# Adaptive Fuzzy Control for Linear Motion Stage

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ABSTRACT: Motion control is an essential part of industrial machinery and manufacturing systems. In this paper, the adaptive fuzzy controller is designed to the position control of X-Y linear stage for precision motion and trajectory tracking control. The direct fuzzy controller controller is developed to demonstrate the feasibility to track a reference trajectory. The Lyapunov stability theorem has been used to testify the asymptotic stability of the whole system and the free parameters of the adaptive fuzzy controller can be tuned on-line by an output feedback control law. The simulation results are presented to verify the tracking performances of the proposed controllers.

**Keywords:** Adaptive fuzzy control, X-Y linear stage, precision control.

## 1 INTRODUCTION

Recently, the linear motion stage has become a vitally important part of precision industry. Due to high stiffness and resolution properties, lead-screw architectures are usually employed in linear table which is broadly used in industry automation equipment, semiconductor manufacturing processes and pharmaceutical applications. This mechanism often exists several kinds of disturbances, such that nonlinear friction and uncertainties, that will affect the precision and tracking performance of controlled system.

The adaptive fuzzy controllers [2, 3, 5] have shown better capability in situations where the system model with uncertainties and unknown variations. Wang had proposed stable direct and indirect adaptive fuzzy controllers for nonlinear systems [3]. Yin and Lee [4] proposed a fuzzy model-reference adaptive controller for the system with unknown parameters. The approximation error in the adaptive fuzzy control systems is generally inevitable [1, 2]. Also, in the practical applications, systems variable are continually by external disturbances and friction force. Thus, the adaptive fuzzy controller should be designed to be robust to cope with approximation error. So, the aim of our research is to design a controller that ensures to have a smaller tracking error.

In this paper, we proposed direct adaptive fuzzy control method that guarantees the tracking error is smaller. The plant model with a friction effect is considered here. First, we design the direct fuzzy controller that able to deal with two-axis on motion stage. Second, we need to know the all parameters of every axis motor that can make up the motion stage model. Final, based on the proposed architecture, we can obtain the trajectories of X-Y stage.

## 2 MODELING OF TWO-AXIS STAGE

The dynamic equation the single-axis mechanism with the friction model include is

$$\ddot{\theta}_r = -\frac{B_i}{J_i}\dot{\theta}_r + \frac{T_{ei}}{J_i} - \frac{1}{J_i}(T_{Li} + T_{fi}(v_i)) \qquad (1)$$

where $i = x, y$ ( $x$ and $y$ denote the axis), $J_i$ is the moment of inertia, $B_i$ is the damping coefficient; $T_{Li}$ is the external disturbance term including cross-coupled interference and $T_{fi}(v_i)$ is the friction torque, $v_i$ is the linear velocity of X- and Y-axis. Considering Coulomb friction, viscous friction and Stribeck effect, the friction torque can be formulated as follows:

$$T_{fi}(v_i) = F_{Ci}\,\mathrm{sgn}(v_i) + (F_{Si} - F_{Ci})e^{-\left(\frac{v_i}{v_{Si}}\right)^2}\mathrm{sgn}(v_i) + K_{Vi}v_i \qquad (2)$$

where $F_{Ci}$ is the Coulomb friction, $F_{Si}$ is the static friction, $v_{si}$ is the Stribeck velocity parameter, $K_{vi}$ is the coefficient of viscous friction, $\mathrm{sgn}(\cdot)$ is a sign function. All the parameters in (2) are time-varying.

Base on realization of Field-oriented control, the dynamic equation can be simplified as following:

$$T_{ei} = K_{ti} i_q^* \qquad (3.1)$$

$$k_t = 3ML_{md}I_{fd}/2 \qquad (3.2)$$

where $M$ denotes the number of primary pole pairs, $L_{md}$ is $d$ axis inductances, $\mathrm{I}_{fd}$ denotes equivalent $d$ -axis magnetizing current, $K_{ti}$ is the denote torque constant.

## 3 ROBUST ADAPTIVE FUZZY CONTROL

We consider the research equation in the following form.

$$x^{(n)} = f(x, \dot{x}, \ldots, x^{(n-1)}) + bu \\ y = x \quad (4)$$

where $x$ is the position variable of stage, $f(\bullet)$ is unknown function and $b$ is unknown positive constant. We adopt $If-THEN$ fuzzy rule to describe the control rule,

$IF$ $x_1$ is $P_1^r$ THEN …THEN $x_n$ is $P_n^r$, and $u$ is $Q^r$ (5)

where $P_n^r$ and $Q^r$ are fuzzy sets in R.

