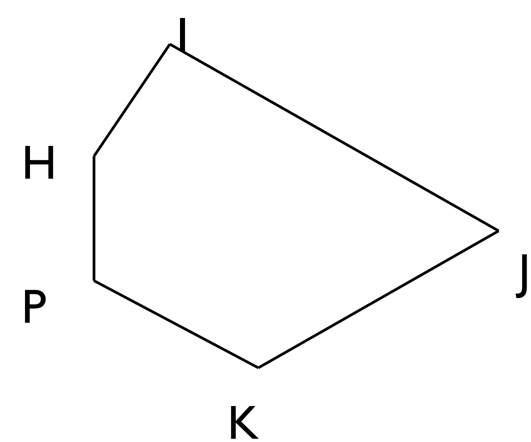
- When two figures are congruent, there is a correspondence between their angles and sides such that corresponding angles are congruent and corresponding sides are congruent.
- In other words, they have matching angles and matching sides.

Naming & Comparing Polygons

- List vertices in order, either clockwise or counterclockwise.
- When comparing 2 polygons, begin at corresponding vertices; name the vertices in order and; go in the same direction.
- By doing this you can identify corresponding parts.



DCBAE



IJKPH

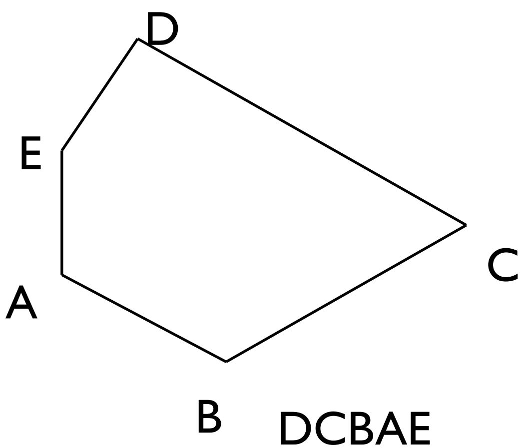
$\angle D$ corresponds to $\angle I$

$\overline{AE}$ corresponds to $\overline{PH}$

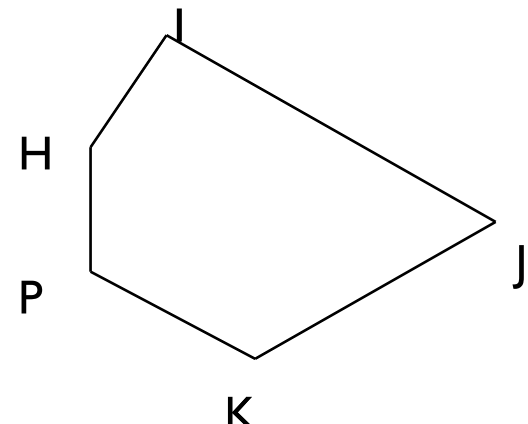
Name Corresponding Parts

- Name all the angles that correspond:

∠ D corresponds to ∠ I
∠ C corresponds to ∠ J
∠ B corresponds to ∠ K
∠ A corresponds to ∠ P
∠ E corresponds to ∠ H



DCBAE



IJKPH

- Name all the segments that correspond:

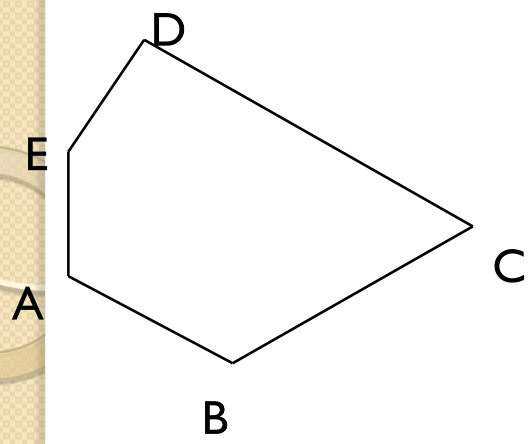
$\overline{DC}$ corresponds to $\overline{IJ}$
$\overline{CB}$ corresponds to $\overline{JK}$
$\overline{BA}$ corresponds to $\overline{KP}$
$\overline{AE}$ corresponds to $\overline{PH}$
$\overline{ED}$ corresponds to $\overline{HI}$

How many corresponding angles are there?

5

How many corresponding sides are there?

5



How many ways can you name pentagon DCBAE?

10

Do it.

Pick a vertex and go clockwise

DCBAE

CBAED

BAEDC

AEDCB

EDCBA

Pick a vertex and go counterclockwise

DEABC

CDEAB

BCDEA

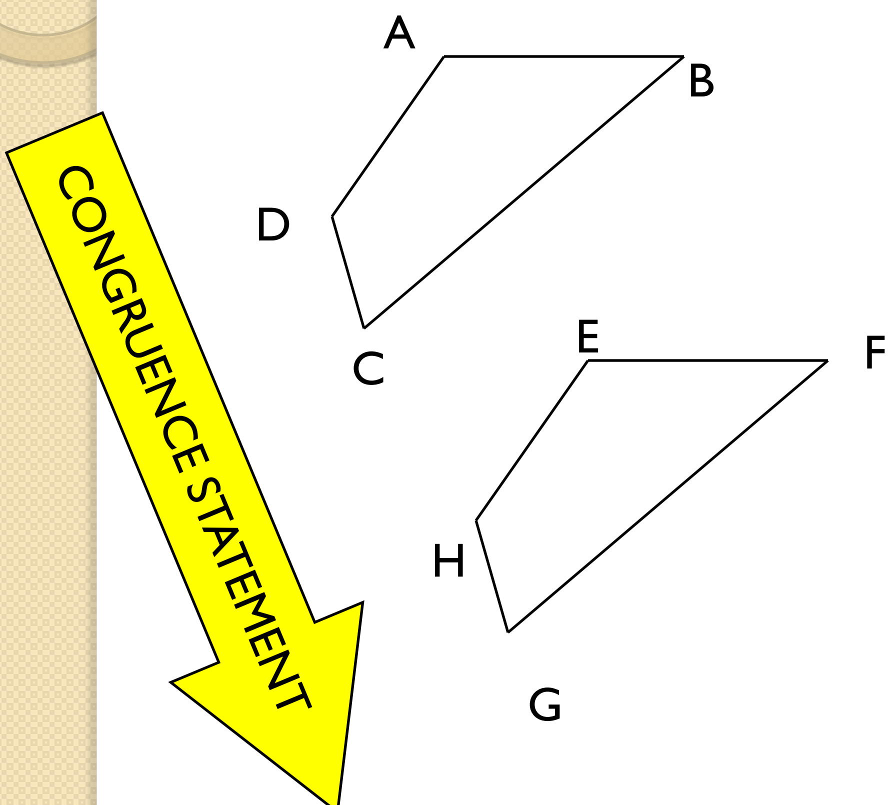
ABCDE

EABCD

Polygon Congruence Postulate

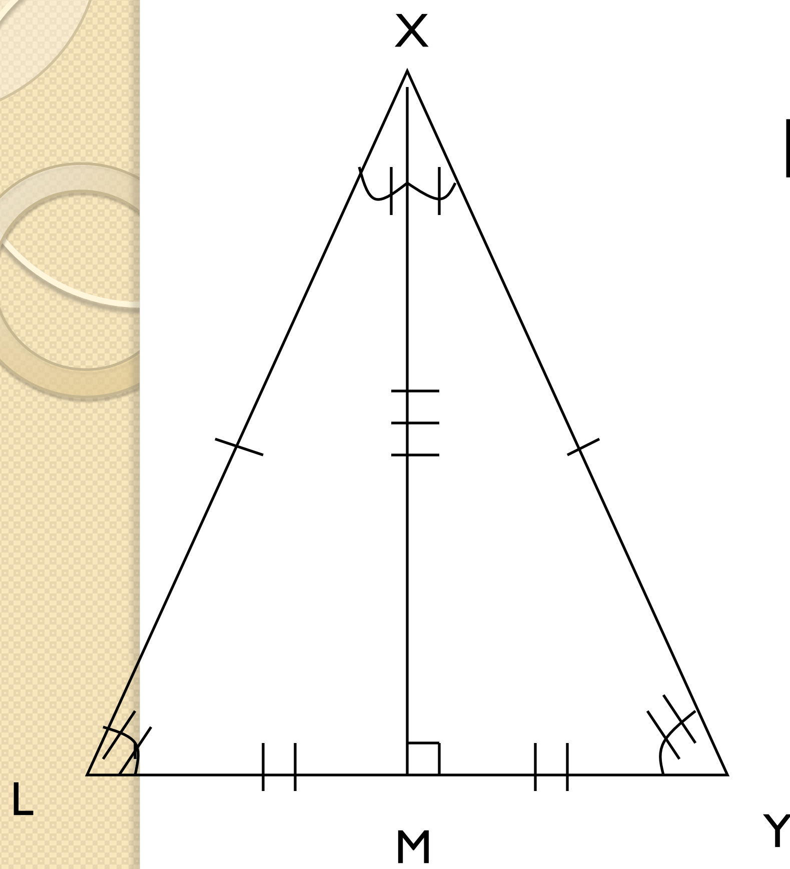
If each pair of corresponding angles is congruent, and each pair of corresponding sides is congruent, then the two polygons are congruent.

Congruence Statements



ABCD ≅ EFGH

- Given: These polygons are congruent.
- Remember, if they are congruent, they are EXACTLY the same.
- That means that all of the corresponding angles are congruent and all of the corresponding sides are congruent.
- DO NOT say that 'all the sides are congruent" and "all the angles are congruent", because they are not.



Prove: ΔLXM ≅ ΔYXM

Statements	Reasons
$\overline{XY} \cong \overline{XL}$	Given
$\overline{LM} \cong \overline{YM}$ Given	
$\overline{XM} \cong \overline{XM}$	Reflexive Property
∠LXM ≅ ∠YXM	Given
∠L ≅ ∠Y	Given
∠XMY ≅ ∠SML	Right angles
ΔLXM ≅ ΔYXM	Polygon Congruence Postulate

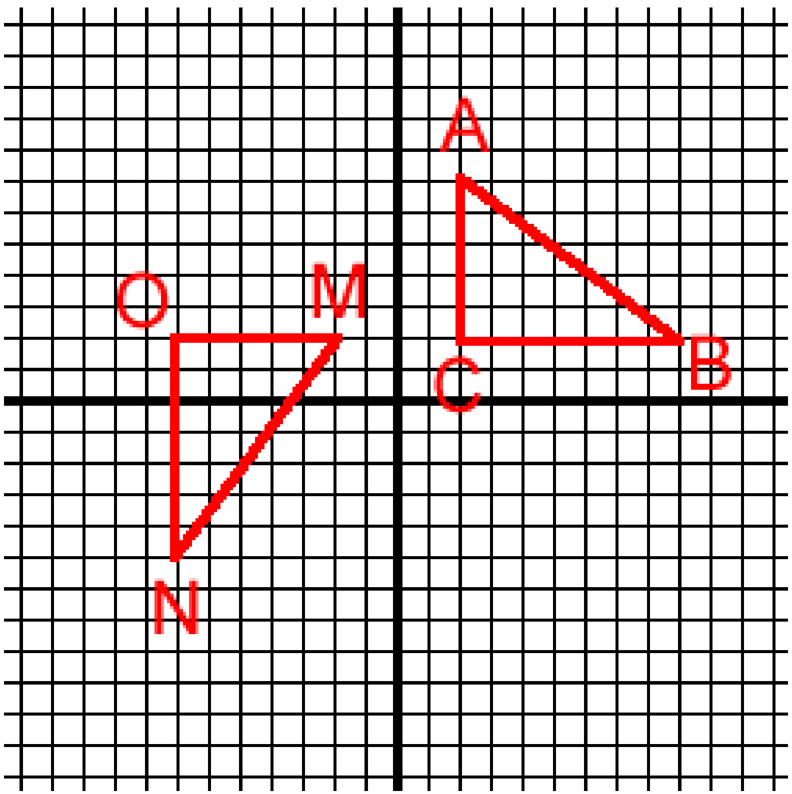
Proving Triangles Congruent

SSS - Postulate

If all the sides of one triangle are congruent to all of the sides of a second triangle, then the triangles are congruent. (SSS)

Example #1 – SSS – Postulate

Use the SSS Postulate to show the two triangles are congruent. Find the length of each side.