We define $\mathbf{k} = (k_n, \ldots, k_1)^T$ to let polynomial $s^n + k_1 e^{(n-1)} + \cdots + k_n$ all root in left plane. The control law can be defined as following:

$$u^* = \frac{1}{b}\left[-f(x) + y_m^{(n)} + \mathbf{K}^T \mathbf{e}\right] \quad (6)$$

From (4) and (6), the close dynamic equation is

$$e^{(n)} + k_1 e^{(n-1)} + \cdots + k_n e = 0 \quad (7)$$

Base on choosing $k$ value, we can get the error which will be approach zero when the time approaches infinity. However, designing a control law $u = u(x/\theta)$ and a adjust parameter base on direct adaptive fuzzy control, it can make output value as similar as possible for ideal output. We design a direct fuzzy control as following:

$$u = u_D(x/\theta) \quad (8)$$

where $u_D$ is a fuzzy system and $\theta$ is the adjustable parameter sets. Fuzzy system can be constructed by two steps:

Step 1: Define $m_i$ fuzzy sets $A_i^{l_i}$ for variable $x_i (i = 1, 2, \cdots, n)$

Step 2: Use $\prod_{i=1}^{n} m_i$ fuzzy rules to construct fuzzy system $u_D(x/\theta)$ as following

$IF$ $x_1$ is $A_1^{l_1}$ THEN … THEN $x_n$ is $A_n^{l_n}$, and $u_D$ is $S^{l_1, \cdots, l_n}$

where $l_1 = 1, 2, \cdots, m_i$ and $i = 1, 2, \cdots, n$.

We adopt single-valued fuzzy control, product inference engine and center average defuzzifier to design fuzzy control as following:

$$u_D(x/\theta) = \frac{\sum_{l_1=1}^{m_1} \cdots \sum_{l_n}^{m_n} \bar{y}_u^{l_1 \cdots l_n} \left( \prod_{i=1}^{n} \mu_{A_i}^{l_i}(x_i) \right)}{\sum_{l_1=1}^{m_1} \cdots \sum_{l_n=1}^{m_n} \left( \prod_{i=1}^{n} \mu_{A_i}^{l_i}(x_i) \right)} \quad (9)$$

where $\bar{y}_u^{l_1 \cdots l_n}$ is a free parameter and the fuzzy control is as following:

$$u_D = \theta^T \xi(x) \quad (10)$$

where $\xi(x)$ is $\prod_{i=1}^{n} m_i$ rank vector, it's element as following:

$$\xi_{l_1 \cdots l_n}(x) = \frac{\prod_{i=1}^{n} \mu_{A_i}^{l_i}(x_i)}{\sum_{l_1=1}^{m_1} \cdots \sum_{l_n=1}^{m_n} \left( \prod_{i=1}^{n} \mu_{A_i}^{l_i}(x_i) \right)} \quad (11)$$

From (8) to (6), the equation is as following:

$$x^{(n)} = -bu^* + y_m^{(n)} + \mathbf{K}^T \mathbf{e} + bu \\ e^{(n)} = -\mathbf{K}^T \mathbf{e} + b[u^* - u_D(x/\theta)] \quad (12)$$

We define $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -k_2 & -k_1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ b \end{bmatrix}$. So the close-loop dynamic equation can be written in vector form.

$$\dot{\mathbf{e}} = \mathbf{Ae} + \mathbf{b}[u^* - u_D(x/\theta)] \quad (13)$$

Defining optimal parameter is as following

$$\theta^* = \arg \min_{\theta \in R^{\prod_{i=1}^{n} m_i}} \left[ \sup_{x \in R^n} | u_D(x/\theta) - u^* | \right] \quad (14)$$

Defining minimum approach error is as following

$$w = u_D(x/\theta^*) - u^* \quad (15)$$

The close-loop dynamic equation can be written as following:

$$\dot{e} = \mathbf{Ae} + \mathbf{b}(u_D(x/\theta) - u_D(x/\theta)) - \mathbf{b}(u_D(x/\theta) - u^*) \\ = \mathbf{Ae} + \mathbf{b}(\theta^* - \theta)^T \xi(x) - \mathbf{b}w \quad (16)$$

We define the many equations, but we don't know whether the system is stable. So we use the Lyapunov theory to decide the system stable. First, Lyapunov equation is defined as following:

$$\mathrm{V} = \frac{1}{2}\mathbf{e}^T \mathbf{Pe} + \frac{\mathbf{b}}{2r}(\boldsymbol{\theta}^* - \boldsymbol{\theta})^T (\boldsymbol{\theta}^* - \boldsymbol{\theta}) \quad (17)$$

where $r$ is a positive constant and $\mathbf{P}$ is a positive definite matrix. But $\mathbf{P}$ needs to satisfy that $\mathbf{A}^T\mathbf{P} + \mathbf{PA} = -\mathbf{Q}$, where $\mathbf{Q}$ is a arbitrarily positive definite matrix. Second, the differential operator is used in Lyapunov equation. That is

$$\dot{\mathrm{V}} = \frac{1}{2}\dot{\mathbf{e}}^T \mathbf{Pe} + \frac{1}{2}\mathbf{e}^T \mathbf{P}\dot{\mathbf{e}} - \frac{\mathbf{b}}{r}(\boldsymbol{\theta}^* - \boldsymbol{\theta})^T \dot{\boldsymbol{\theta}} \quad (18) \\ = -\frac{1}{2}\mathbf{e}^T \mathbf{Qe} + \mathbf{e}^T \mathbf{Pb}\left[(\boldsymbol{\theta}^* - \boldsymbol{\theta})^T \xi(x) - w\right] - \frac{\mathbf{b}}{r}(\boldsymbol{\theta}^* - \boldsymbol{\theta})^T \dot{\boldsymbol{\theta}}$$

We choose last one row of $\mathbf{P}$ matrix is $\mathbf{p}_n$, so we can know $\mathbf{e}^T \mathbf{Pb} = \mathbf{e}^T \mathbf{p}_n \mathbf{b}$. The differential Lyapunov equation can be rewritten as following:

$$\dot{\mathrm{V}} = -\frac{1}{2}\mathbf{e}^T \mathbf{Qe} + \frac{\mathbf{b}}{r}(\boldsymbol{\theta}^* - \boldsymbol{\theta})^T [r\mathbf{e}^T \mathbf{p}_n \xi(\mathrm{x}) - \dot{\boldsymbol{\theta}}] - \mathbf{e}^T \mathbf{p}_n \mathbf{b} w \quad (19)$$

In order to $\dot{V} < 0$, we adopt the adaptive law $\dot{\boldsymbol{\theta}} = r\mathbf{e}^T\mathbf{p}_n\boldsymbol{\xi}(x)$. The differential Lyapunov equation can be rewritten as following:

$$\dot{V} = -\frac{1}{2}\mathbf{e}^T\mathbf{Q}\mathbf{e} - \mathbf{e}^T\mathbf{p}_n\mathbf{b}w < 0 \qquad (20)$$

where $\mathbf{Q} > 0$, we can design many rules by fuzzy system to let $w$ smaller. Base on $\left|\mathbf{e}^T\mathbf{p}_n\mathbf{b}w\right| < \frac{1}{2}\mathbf{e}^T\mathbf{Q}$, The differential Lyapunov equation less than zero.

## 4 SIMULANTION RESULTS

In this section, direct adaptive fuzzy controller is presented to verify the smaller tracking error of proposed control methods. The X-Y table system parameters are assumed in Table 1.

TABLE1 X-Y STAGE MODEL PARAMETERS

| X-Y stage model | Model parameters |
|---|---|
| X-axis model | $K_{tx} = 0.96 Nm/A$<br>$J_x = 2.9\times10^{-3} N m\sec^2$<br>$B_x = 0.1003 N m\sec/rad$ |
| Y-axis model | $K_{ty} = 0.96 Nm/A$<br>$J_y = 2.79\times10^{-3} N m\sec^2$<br>$B_y = 0.1015 N m\sec/rad$ |
| Stribeck Friction | $F_{ci} = 0.15N$, $F_{si} = 0.09N$<br>$V_{si} = 10 m/s$, $\alpha = 0.02$ |

The membership functions are selected as follows: $\mu_{N3}(x_i) = 1/(1+\exp(5(x_i+2)))$, $\mu_{N2}(x_i) = \exp(-(x_i+1.5)^2)$, $\mu_{N1}(x_i) = \exp(-(x_i+0.5)^2)$, $\mu_{P1}(x_i) = \exp(-(x_i-0.5)^2)$, $\mu_{P2}(x_i) = \exp(-(x_i-1.5)^2)$, and $\mu_{P3}(x_i) = 1/(1+\exp(-5(x_i-2)))$, spanning in the interval $[-3,3]$. Let the initial state $x(0) = [1 \quad 0]$, and all initial value of $\boldsymbol{\theta}(0)$ is set to zero vector. The parameters are chosen as $k_1 = 1$, $k_2 = 10$, $r = 50$ and $\mathbf{Q} = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}$.