AC = 5

BC = 7

AB = $\sqrt{5^2+7^2}=\sqrt{74}$

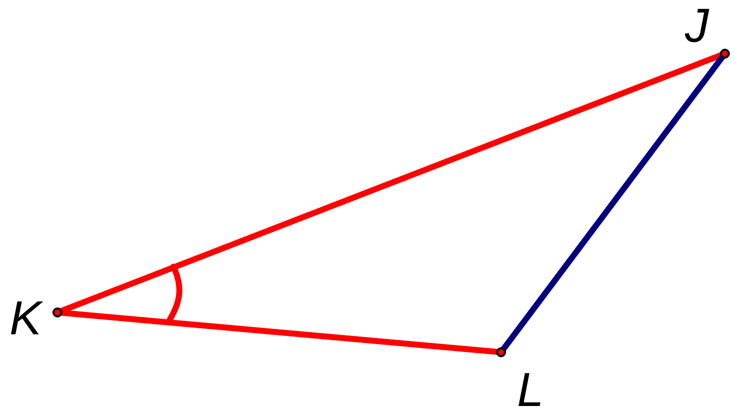
MO = 5

NO = 7

MN = $\sqrt{5^2+7^2}=\sqrt{74}$

$VABC \cong VMNO$

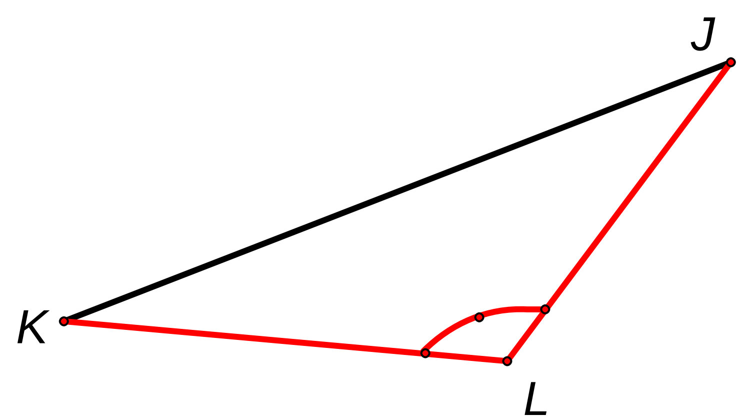
Definition – Included Angle



∠K is the angle between JK and KL. It is called the *included angle* of sides JK and KL.

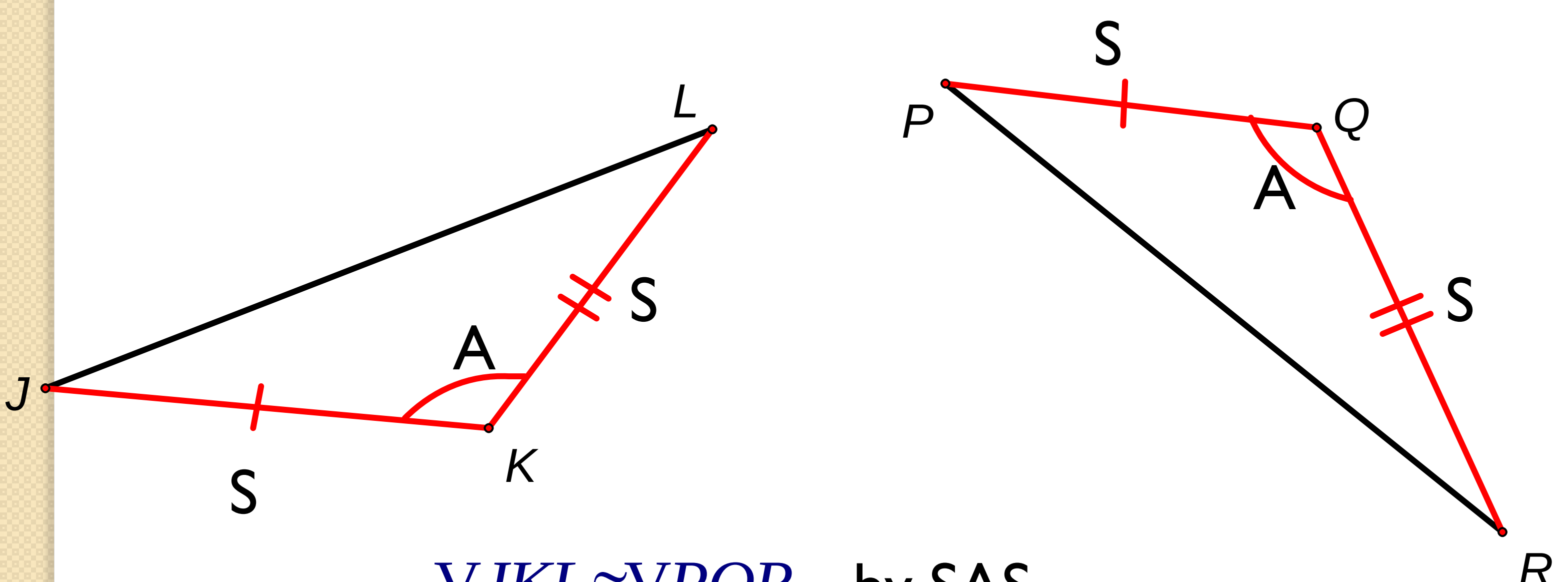
What is the included angle for sides KL and JL?

∠L



SAS - Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent. (SAS)

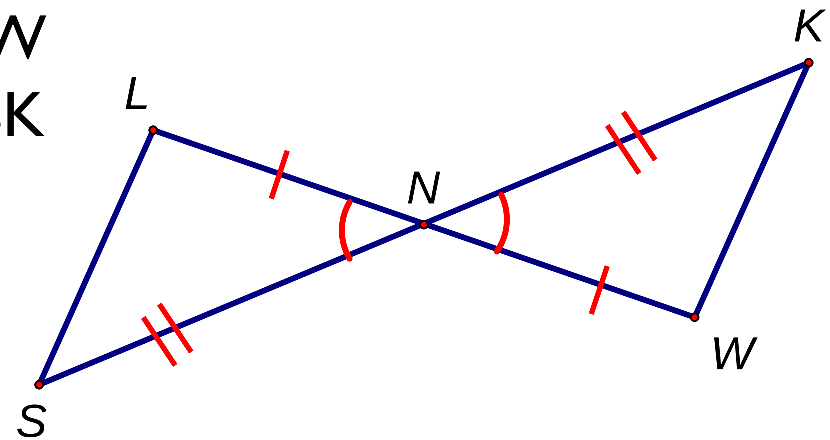


VJKL≅VPQR by SAS

Example #2 – SAS – Postulate

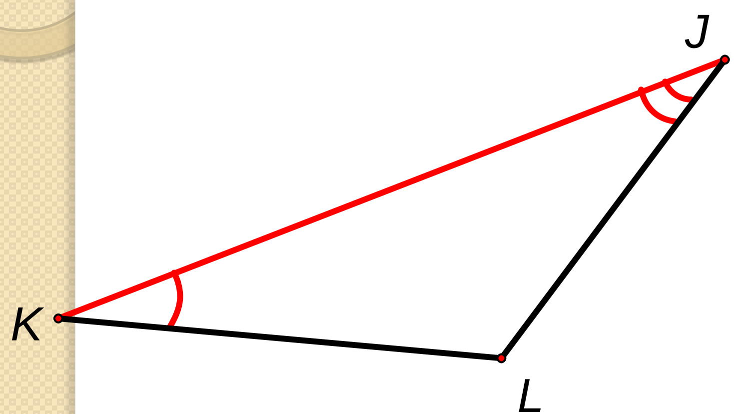
Given: N is the midpoint of LW
N is the midpoint of SK

Prove: $VLNS \cong VWNK$



N is the midpoint of LW N is the midpoint of SK	Given
$\overline{LN} \cong \overline{NW}$, $\overline{SN} \cong \overline{NK}$	Definition of Midpoint
$\angle LNS \cong \angle WNK$	Vertical Angles are congruent
$VLNS \cong VWNK$	SAS Postulate

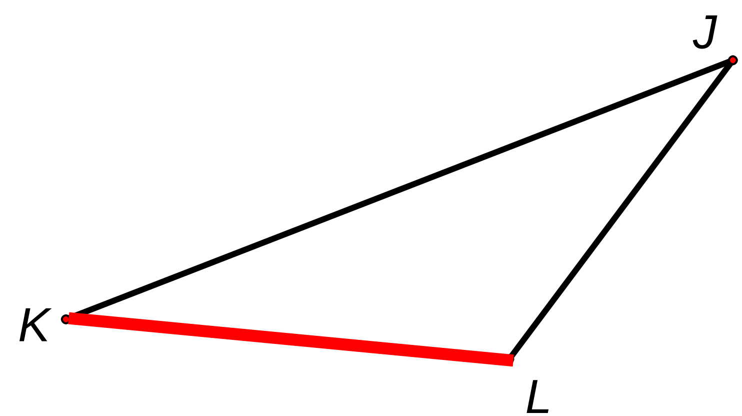
Definition – Included Side



$\overline{JK}$ is the side between $\angle J$ and $\angle K$. It is called the <u>*included side*</u> of angles J and K.

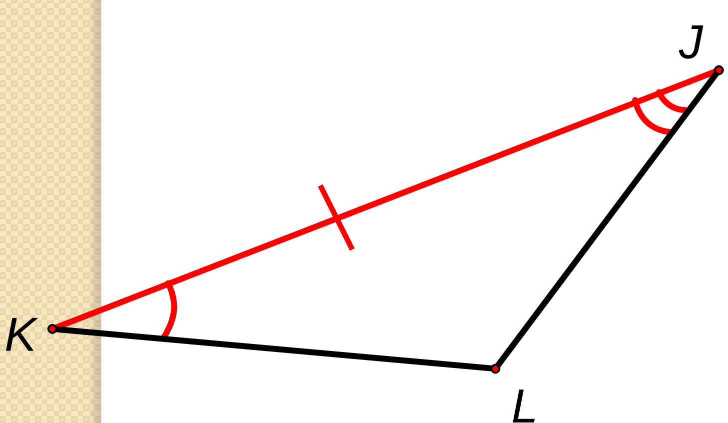
What is the included side for angles K and L?

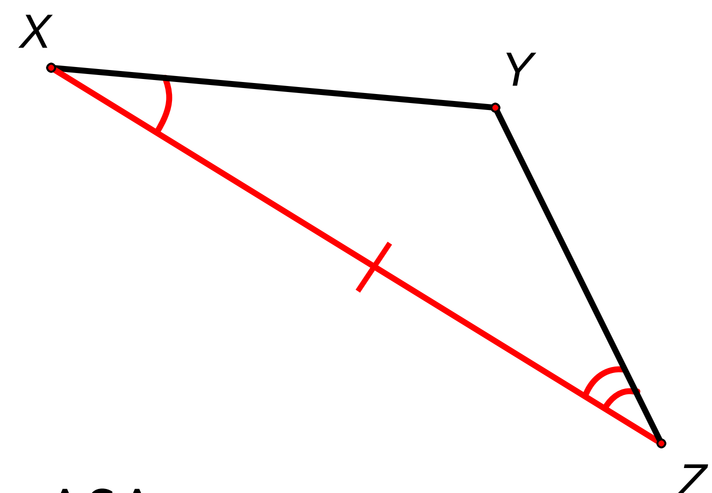
$\overline{KL}$



ASA - Postulate

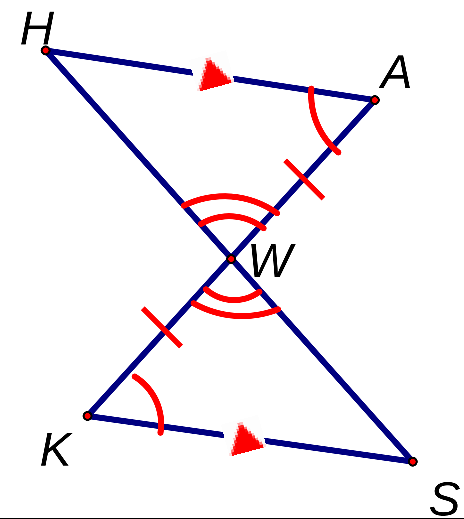
If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the triangles are congruent. (ASA)





$VJKL \cong VZXY$ by ASA

Example #3 – ASA – Postulate



Given: $\overline{HA} \parallel \overline{KS}$

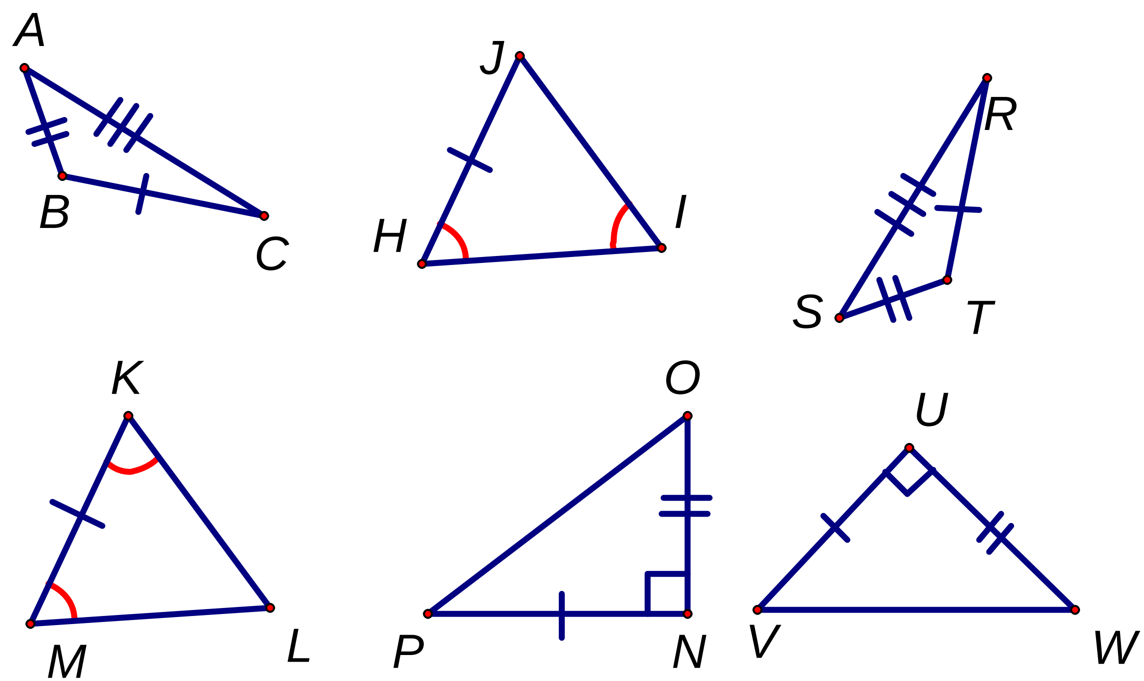
$\overline{AW} \cong \overline{WK}$

Prove: $VHAW \cong VSKW$

$\overline{HA} \parallel \overline{KS}$, $\overline{AW} \cong \overline{WK}$	Given
$\angle HAW \cong \angle SKW$	Alt. Int. Angles are congruent
$\angle HWA \cong \angle SWK$	Vertical Angles are congruent
$VHAW \cong VSKW$	ASA Postulate

Identify the Congruent Triangles

Identify the congruent triangles (if any). State the postulate by which the triangles are congruent.



VABC≅VSTR by SSS

VPNO≅VVUW by SAS

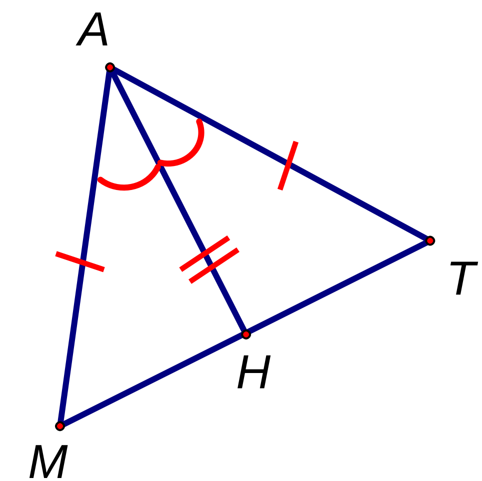
Note: VJHI is not SSS, SAS, or ASA.

Example #4—Paragraph Proof

Given: $\triangle MAT$ is isosceles with vertex bisected by AH.

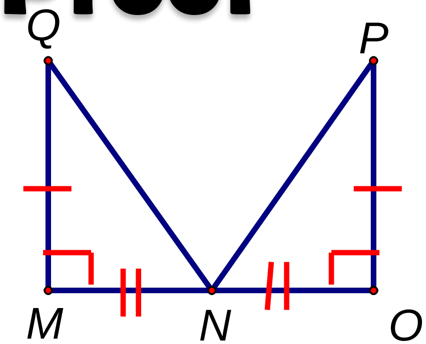
Prove:

$\overline{MH} \cong \overline{HT}$



- Sides MA and AT are congruent by the definition of an isosceles triangle.
- Angle MAH is congruent to angle TAH by the definition of an angle bisector.
- Side AH is congruent to side AH by the reflexive property.
- Triangle MAH is congruent to triangle TAH by SAS.
- So, side MH is congruent to side HT.

Example #5 – Column Proof



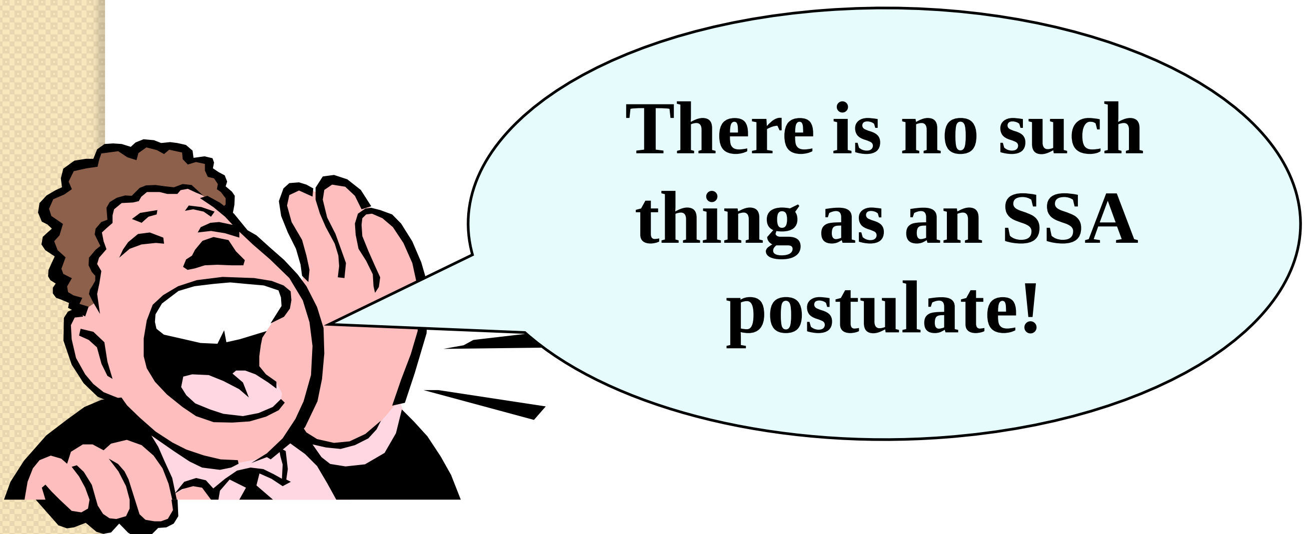
Given: $\overline{QM} \parallel \overline{PO}$, $\overline{QM} \perp \overline{MO}$

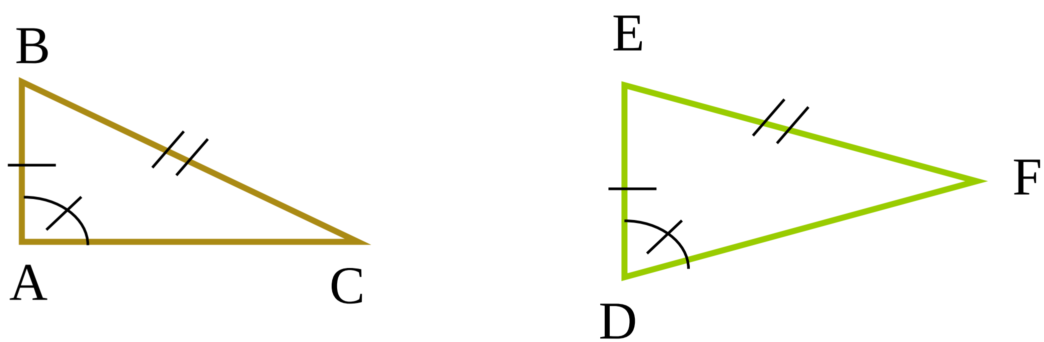
$\overline{QM} \cong \overline{PO}$, $\overline{MO}$ has midpoint N

Prove: $\overline{QN} \cong \overline{PN}$

$\overline{QM} \parallel \overline{PO}$, $\overline{QM} \perp \overline{MO}$ $\overline{QM} \cong \overline{PO}$	Given
$\overline{PO} \perp \overline{MO}$	A line $\perp$ to one of two $\parallel$ lines is $\perp$ to the other line.
$m\angle QMN = 90^o$ $m\angle PON = 90^o$	Perpendicular lines intersect at 4 right angles.
$\angle QMN \cong \angle PON$	Substitution, Def of Congruent Angles
$\overline{MO} \cong \overline{ON}$	Definition of Midpoint
$VQMN \cong VPON$	SAS
$\overline{QN} \cong \overline{PN}$	Proven

Warning: No SSA Postulate

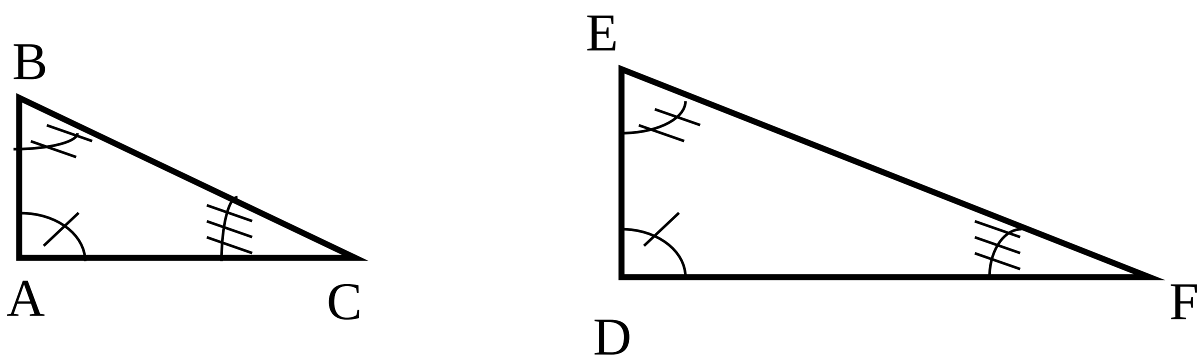




NOT CONGRUENT

Warning: No AAA Postulate

There is no such thing as an AAA postulate!