The circle contour is utilized in our simulation. The math model can be described as following:

$$X_i = R\cos(\varphi_i)$$
$$Y_i = R\sin(\varphi_i)$$
$$\varphi_i = \varphi_{i-1} + \Delta\varphi$$

where $R$ is radius of the circle, $\Delta\varphi$ is the variable value of angle, $X_i$ is the motion command of the X-axis, $Y_i$ is the motion command of the Y-axis. The reference signal for X-axis is assumed as $X_i(t) = 1*\sin(t)$, Y-axis is assumed as $Y_i(t) = 1*\sin(t)$, and the external disturbance is set to 10 in our simulation.

The simulation results of the adaptive fuzzy control method are obtained to confirm the tracking characteristics. Two indexes are employed to demonstrate the performances. They are the average tracking error and tracking error standard deviation. They are defined as:

(1) Average tracking error:

$$m = \sum_{k=1}^{n} \frac{E(k)}{n} \qquad (21.1)$$

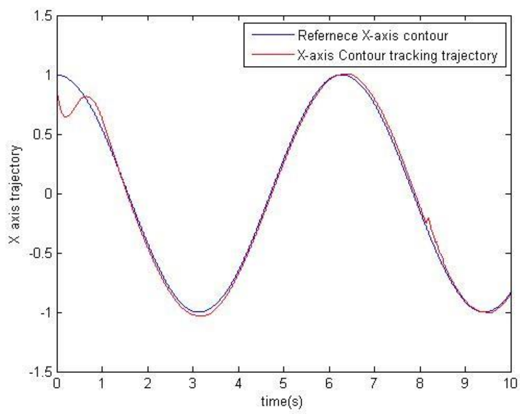
with $E(k) = \sqrt{e_x^2(k) + e_y^2(k)}$, $i = x, y$

where $e_x(k)$ is the tracking error in X-axis, $e_y(k)$ is the tracking error in Y-axis, $n$ is the total simulation points.

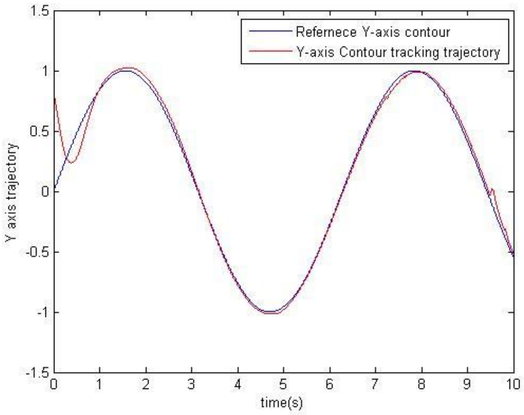
(2) Tracking error standard deviation:

$$E_s = \sqrt{\frac{\sum_{k=1}^{n}(E(k)-m)^2}{n}} \qquad (21.2)$$

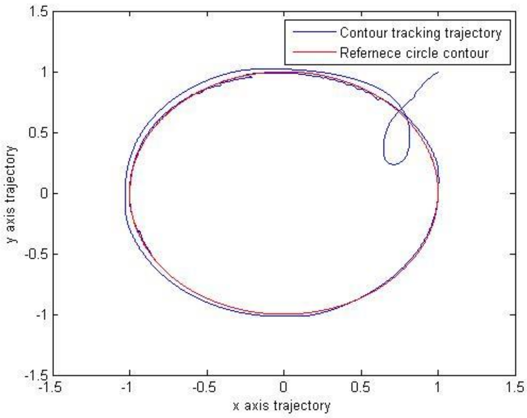
Figure 1(a) shows the X-axis tracking response. It can be observed that the displacement error can be reduced significantly at 1 second. On average, the tracking error of X-axis is 0.0488, and the standard deviation of error is 0.0647. The Figure 1(b) shows the Y-axis tracking response, and its plant model is different from X-axis one. It can be seen that the tracking error can be substantially improved and the error is diminished at 2 second. On average, the tracking error of Y-axis is 0.0530, and the standard deviation of error is 0.1347.Figure 1(c) shows the contour trajectory of X-Y stage. In the beginning, trajectory route has deviated from the path of the reference. After two seconds, the adaptive fuzzy control architecture can accomplish the better tracking performance and trajectory error converges to zero at steady state. On average, the mean trajectory error is 0.079 and the standard deviation of trajectory error is 0.119.




(a)




(b)




(c)

Figure 1. Simulation results of computed-torque controller due to circle contour. (a) contour tracking trajectory (b) trajectories of X and Y axes.

## 5 CONCLUSIONS

The precision motion X-Y stage and adaptive fuzzy controller have been designed in this paper. The nonlinear friction effect and external disturbance behavior are including in the system model. Based on the Lyapunov theorem, adaptive laws of fuzzy coefficients can be obtained and utilized for contour tracking application. The simulation results show that the proposed adaptive fuzzy controller can performs well in control the X-Y stage system.

## 6 REFERENCES

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