NOT CONGRUENT

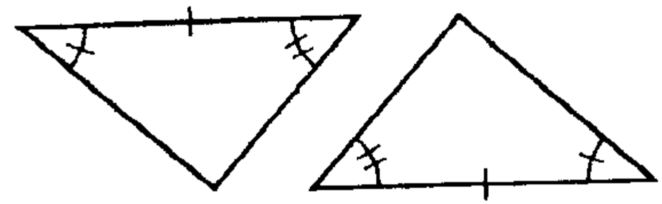
The Congruence Postulates

- ☞ SSS correspondence
- ☞ ASA correspondence
- ☞ SAS correspondence
- ☞ AAS correspondence
- ☞ ~~SSA correspondence~~
- ☞ ~~AAA correspondence~~

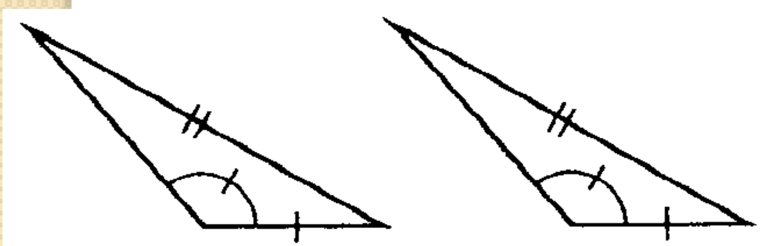
Name That Postulate

(when possible)

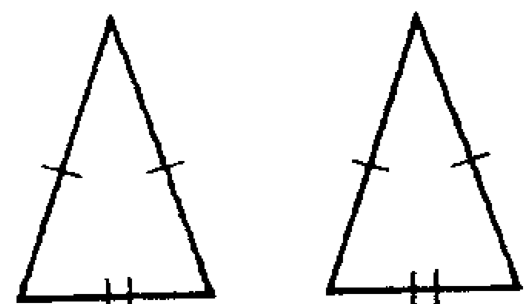
SAS



ASA



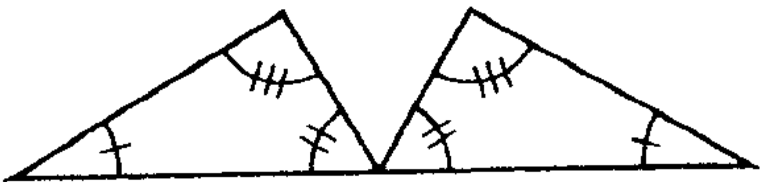
~~**SSA**~~



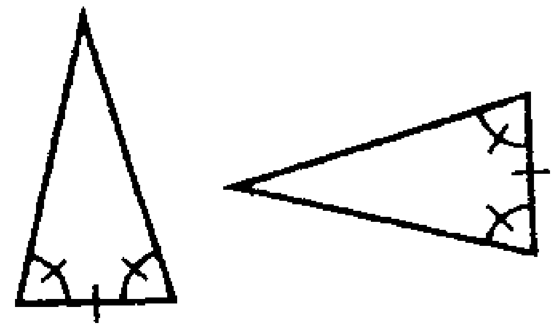
SSS

Name That Postulate

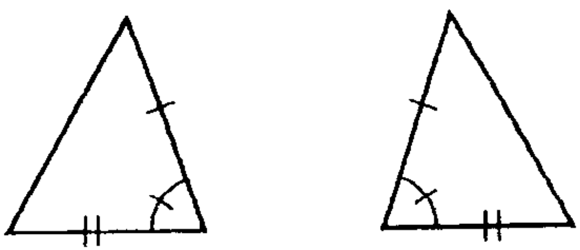
(when possible)



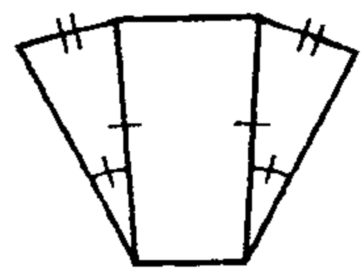
~~AAA~~



ASA



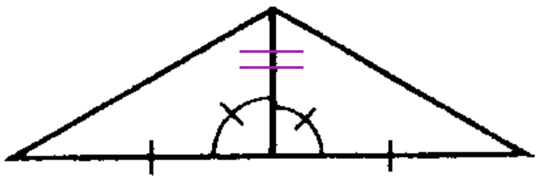
SAS



~~SSA~~

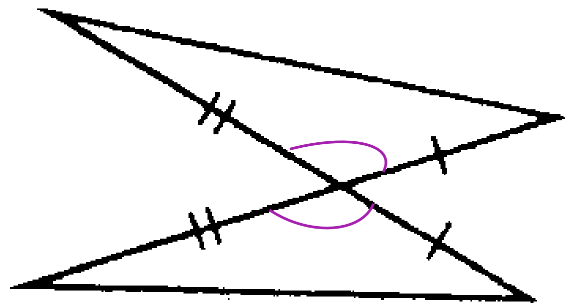
Name That Postulate

(when possible)



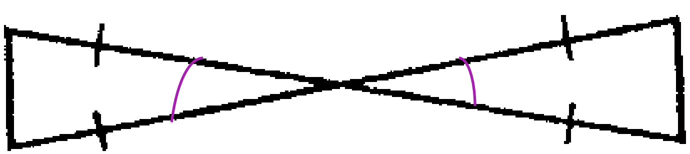
Reflexive Property

SAS



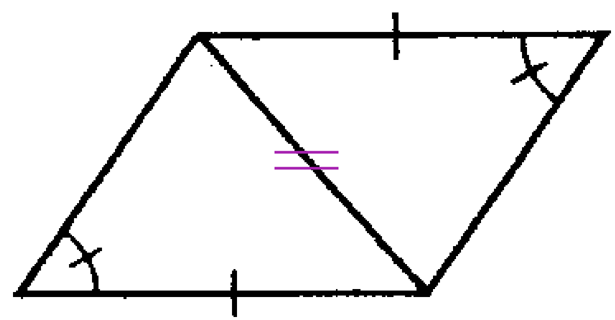
Vertical Angle

SAS



Vertical Angles

SAS

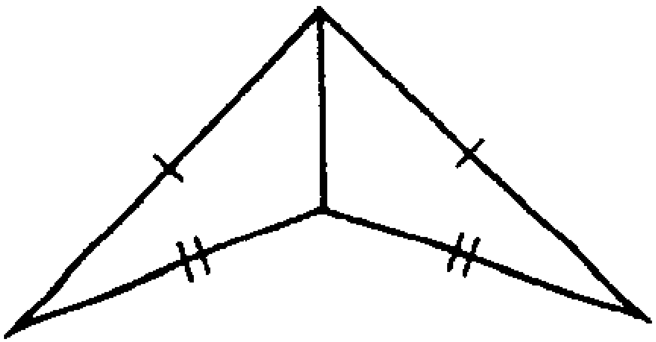


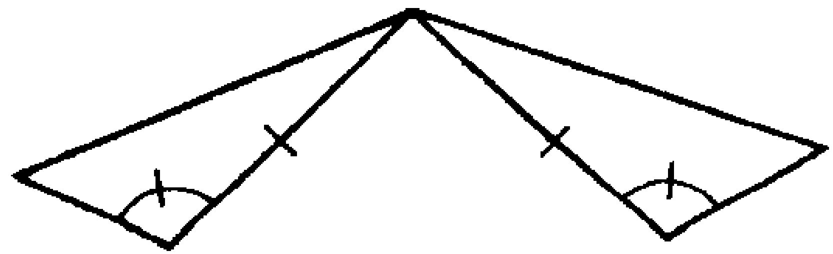
Reflexive Property

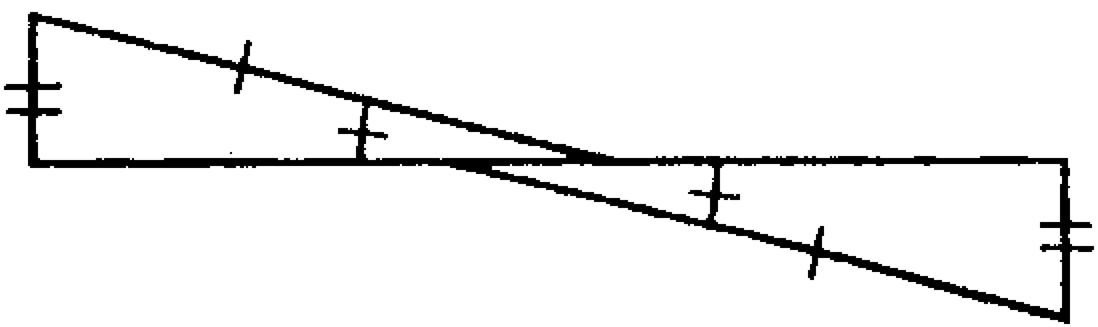
~~**SSA**~~

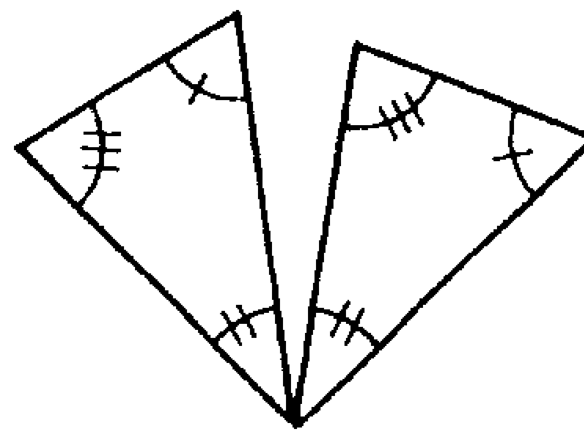
Give name the Postulate

(when possible)



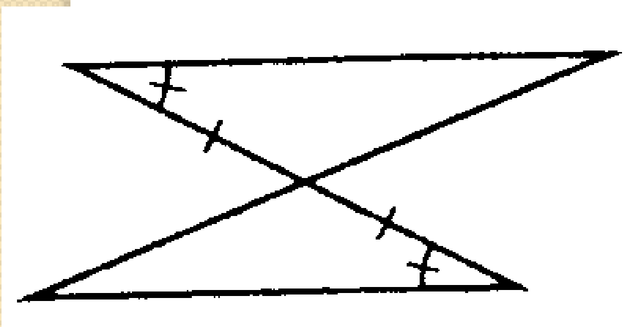


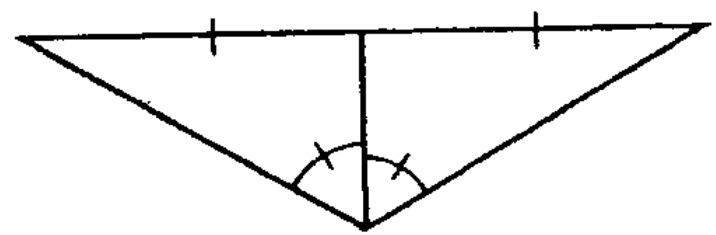


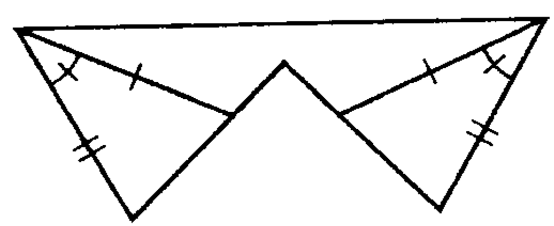


Give name the Postulate

(when possible)







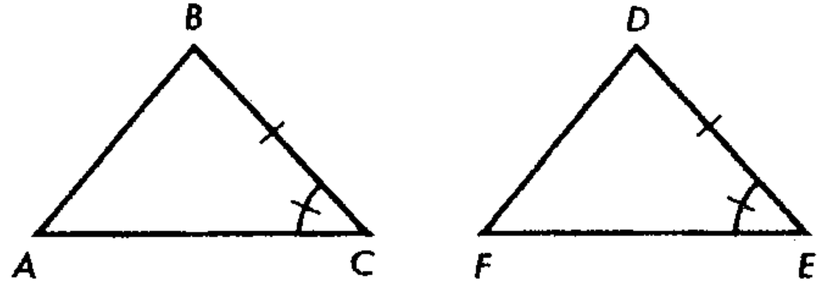
Let's Practice

Indicate the additional information needed to enable us to apply the specified congruence postulate.

For ASA: $\angle B \cong \angle D$

For SAS: $\overline{AC} \cong \overline{FE}$

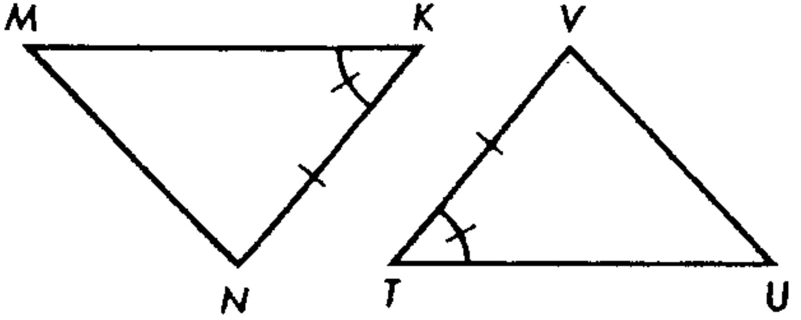
For AAS: $\angle A \cong \angle F$



Lets Practice

Indicate the additional information needed to enable us to apply the specified congruence postulate.

For ASA:



For SAS:

For AAS:

Properties of Congruent Triangles

- Reflexive:
  - Every triangle is congruent to itself.
- Symmetric:
  - If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$
- Transitive:
  - If $\triangle ABC \cong \triangle DEF$, and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$

Summary

Conditions for Triangles to be Congruent

- **A** *Three Sides Equal*
- **B** *Two Sides and Their Included Angle Equal*
- **C** *Two Angles and One Side Equal*
- **D** *Two Right-angled Triangles with Equal Hypotenuses and Another Pair of Equal Sides*

Summary

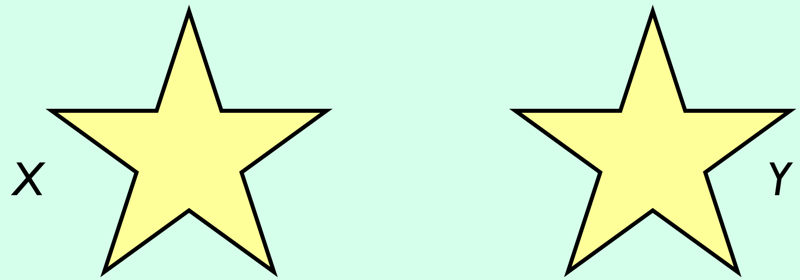
- Triangles may be proved congruent by
  Side – Side – Side (SSS) Postulate
  Side – Angle – Side (SAS) Postulate
  Angle – Side – Angle (ASA) Postulate
  Angle – Angle – Side (AAS) Postulate
  Hypotenuse – Leg (HL) Postulate
- Parts of triangles may be shown to be congruent by Congruent Parts of Congruent Triangles are Congruent (CPCTC)

Key Concept 1

A) Congruent Figures

1. Two figures having the same shape and the same size are called **congruent figures**.

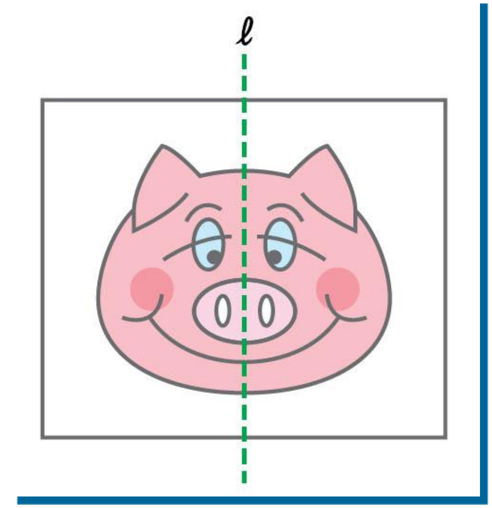
   E.g. The figures *X* and *Y* as shown are congruent.



2. If two figures are congruent, then they will fit exactly on each other.

Quick Example

The figure on the right shows a symmetric figure with *l* being the axis of symmetry. Find out if there are any congruent figures.



Solution

The line *l* divides the figure into 2 congruent figures,

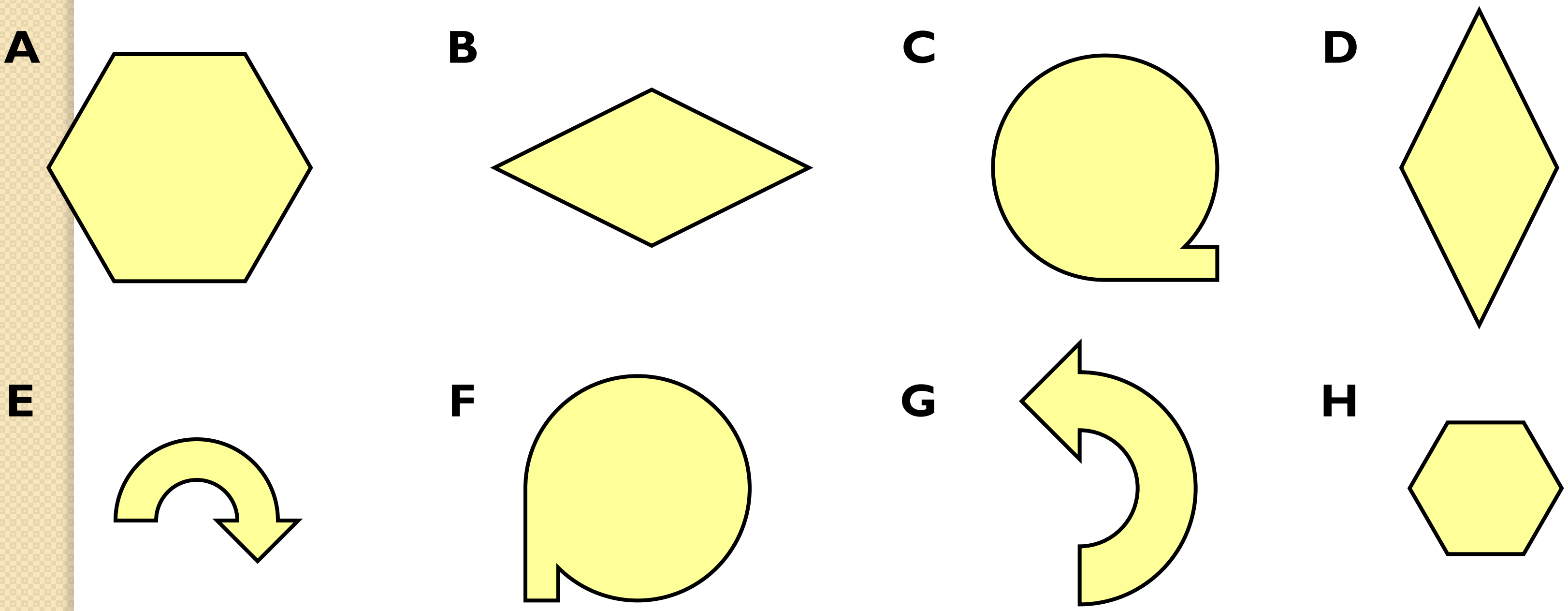
i.e.  and  are congruent figures.

Therefore, there are two congruent figures.

The Meaning of Congruence

Extra Example I

Find out by inspection the congruent figures among the following.



Solution

B, **D** ; **C**, **F**

Key Concept 2

B) Transformation and Congruence

- When a figure is translated, rotated or reflected, the image produced is congruent to the original figure. When a figure is enlarged or reduced, the image produced will **NOT** be congruent to the original one.

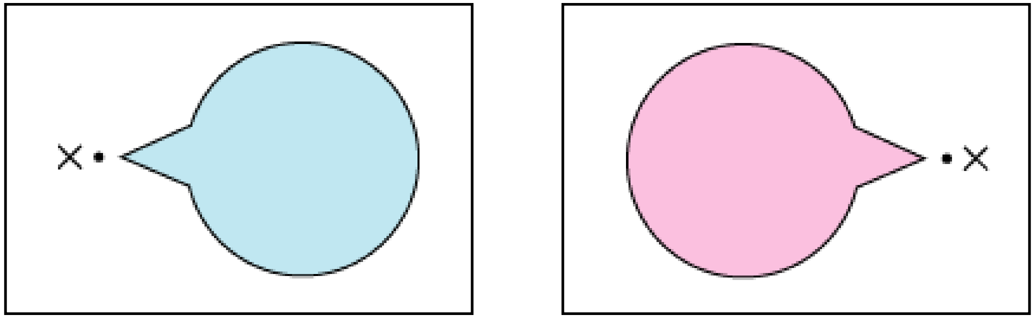
Extra Example II

In each of the following pairs of figures, the red one is obtained by transforming the blue one about the fixed point X. Determine

(i) which type of transformation (translation, rotation, reflection, enlargement, reduction) it is,

(ii) whether the two figures are congruent or not.

(a)

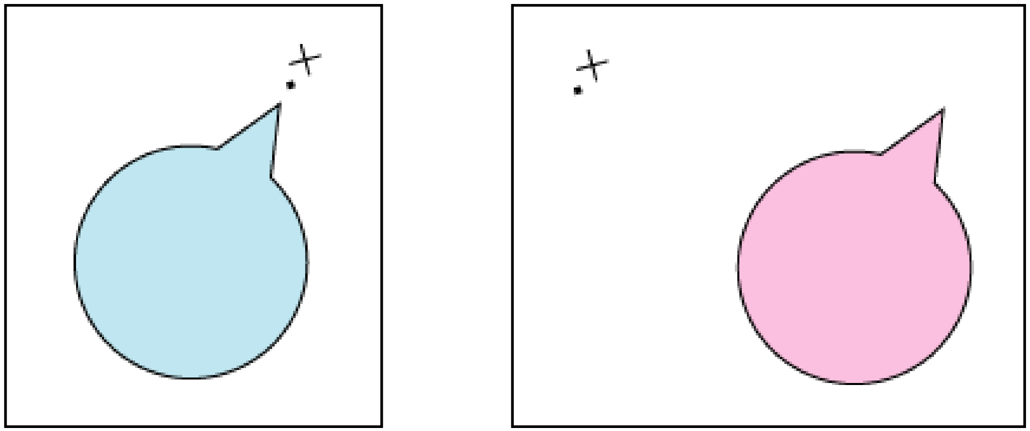


(i) Reflection

(ii) Yes

The Meaning of Congruence

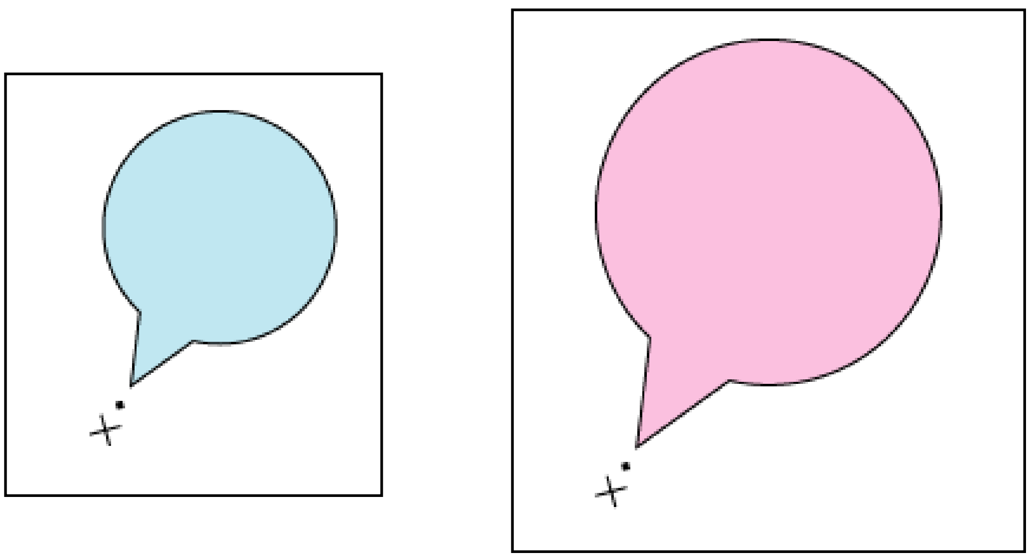
(b)



(i) Translation

(ii) Yes

(c)

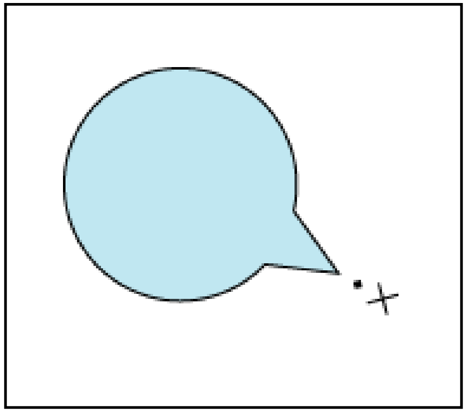


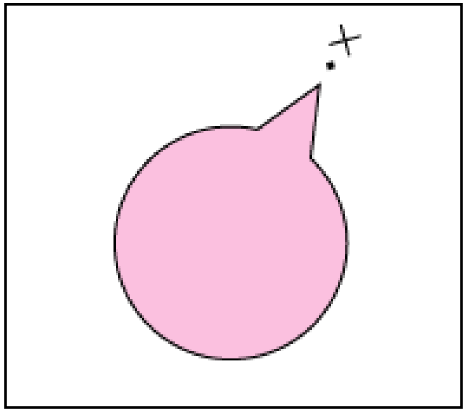
(i) Enlargement

(ii) No

The Meaning of Congruence

(d)

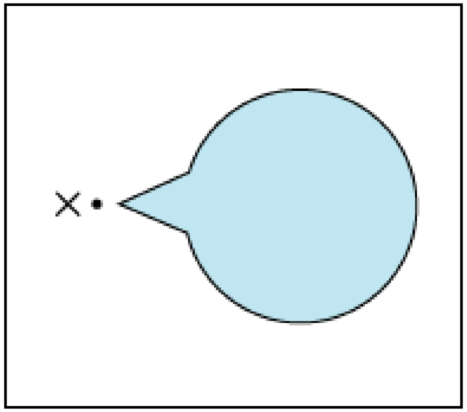


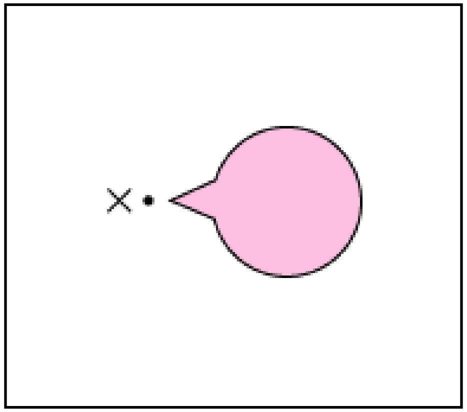


(i) Rotation

(ii) Yes

(e)





(i) Reduction

(ii) No

Key Concept 3

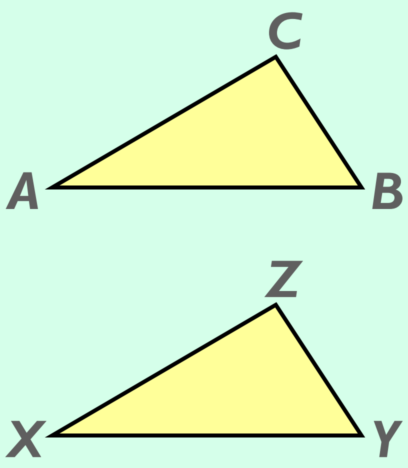
C) Congruent Triangles

- When two triangles are congruent, all their **corresponding sides** and **corresponding angles** are equal.

E.g. In the figure, if $\triangle ABC \cong \triangle XYZ$,

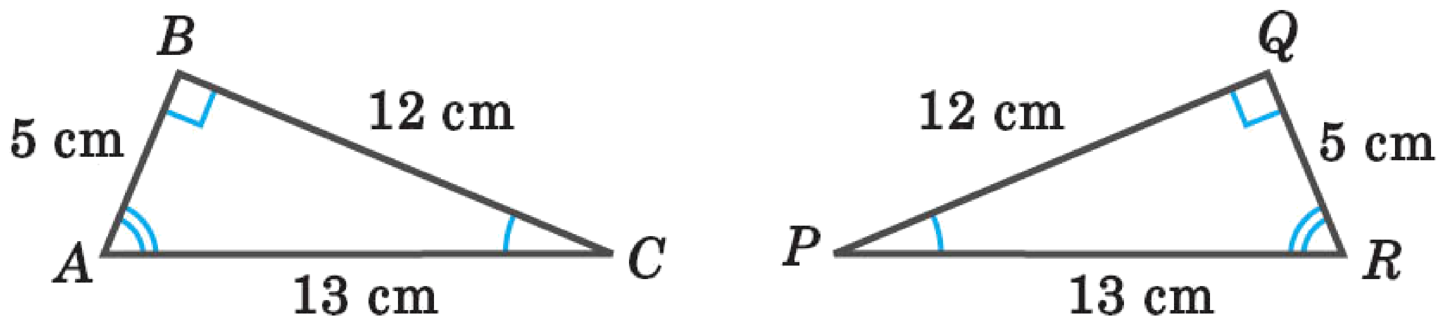
then

$\angle A = \angle X$, $\angle B = \angle Y$, $\angle C = \angle Z$, and $AB = XY$, $BC = YZ$, $CA = ZX$.



Quick Example

Name a pair of congruent triangles in the figure.

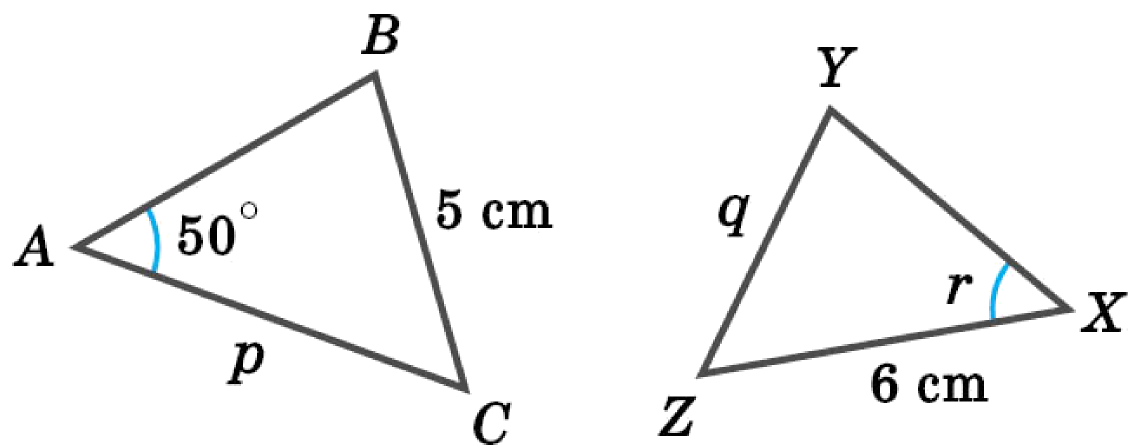


Solution

From the figure, we see that $\triangle ABC \cong \triangle RQP$.

Quick Example

Given that $\triangle ABC \cong \triangle XYZ$ in the figure, find the unknowns p, q and r.



Solution

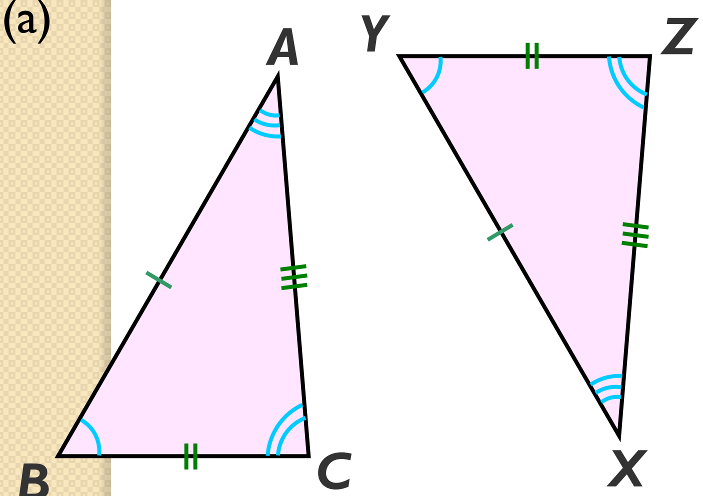
$\because$ For two congruent triangles, their corresponding sides and angles are equal.

$\therefore$ $p = 6$ cm, $q = 5$ cm, $r = 50°$

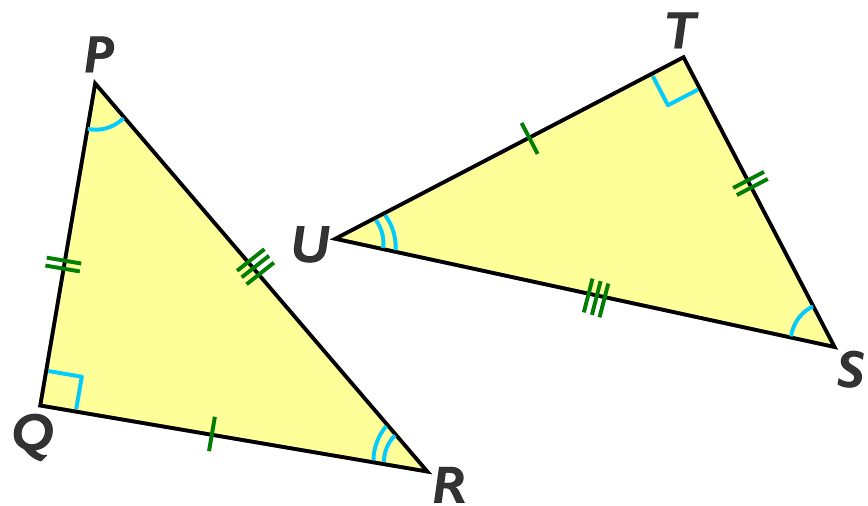
Extra Example III

Write down the congruent triangles in each of the following.

(a)



(b)



Solution

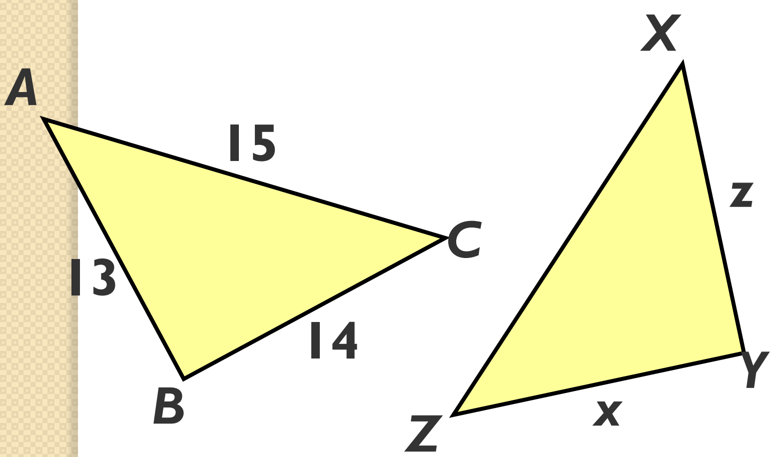
(a) $\triangle ABC \cong \triangle XYZ$

(b) $\triangle PQR \cong \triangle STU$

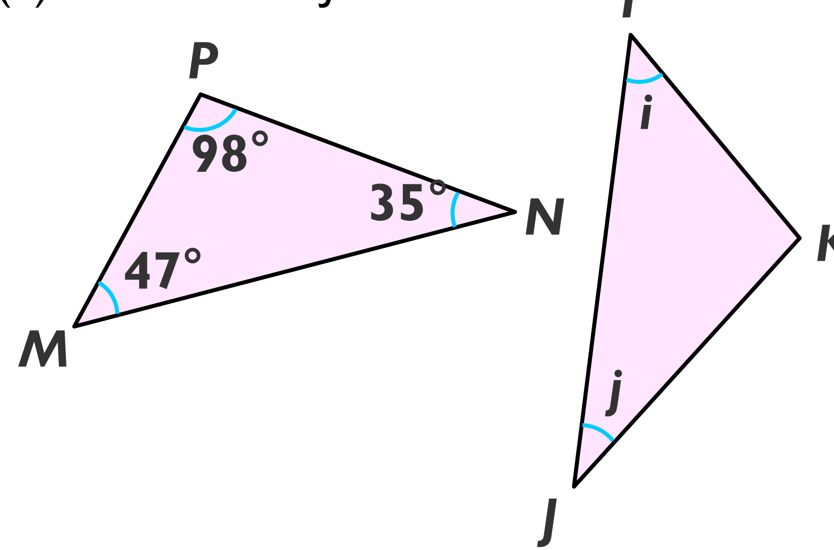
Extra Example IV

Find the unknowns (denoted by small letters) in each of the following.

(a) $\triangle ABC \cong \triangle XYZ$



(b) $\triangle MNP \cong \triangle IJK$



Solution

(a) $x = \underline{\underline{14}}$, $z = \underline{\underline{13}}$

(b) $j = \underline{\underline{35^\circ}}$, $i = \underline{\underline{47^\circ}}$

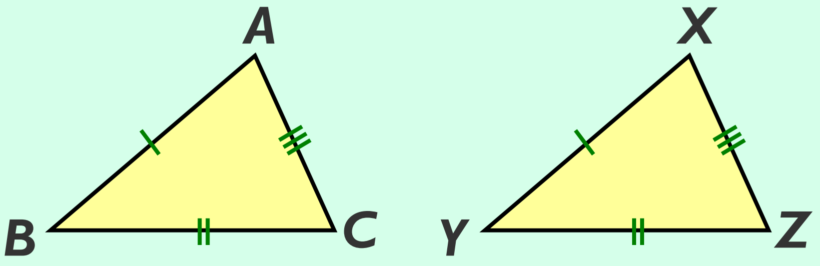
Key Concept 1

A) Three Sides Equal

- If $AB = XY$, $BC = YZ$ and $CA = ZX$,

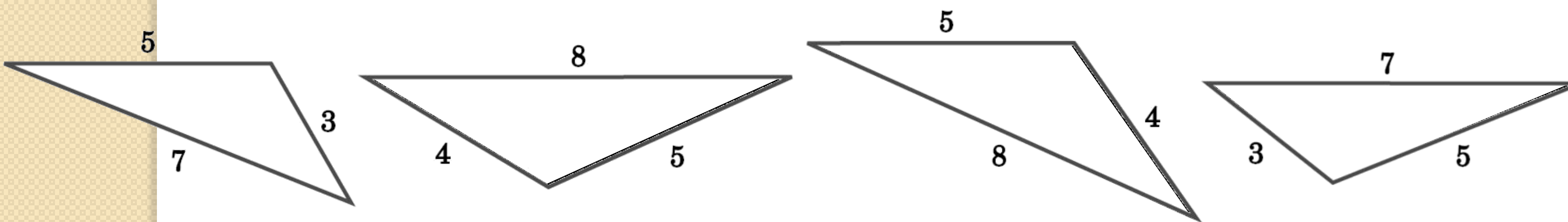
  then $\triangle ABC \cong \triangle XYZ$.

  【*Reference: SSS*】



Quick Example

Determine which pair(s) of triangles in the following are congruent.



(I) (II) (III) (IV)

Solution

In the figure, because of *SSS*,

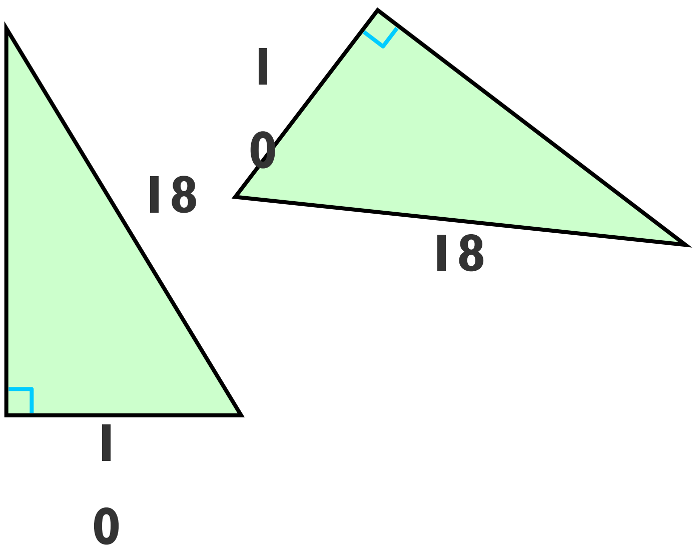
(I) and (IV) are a pair of congruent triangles;

(II) and (III) are another pair of congruent triangles.

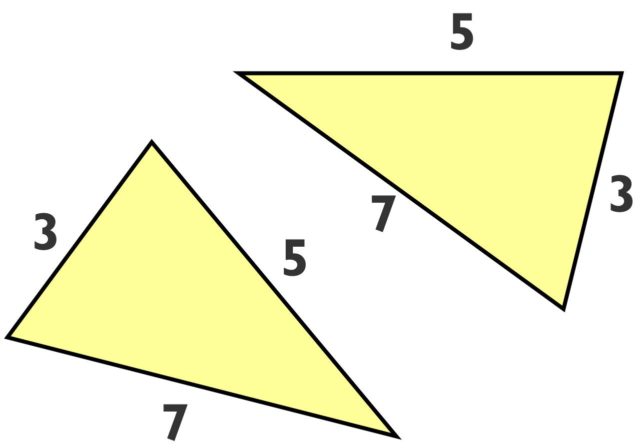
Extra Example V

Each of the following pairs of triangles are congruent. Which of them are congruent because of *SSS*?

A



B



Solution

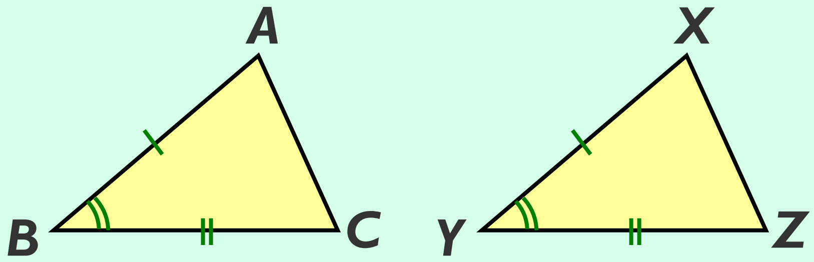
B

Key Concept 2

B) Two Sides and Their Included Angle Equal

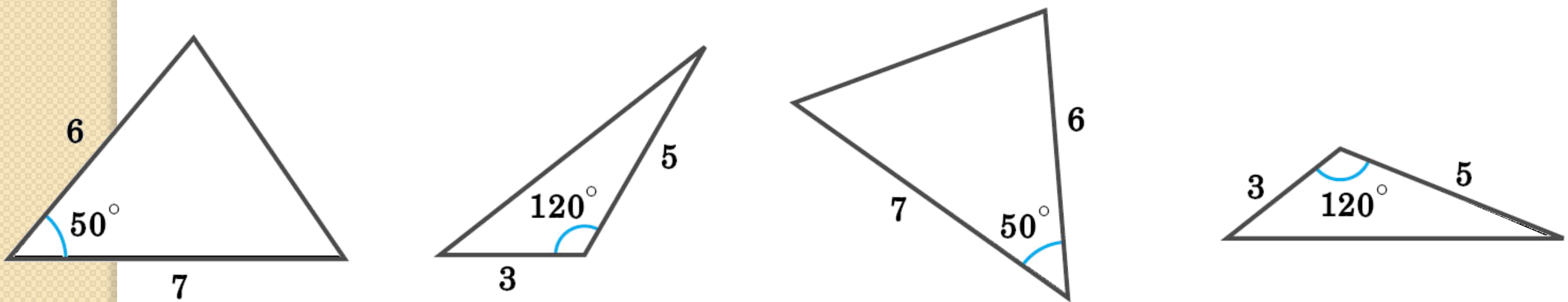
- If $AB = XY$, $\angle B = \angle Y$ and $BC = YZ$, then $\triangle ABC \cong \triangle XYZ$.

【*Reference: SAS*】



Quick Example

Determine which pair(s) of triangles in the following are congruent.



(I) (II) (III) (IV)

Solution

In the figure, because of *SAS*,

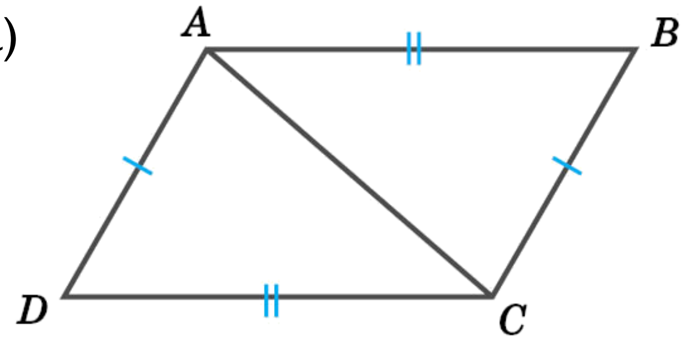
(I) and (III) are a pair of congruent triangles;

(II) and (IV) are another pair of congruent triangles.

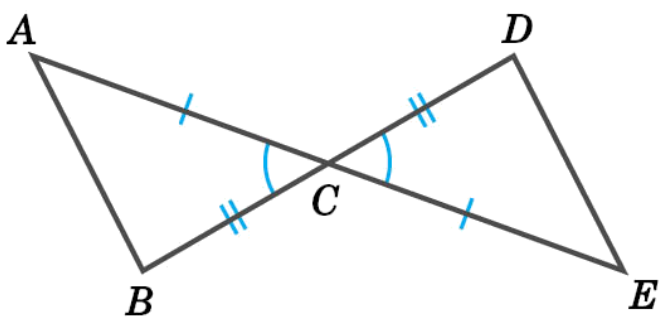
Example 1

Level 2 In each of the following figures, equal sides and equal angles are indicated with the same markings. Write down a pair of congruent triangles, and give reasons.

(a)



(b)



Solution

(a) $\triangle ABC \cong \triangle CDA$ (SSS)

(b) $\triangle ACB \cong \triangle ECD$ (SAS)

Fulfill Exercise Objective

❖ Identify congruent triangles from given diagram and give reasons.

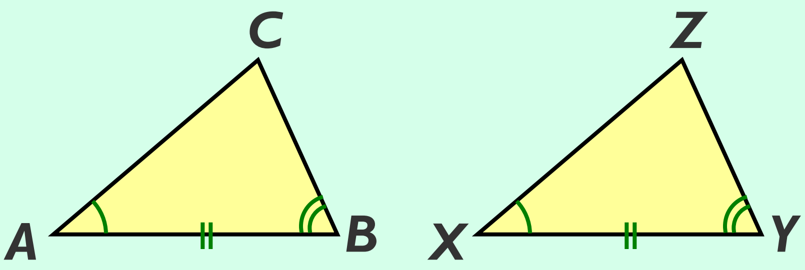
Key Concept 3

C) Two Angles and One Side Equal

1. If $\angle A = \angle X$, $AB = XY$ and $\angle B = \angle Y$,

   then $\triangle ABC \cong \triangle XYZ$.

   【*Reference:ASA*】



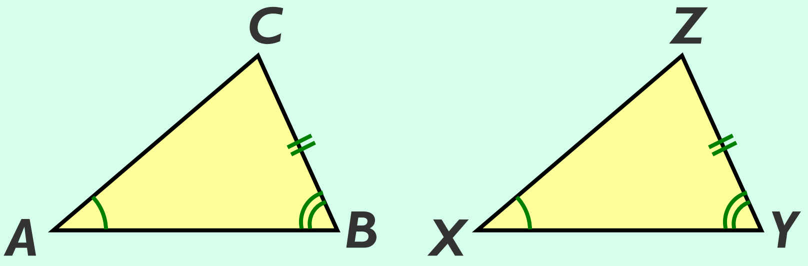
Key Concept 4

C) Two Angles and One Side Equal

2. If $\angle A = \angle X$, $\angle B = \angle Y$ and $BC = YZ$,

then $\triangle ABC \cong \triangle XYZ$.

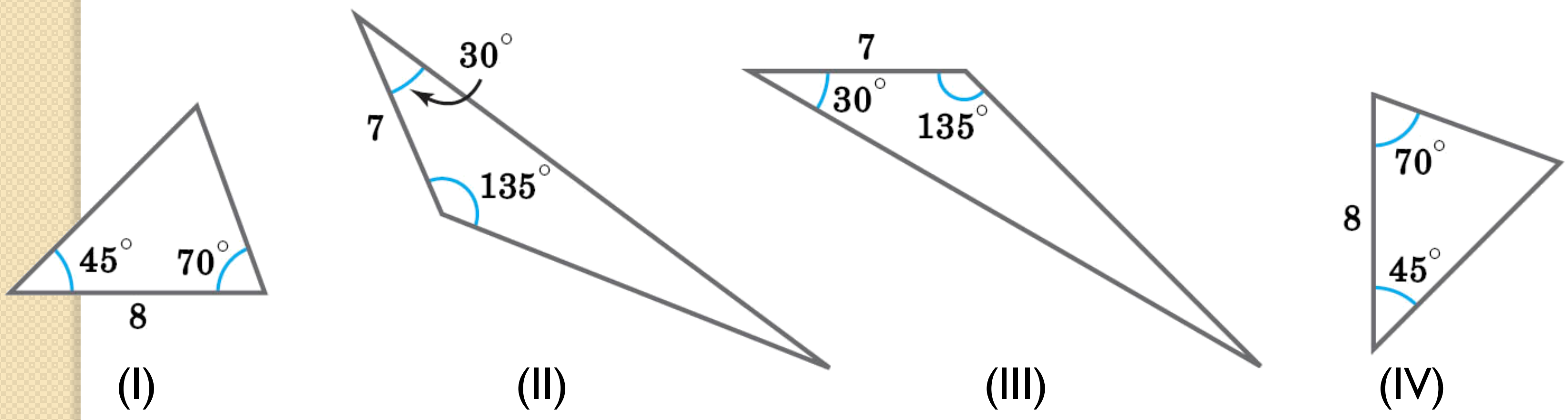
【*Reference:AAS*】



Conditions for Triangles to be Congruent

Quick Example

Determine which pair(s) of triangles in the following are congruent.



Solution

In the figure, because of *ASA*,

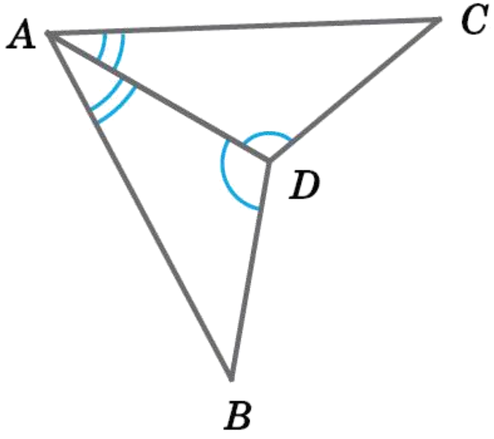
(I) and (IV) are a pair of congruent triangles;

(II) and (III) are another pair of congruent triangles.

Example 2

Level 2

In the figure, equal angles are indicated with the same markings. Write down a pair of congruent triangles, and give reasons.



Solution

$\triangle ABD \cong \triangle ACD$ (ASA)

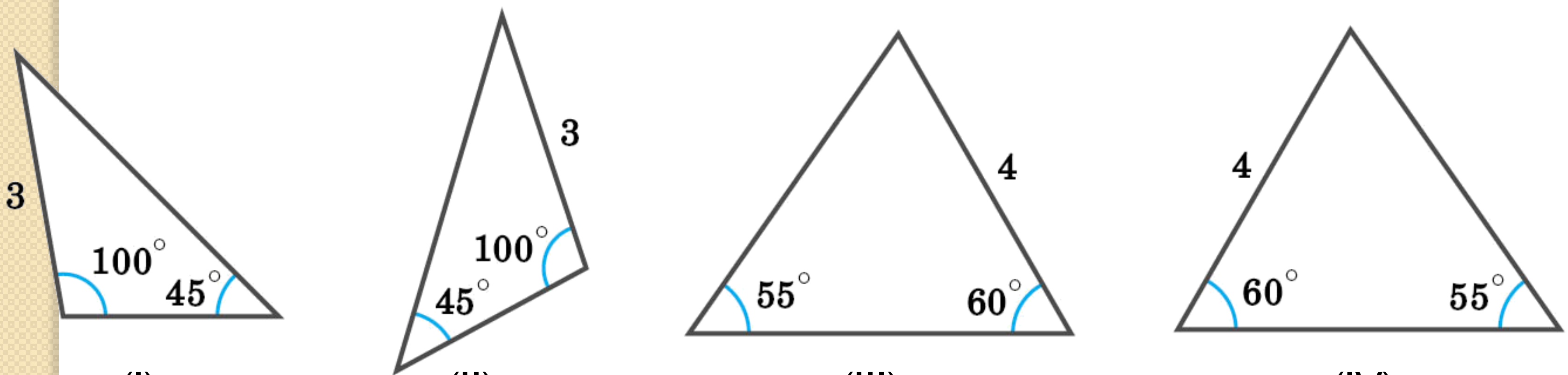
Fulfill Exercise Objective

❖ Identify congruent triangles from given diagram and given reasons.

Conditions for Triangles to be Congruent

Quick Example

Determine which pair(s) of triangles in the following are congruent.



(I) (II) (III) (IV)

Solution

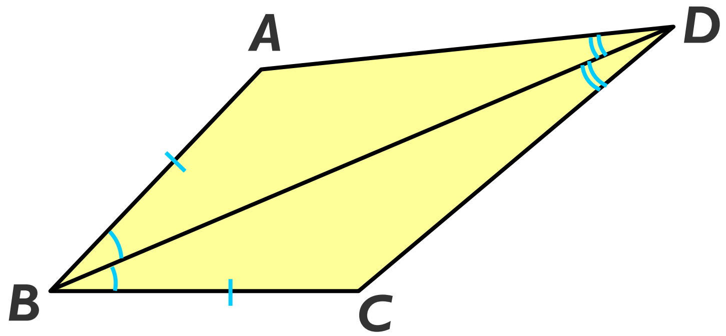
In the figure, because of *AAS*,

(I) and (II) are a pair of congruent triangles;

(III) and (IV) are another pair of congruent triangles.

Extra Example VI

In the figure, equal angles are indicated with the same markings. Write down a pair of congruent triangles, and give reasons.



Solution

$\triangle ABD \cong \triangle CBD$ (AAS)

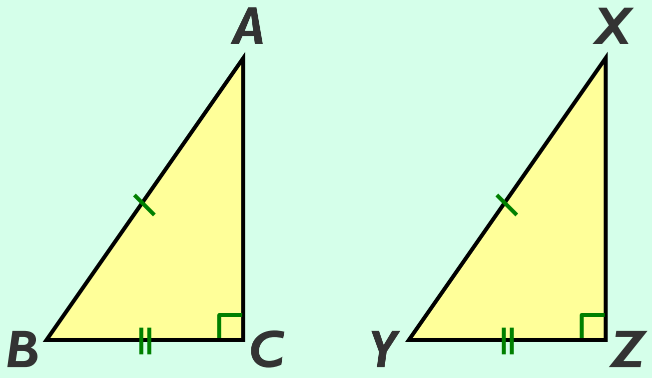
Key Concept 5

D) Two Right-angled Triangles with Equal Hypotenuses and Another Pair of Equal Sides

- If $\angle C = \angle Z = 90°$, $AB = XY$ and $BC = YZ$,

  then $\triangle ABC \cong \triangle XYZ$.

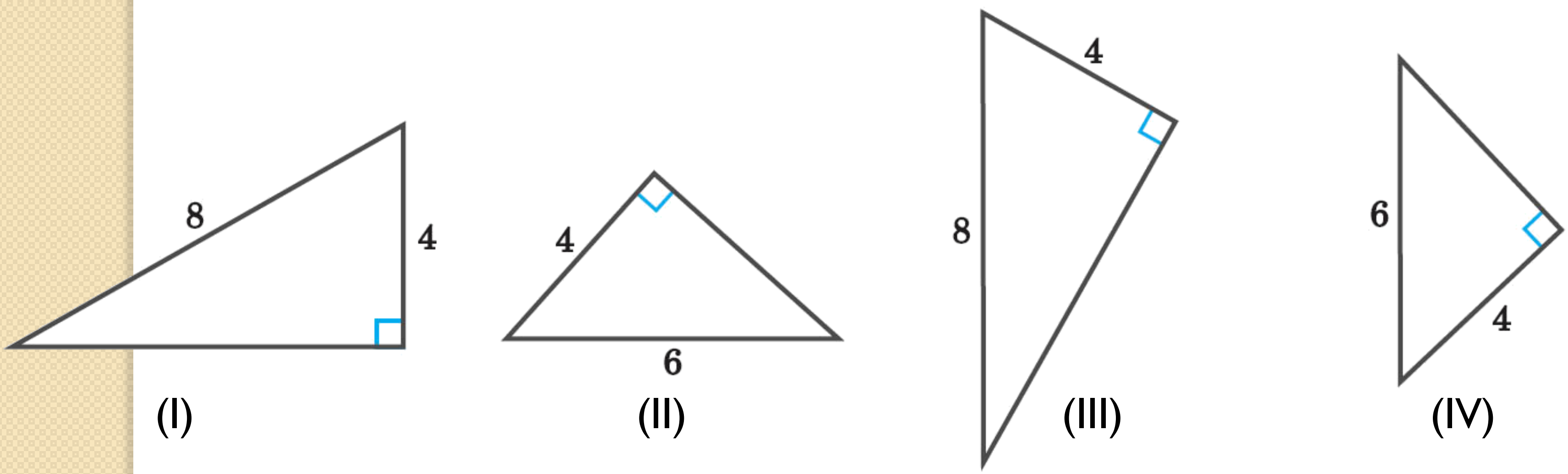
  【*Reference: RHS*】



Conditions for Triangles to be Congruent

Quick Example

Determine which of the following pair(s) of triangles are congruent.



Solution

In the figure, because of *RHS*,

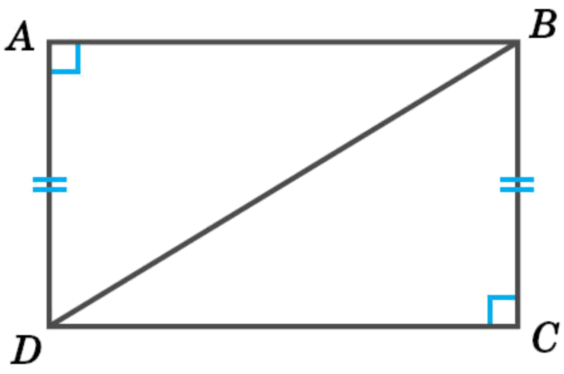
(I) and (III) are a pair of congruent triangles;

(II) and (IV) are another pair of congruent triangles.

Example 3

Level 1

In the figure, $\angle DAB$ and $\angle BCD$ are both right angles and $AD = BC$. Judge whether $\triangle ABD$ and $\triangle CDB$ are congruent, and give reasons.



Solution

Yes, $\triangle ABD \cong \triangle CDB$ (*RHS*)

Fulfill Exercise Objective

- Determine whether two given triangles are congruent.

Exercises

1. Prove that any point at perpendicular bisector of a line segment is equidistant to both ends of the line segment
2. Prove that the intersection of three perpendicular bisector of a triangle is a center of outside circle the triangle
3. Prove that any point at angle bisector of a angle is equidistant to both rays of the angle
4. Prove that the intersection of three angle bisector of a triangle is a center of inside circle of the triangle